

UNIT

3

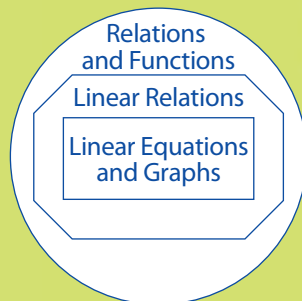
Relations and Functions

Canada requires all packaged foods to display the nutrition facts in both English and French. How might you use the information found on these labels? How can you determine the number of grams of protein in a meal if you double or triple the portion size?

Analysing the graph of a linear relation can provide information about the relationship and the particular situation. For example, consider a graph of the linear relation between the number of potatoes and the total mass of potatoes. The number of potatoes plotted would not increase to infinity. It would be equal to the maximum number that can fit on the truck. In this unit, you will learn how to interpret graphs of real-life situations. You will use several methods of graphing linear relations. You will also determine the equation of a line in different forms.

Your Relations and Functions Organizer

You can use this organizer to see how the concepts in this unit are connected. You will see this organizer on the first page of each chapter. The concepts covered in the chapter will be highlighted.





Nutrition Facts	
Valeur nutritive	
Per 50 g (28 chips) / par 50 g (28 croustilles)	
Amount	% Daily Value
Teneur	% valeur quotidienne
Calories / Calories	250
Fat / Lipides	11 g
Saturated / saturés	1 g
+ Trans / trans	0 g
Cholesterol / Cholestérol	0 mg
Sodium / Sodium	80 mg
Carbohydrate / Glucides	33 g
Fibre / Fibres	3 g
Sugars / Sucres	0 g
Protein / Protéines	4 g
Vitamin A / Vitamine A	0 %
Vitamin C / Vitamine C	0 %
Calcium / Calcium	4 %
Iron / Fer	6 %

Looking Ahead

In this unit, you will solve problems involving ...

- graphs of linear relations
- domain and range values for a situation
- function notation
- slope and rate of change
- linear equations written in various forms

Unit 3 Project

Forensic Discovery

Gold was discovered in 1896 at Rabbit Creek (later named Bonanza Creek), YT. This discovery started the Klondike Gold Rush. Thousands of amateur gold seekers from the south raced to the Klondike River, near Dawson City. There were many routes to the gold fields. Most people had to travel over the difficult and dangerous White Pass Trail or the Chilkoot Trail. Only about 30 000 people completed the journey. Their spirit of adventure and determination contributed to the social and economic development of western Canada. The Klondike Gold Rush ended as quickly as it started. Many people left in 1899 when rumours spread of a gold strike at Nome, Alaska.

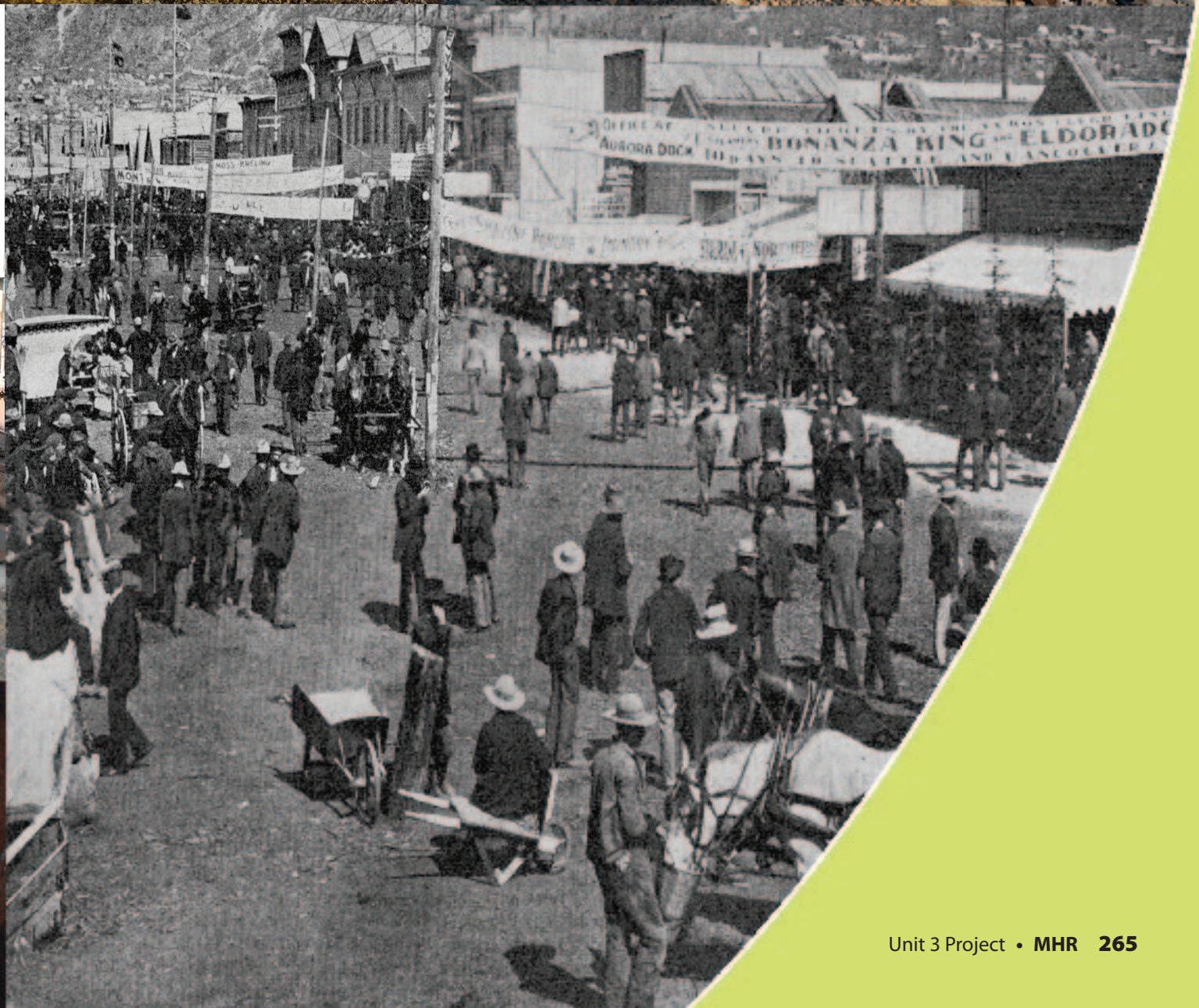
In the Unit 3 project, you will act as a modern forensic archaeologist. Your first case is to solve the mystery of what happened to three gold seekers and possibly find their gold treasure.

Unit Project questions and activities are included in Chapters 6 and 7. As you move through Chapter 6, you will learn about and use functions that contribute to forensic and related scientific inquiry. As you move through Chapter 7, you will collect and graph data and develop equations related to forensic science.

While completing your project, you will ...

- collect data and use functions and graphs to relate a person's height to the length of the humerus bone (Chapters 6 and 7)
- use graphs and functions related to radiocarbon decay to determine the age of fossils (Chapter 6)
- develop an equation relating temperature and altitude and use it to determine specific values (Chapter 7)





CHAPTER

6

Linear Relations and Functions

Our world is becoming increasingly complex. Large amounts of information, on virtually every topic imaginable, are readily available at our fingertips. The ability to interpret and analyse data has never been more necessary. There are many interesting relationships that you can explore. Have you ever wondered, “What would happen if ... ?” or “How does changing this affect ... ?” This is where the study of relations begins.

Big Ideas

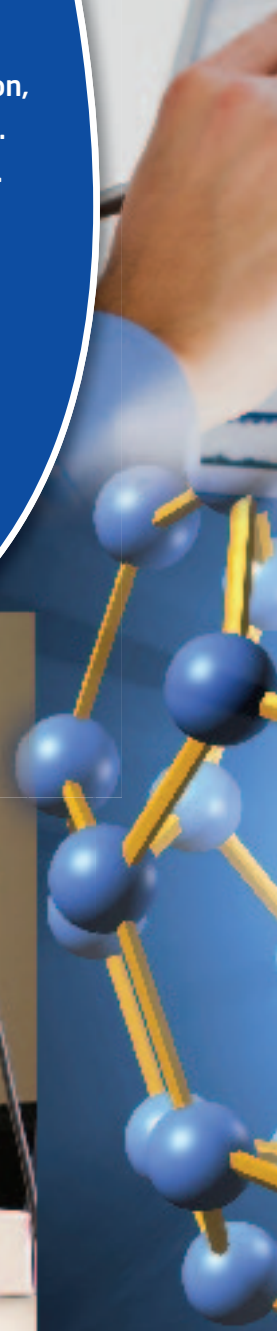
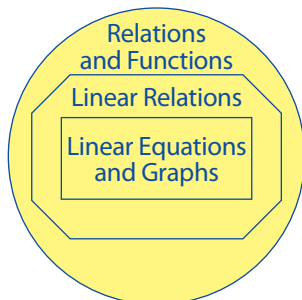
When you have completed this chapter, you will be able to ...

- create graphs that represent different situations and interpret graphs
- apply the various characteristics of linear relations to graphing
- determine an acceptable range of values for a situation
- work with function notation in a variety of ways
- work with slopes and solve problems involving rates of change

Key Terms

relation
linear relation
non-linear relation
discrete data
continuous data
independent variable
dependent variable
domain
range
function
function notation
vertical line test
slope

Your Relations and Functions Organizer



Forensic Scientist

The field of forensic science has been popularized on television and in movies and books. Forensic scientists play an important role in our society. They process and interpret physical evidence for the justice system. Forensic scientists can specialize in a variety of areas. These include accounting, anthropology, biology, chemistry, geology, medicine, and dentistry.

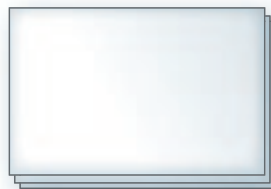
Web Link

To learn more about forensic scientists, go to www.mhrmath10.ca and follow the links.

FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 6.

- 1 Take three sheets of 11 × 17 paper.



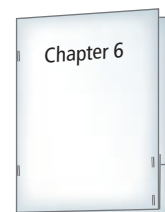
- 2 Make a pocket on each by folding the long bottom edge up 6 cm.



- 3 Stack all three sheets and then fold them in half to create a booklet.

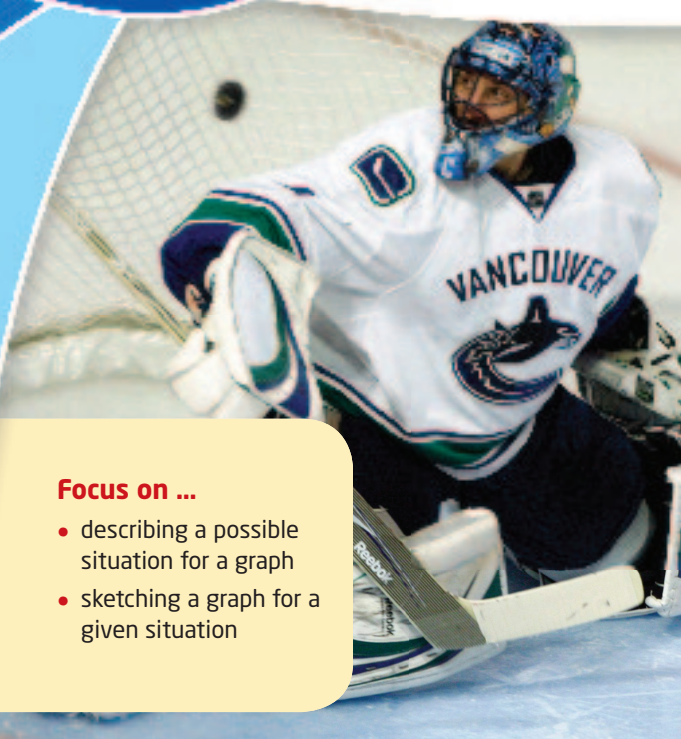


- 4 To complete the booklet, staple the spine and staple the sides of each pocket.



6.1

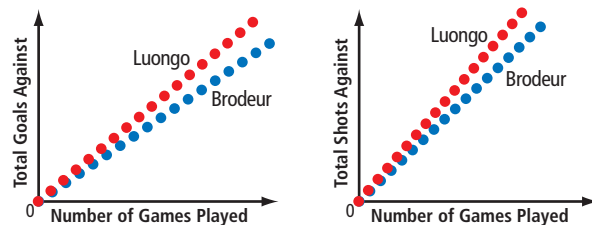
Graphs of Relations



Focus on ...

- describing a possible situation for a graph
- sketching a graph for a given situation

Graphs are often used to visually represent the relationship between two or more things. The graphs shown are derived from the career statistics for two NHL goalies, Martin Brodeur and Roberto Luongo. One graph shows the number of goals scored against each goalie. The other shows the number of shots each faced. If you compare the graphs, what conclusions might you make?



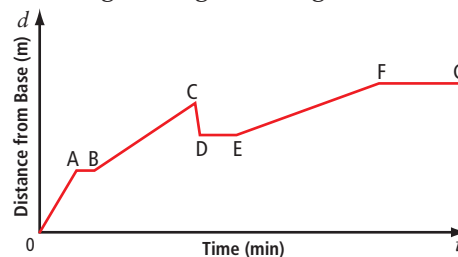
Materials

- ruler
- grid paper



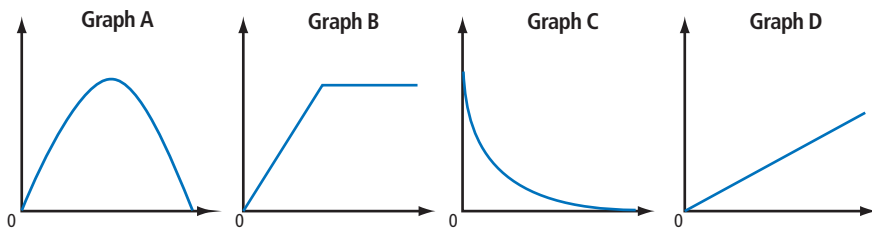
Investigate Describing and Sketching Graphs

1. a) Work in pairs. The graph shows the distance a rock climber is from the base of a cliff as time passes. Using the words *climbing*, *resting*, or *descending*, describe what the climber is doing during each segment shown. Explain your choice.



- b) Is there more than one interpretation of the climber's actions during the times indicated by segments AB, CD, DE, and FG?
- c) For any section that you listed as "climbing," how would you change the graph to show that the person is climbing faster? Explain your reasoning.
- d) What would you add to the graph to show the climber's return to the bottom of the cliff?

2. Work in pairs. Match each graph with a situation from the list. Explain your choice. Suggest titles for each axis to show the quantities being compared.



- a) the temperature of a cup of hot chocolate over time
 b) a car accelerating to a constant speed
 c) the distance a person walks during a hike
 d) the height of a soccer ball kicked across a field
3. Work in small groups. Create a speed-time graph for the following scenario. Put speed on the vertical axis and time on the horizontal axis. Clearly describe each section of your graph. Then, pass your work to another group.
- Connor is riding his skateboard along a path. Almost immediately after leaving home, Connor travels down a short steep hill. At the bottom, the path makes a turn. The remainder of the trip is on relatively flat land. Connor kicks to keep moving. He then stops before a railway crossing. He also practises a few tricks along the way. He completes a basic “ollie” and performs a second ollie over a speed bump. Finally, after travelling at a constant rate for the last part of the trip, Connor arrives at his destination.
4. a) Review and discuss another group’s graph.
 b) How is it similar to yours? How is it different from yours?
5. **Reflect and Respond** How might each situation be shown on a graph?
- a) one quantity is changing at a constant rate in relation to the other quantity
 b) the rate of change is constant and the change is happening quickly
 c) one quantity is not changing
 d) a change in one quantity is not constant

WWW Web Link

To conduct an interactive investigation of distance-time graphs, go to www.mhrmath10.ca and follow the links.

Did You Know?

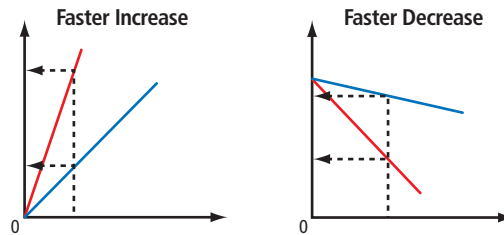
An ollie is a jump that is performed by tapping the tail of the skateboard to the ground. It was invented by Alan “Ollie” Gelfand in 1978. Many tricks start with, or incorporate, an ollie.



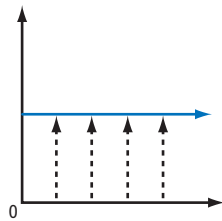
Link the Ideas

A graph is an effective way to show the relationship between two quantities. A constant rate of change is represented graphically by a straight line. The steepness of the line indicates the rate at which one quantity is changing in relation to the other.

A steeper line indicates a faster rate of vertical change on the red line than on the blue line. This change may indicate an increase or a decrease.

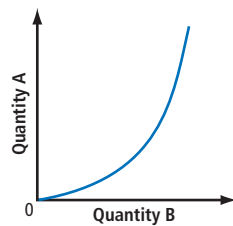


A horizontal line means that there is no rate of change.

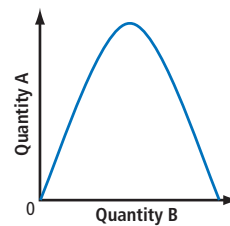


Every value on the horizontal axis is related to the same value on the vertical axis.

Not all relationships are represented by a straight line. A curve shows that the rate of change is not constant.



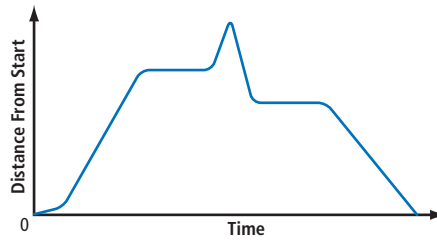
As quantity B increases, the increase in quantity A is gradual at first. It then becomes much greater.



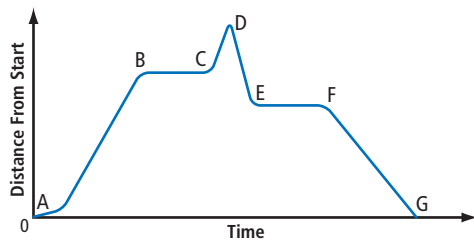
As quantity B increases, the increase in quantity A slows until quantity A reaches a maximum value. Then, quantity A decreases.

Example 1 Interpret a Graph

Wakeboarding has grown to be a popular water sport. The graph shows the distance that a wakeboarder is from her starting point on Last Mountain Lake in Saskatchewan. Describe what the boarder is doing.



Solution



AB: Since the distance is increasing, the wakeboard rider is moving away from her starting point. The change in distance starts slowly at first. It then reaches a constant rate.

Why does the distance increase slowly at first?

BC: Since the distance is not changing, the rider has either stopped or is on a path that keeps her at a constant distance from the starting point.

What kind of path would allow this?

CD: The change in distance increases so that the wakeboard rider is moving away from her starting point at a quicker rate.

DE: Since the distance is decreasing quickly, the rider is moving toward the starting point at a fast rate.

EF: Since the distance is not changing, the rider has either stopped or is on a path that keeps her at a constant distance from the starting point.

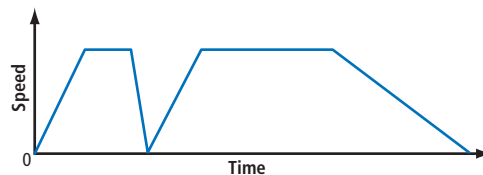
FG: The distance is decreasing to zero. The rider is returning to the starting point at a constant rate.

How do you know the rate is constant?

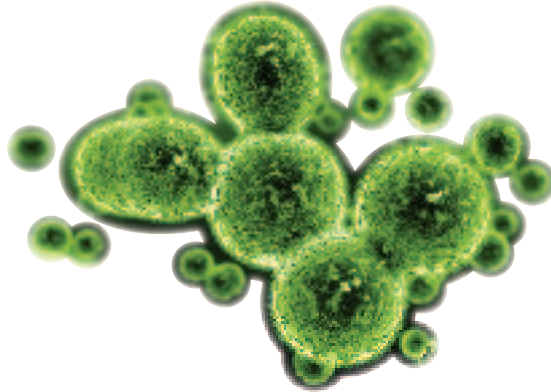


Your Turn

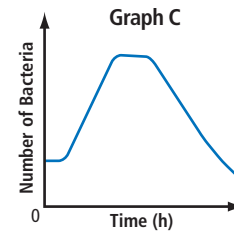
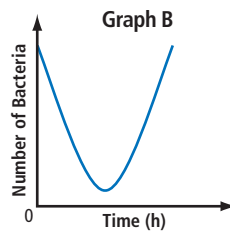
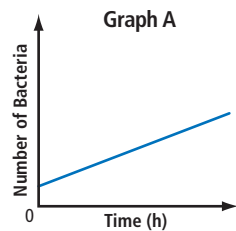
The graph shows the speed of the boat that is pulling a wakeboarder. Describe what the boat is doing.



Example 2 Interpret a Graph



Which graph best represents bacteria growth if the bacteria's food supply is limited? Explain your choice.



Solution

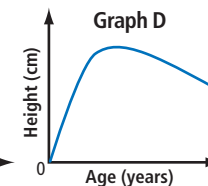
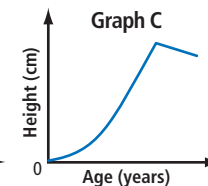
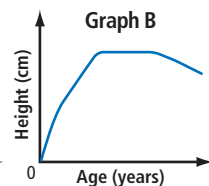
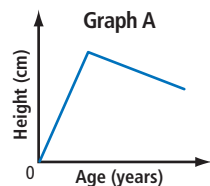
If the food supply is limited, the bacteria eventually will run out of food and die off. Graph A can be ruled out since it indicates continued growth.

Graph B is also not the correct choice. It shows the number of bacteria decreasing at the start while the food supply is high, reaching a low point, and then increasing.

Graph C is the correct choice. The increase in bacteria is initially slow but then goes through a period of rapid growth. The number remains stable for a while. Then the bacteria die off because there is no more food.

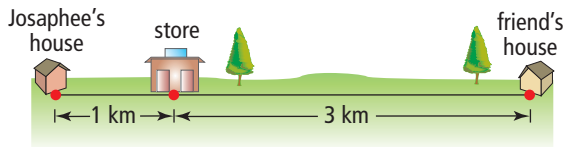
Your Turn

Which graph best represents a person's height as the person ages? Explain your choice.

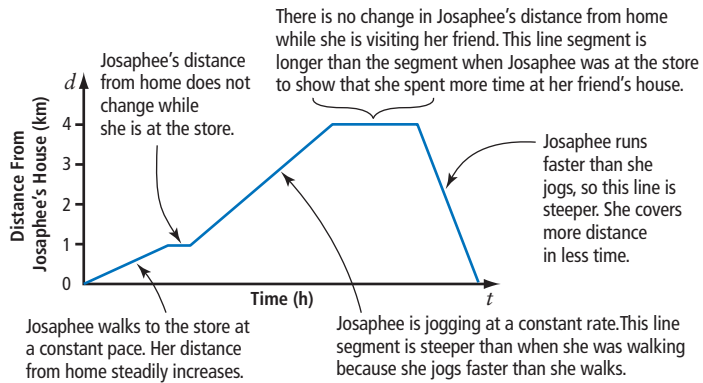


Example 3 Graph a Situation

Josaphee leaves her home and walks to the store. After buying a drink, she slowly jogs to her friend's house. Josaphee visits with her friend for a while and then runs directly home. Using the distances shown, draw a distance-time graph that shows Josaphee's distance from her house. Explain each section of your graph.



Solution



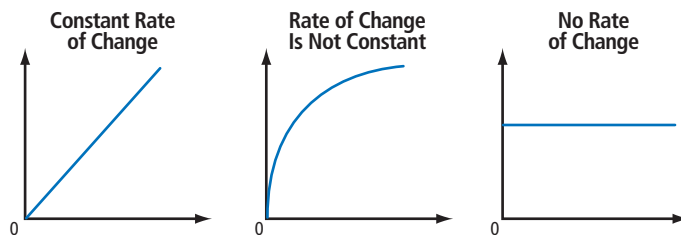
To make a distance-time graph, Josaphee's distance from her home is placed on the vertical axis. Time is placed on the horizontal axis.

Your Turn

For the same scenario and using the distances shown, draw a distance-time graph that shows Josaphee's distance from the store. Explain each section of your graph.

Key Ideas

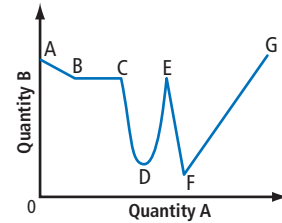
- When comparing two quantities, straight lines are used to indicate a constant change in the relationship. Curves are used when the rate of change is not constant. Horizontal lines are used if one quantity is not changing relative to a change in the other quantity.



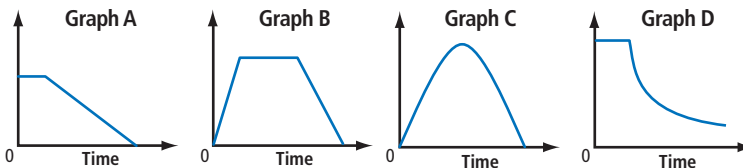
Check Your Understanding

Practise

1. The graph shows how quantity B is changing relative to quantity A. Describe each section of the graph as representing a constant increase, a constant decrease, an increase that is not constant, a decrease that is not constant, or no change. Explain your answers.



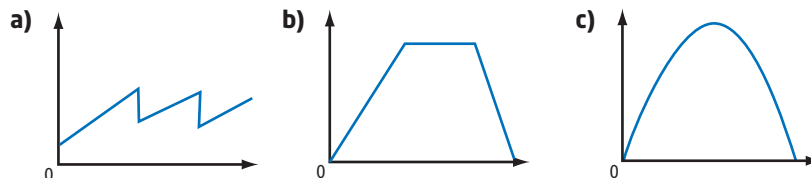
2. a) Match each scenario with its appropriate graph.
- the speed of a train as it arrives at a station
 - a football's distance above ground as the ball is kicked
 - the number of un-popped kernels as a popcorn maker heats up and pops the corn



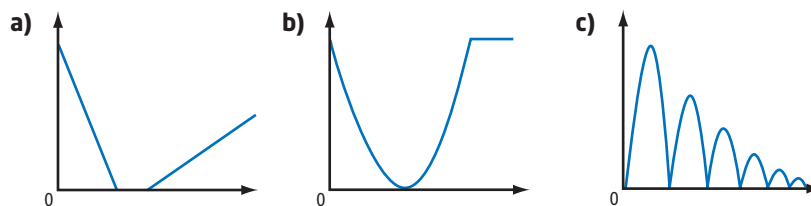
- b) Describe a scenario for the graph that you do not use in part a).

3. Sketch a copy of each graph. Label the axes using the choices given.

Choices for Vertical Axis	Choices for Horizontal Axis
Profit From Sales	Time
Speed of ATV	Ticket Price
Height of Grass	Distance Travelled



4. Describe a possible scenario for each graph. Tell what each axis represents in each case.



Apply

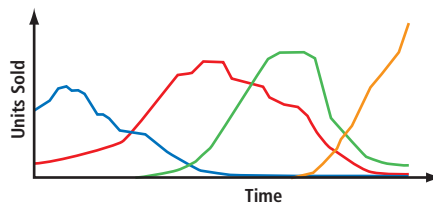
5. Paul boards Vancouver's SkyTrain at the Main Street station. He travels east on the Millennium line toward the Columbia station. He falls asleep around the 29th Avenue station and does not wake up until the Rupert station. Paul decides to stay on the train until Commercial Drive, where he transfers to another eastbound train. He takes this train to the Columbia station. Make a distance-time graph for Paul's journey. Sketch his distance from the Columbia station versus time.



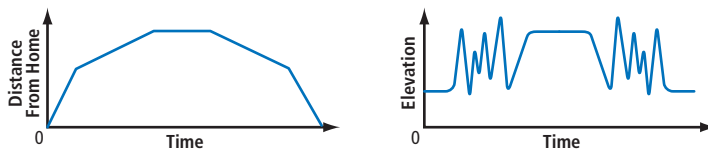
Did You Know?

The first SkyTrain line, the Expo Line, was built in time for the Expo 86 World's Fair. It began operation in December 1985. The trains are made by a Canadian company, Bombardier.

6. Formats for distributing recorded music have changed through the years. Study the multi-line graph. Predict which line represents each format: vinyl albums, cassette tapes, compact discs, and digital downloads. Explain your choices.



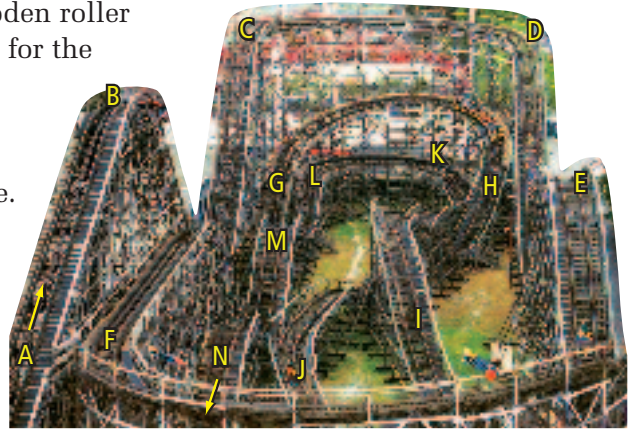
7. Uriash enjoys snowmobiling. The two graphs give information about one of his rides. Use them to describe what Uriash did.



8. The table gives the approximate amount of water needed for various activities. Sketch a graph showing your water usage from the moment you wake up until you go to bed. On the vertical axis, record the amount of water that you use. Record time on the horizontal axis. Include a description.

Water Use	Amount
Toilet flush	6 L
Shower	10 L/min
Bath	68 L
Sink faucet	10 L/min
Dishwasher	27 L/load
Washing machine	99 L/load

9. The Coaster is a wooden roller coaster built in 1958 for the Pacific National Exhibition in Vancouver. Imagine yourself taking a ride. Follow all the ups, downs, twists, and turns. The letters on the track will help you follow the Coaster's path.

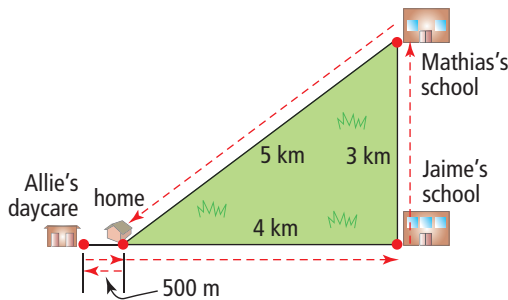


WWW Web Link

To view a video that shows how the Coaster was built or to view a video of a ride on the roller coaster, go to www.mhrmath10.ca and follow the links.

- a) Sketch a height-time graph showing your height above the ground versus time for one complete ride.
- b) Sketch a speed-time graph showing your speed versus time, for one complete ride.

10. After arriving home from work, Cari leaves to pick up her daughter Allie from daycare. She walks to the daycare and then walks home with her daughter. One hour later, Cari and Allie leave home by car to pick up Cari's other children. They go first to Jaime's school and then to Mathias's school. The diagram shows the distances to each location and Cari's route, marked in red.



Sketch a distance-time graph of each scenario. Starting from the moment Cari leaves for the daycare, show

- a) Cari's distance from her home
- b) Allie's distance from home
- c) Jaime's distance from home
- d) Mathias's distance from home

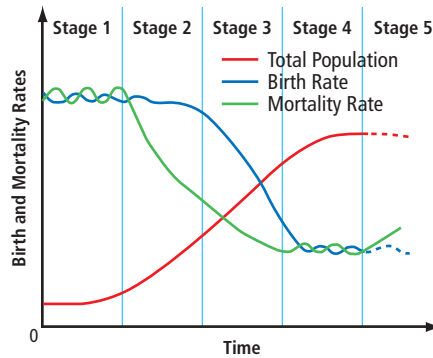
11. Create a speed-time graph for this scenario.

A skydiver jumps from an airplane that is flying at a speed of 160 km/h. In about the first 10 s, the skydiver accelerates to a falling speed of 190 km/h. He stays at this speed because he has adopted the standard flat and stable, or “face to Earth,” position. After another 30 s, the skydiver opens his parachute and quickly slows his descent to about 18 km/h. He maintains this speed until just before reaching the ground. Then he uses his parachute to slow down slightly, allowing him to make a smooth landing.

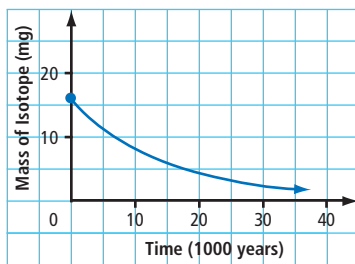


Extend

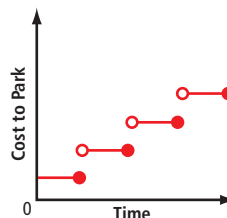
12. Demography is the study of human populations. The graph shows changes in birth and mortality rates over time and their effects on the total population. Study the birth and mortality rates. Describe and explain the changes in the total population at each of the five stages marked.



13. a) Half-life is the time required for half of a sample of a radioactive substance to decay. The graph shows a typical decay curve for an isotope. What is the half-life of this radioactive substance?

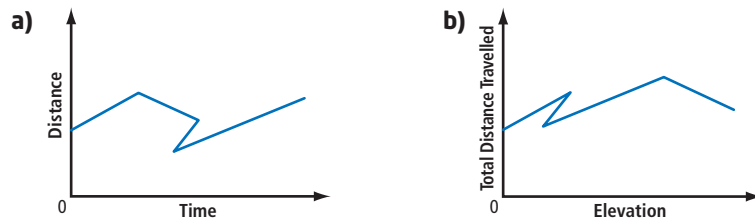


- b) Bismuth-210 has a half-life of 5 days. Make a graph showing a decay curve for this substance. Show the first 20 days of decay.
14. The graph shows fees to park a vehicle in a public parking lot in Calgary. Describe the rate scheme.



Create Connections

15. Explain why each graph represents an impossible situation.



16. Write a story less than one page in length that incorporates four general graphs: speed versus distance, temperature versus time, number of people versus time, and money versus number of people.

Materials

- CBL interface with a motion detector
- computer or graphing calculator with appropriate software

17. **MINI LAB** Use a motion detector connected to a computer or graphing calculator to collect and graph information about movement in your classroom.

Step 1 Load the data-collection program onto your computer or calculator.

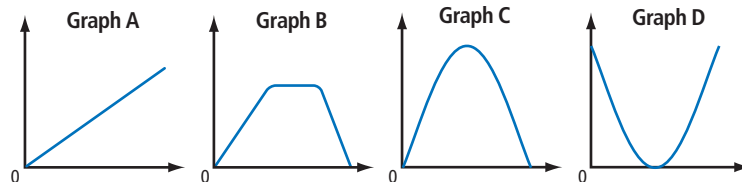
Step 2 Connect the CBL interface to the calculator or computer.

Step 3 Connect the motion detector to any of the sonic ports on the CBL interface.

Step 4 Attach the motion detector to a desk or table. Make sure there is a clear path in front of the detector. The device emits ultrasonic waves that fill a cone-shaped area about 15° to 20° off the centre line of the detector. Keep objects such as desks or chairs out of the cone, because the waves may detect them and record incorrect data.

Step 5 Start the data-collection program.

Step 6 The motion detector measures the time it takes for ultrasonic waves to travel to an object and return to the sensor. As a class, take turns walking in front of the sensor in a manner that produces each of the following graphs.



Step 7 Have a classmate walk in front of the sensor in any way he or she likes. Observe the motion carefully. Create a distance-time graph for the motion. Compare your graph to the one produced on the calculator or computer.

Step 8 Repeat step 7 for other members of the class.

6.2

Linear Relations

Focus on ...

- determining if a relation is linear
- representing linear relations in a variety of ways
- explaining why data points should or should not be connected
- identifying the dependent and independent variables in a relation

Materials

- measuring tape
- ruler
- grid paper or graphing technology

relation

- an association between two quantities
- can be presented in words, as an equation, as ordered pairs, as a table of values, or as a graph

Knowing the relationship between two quantities can be very useful. For example, it benefits a business to know how the number of units it sells is related to the product's price. It is important for a civil engineer to know the relationship between structural design and strength. Medical doctors are interested in the relationship between a patient's body mass index and heart health.

People have long studied relationships. The famous artist and scientist Leonardo da Vinci (1452–1519) studied the proportions in the human body. Why do you think this information is important to an artist?



Leonardo da Vinci

Investigate Relationships in the Human Body

1. Work with a partner.
 - a) Measure and record each other's height, in centimetres.
 - b) Measure and record the length, in centimetres, of each other's foot.
2. How many of your feet does it take to equal your height? How about your partner's?
3.
 - a) As a class, share your results and determine the mean. Using this mean, predict a relationship between a person's height and foot length. Show your answer to the nearest whole foot.
 - b) Write this relationship as an equation. This is your model **relation**.
4. Estimate the shortest length and longest length of the feet of high-school students. Use these estimates to complete a table of values. Use increments of 1 cm for foot length. Determine the corresponding height using your equation from step 3b).

5. Look at the height values in your completed table. Are they appropriate to represent the heights of high-school students? If not, make adjustments to your foot-length estimates.
6.
 - a) Plot your results from step 4 to see a graph of this relationship. Record foot length on the horizontal axis and height on the vertical axis.
 - b) Should you draw a line through the points on the graph? Why or why not?
 - c) Is the resulting graph a straight line or a curve? Explain your answer.
 - d) Determine the difference between each height value in your table. What do you notice?
7. **Reflect and Respond**
 - a) What are different ways to present a relationship?
 - b) If a relation is presented as a table of values, how do you determine if the relation is linear without creating a graph? Is the relationship between a person's height and shoe size linear? Explain your answer.
 - c) Should a graph of this relation have the data points connected? Explain your answer.

Link the Ideas

Relations

A relation can be presented in a variety of ways. For example,

Words

Three times the length of your ear, e , is equal to the length of your face, f , (from chin to hairline).

Equation

$$f = 3e$$

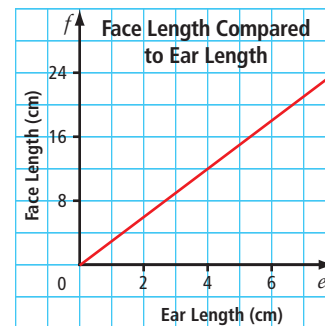
Ordered Pairs

(4, 12), (4.5, 13.5),
(5, 15), (5.5, 16.5),
(6, 18), (6.5, 19.5)

Table of Values

Ear Length, e (cm)	Face Length, f (cm)
4	12
4.5	13.5
5	15
5.5	16.5
6	18
6.5	19.5

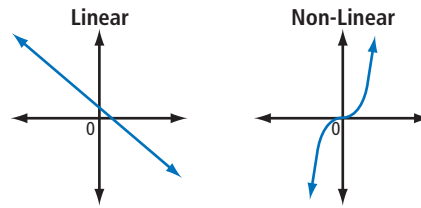
Graph



Linear and Non-Linear Relations

There are a number of ways to determine whether a relation is a **linear relation** or a **non-linear relation**.

Linear relations have graphs that are straight lines.



You can determine whether a relation is linear or non-linear from a table of values. In linear relations, values of y increase or decrease by a constant amount as values of x increase or decrease by a constant amount. Horizontal and vertical lines are exceptions.

Linear Relation		Non-Linear Relation	
x	y	x	y
2	8	2	8
3	11	3	27
4	14	4	64
5	17	5	125

For the Linear Relation table, the change in x is +1 and the change in y is +3 for each row. For the Non-Linear Relation table, the change in x is +1 and the change in y is +19, +37, and +61 for each row.

When a linear relation is written as an equation, it will contain one or two variables and its degree will be 1.

Linear Relations

$$x = 7$$

$$3m + 2n = -12$$

$$y = -\frac{2}{3}x + 5$$

Non-Linear Relations

$$2x + y^2 = 6$$

$$h = k^3$$

$$xy = 3$$

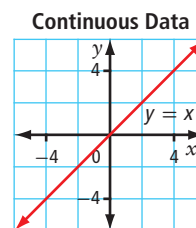
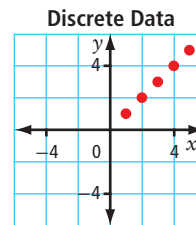
For an expression to have degree 1, what must be the maximum sum of the exponents of the variables for any term?

Discrete or Continuous Data

A graph of **discrete data** can only show points because the values in between them have no meaning. A graph of **continuous data** is a solid line or curve.

For example, a relation is defined by the set of ordered pairs $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$. There are only five data points in the relation. These are discrete data. The graph has five unconnected points.

For the relation defined by the equation $y = x$, there are an infinite number of possible ordered pairs. The points $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$, and $(5, 5)$ satisfy this relation. So do many other points such as $(\frac{3}{2}, \frac{3}{2})$ and $(-3.6, -3.6)$. These represent continuous data. On a graph, you show an infinite set with an unbroken, or continuous, line.



linear relation

- a relation that forms a straight line when the data are plotted on a graph

non-linear relation

- a relation that does not form a straight line when the data are plotted on a graph

discrete data

- data values on a graph that are not connected

continuous data

- data values on a graph that are connected

independent variable

- the variable for which values are selected

dependent variable

- the variable whose values depend on those of the independent variable

Independent and Dependent Variables

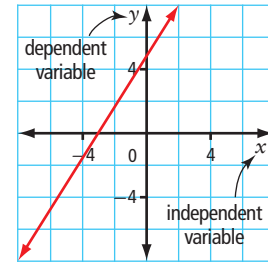
In a relation with two variables, one is the **independent variable** and the other is the **dependent variable**.

When a relation is expressed as a table of values, the values of the independent variable are listed in the first column. The values of the dependent variable are listed in the second column.

x	$y = 3x + 5$
-1	2
0	5
1	8
2	11

↗ Choices for independent variable ↖ Corresponding values of dependent variable

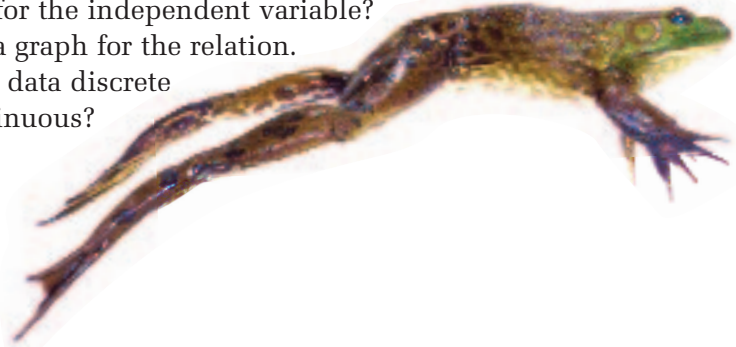
When a relation is expressed as a graph, the values of the dependent variable are plotted along the vertical axis. The values of the independent variable are plotted along the horizontal axis.



Example 1 Describe a Relation in a Variety of Ways

The Canadian National Frog Jumping Championship is part of Les Folies Grenouilles. This annual festival is in St-Pierre-Jolys, MB. The first champion, a frog named Georges, jumped a distance of just over 2 m in a single leap. Assume that Georges could maintain a distance of 2 m on every jump and that the total distance travelled from the start is measured after every jump. Consider the relationship between the number of jumps Georges takes and the total distance the frog travels.

- Identify the relationship as linear or a non-linear. Explain how you know.
- Create a variable to represent each quantity in the relation. Which is the dependent variable? Which is the independent variable?
- Create a table of values for this relation. What are appropriate values for the independent variable?
- Create a graph for the relation. Are the data discrete or continuous?



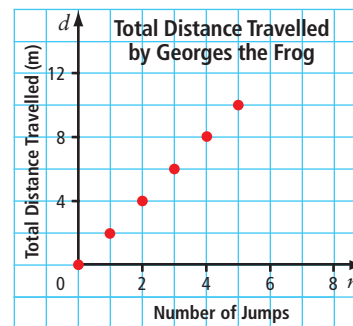
Solution

- a) Since the distance that Georges covers on each leap is the same, the relation is linear.
- b) The total distance travelled depends on how many jumps the frog takes. Let n represent the independent variable, the number of jumps. Let d represent the dependent variable, the distance travelled.
- c) Choose a realistic number of consecutive jumps that Georges might make. For example, the frog could make five jumps.

n	d
0	0
1	2
2	4
3	6
4	8
5	10

Why can the values of n only be whole numbers?

- d) Display the independent variable, n , on the horizontal axis and the dependent variable, d , on the vertical axis. The data are discrete because there are only six possible values in the relation. Georges does not take partial jumps, so values for n such as 1.5 or 2.8 cannot be used.



Your Turn

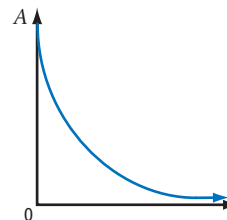
Another popular event at *Les Folies Grenouilles* is the fireworks display. Assume that the event organizers send off 20 firework shells each minute.

- a) Is the relationship between the total number of fireworks and the duration of the event linear or non-linear? Explain how you know.
- b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable?
- c) Create a table of values for this relation. What are appropriate values for the independent variable?
- d) Create a graph for the relation. Are the data discrete or continuous?

Example 2 Determine Whether a Relation Is Linear or Non-linear

Consider each relation. Determine whether the relation is linear. Explain why or why not.

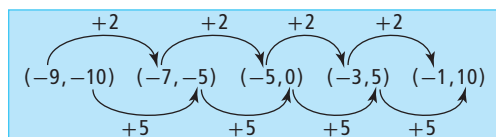
- a) the relation described by $\{\dots, (-9, -10), (-7, -5), (-5, 0), (-3, 5), (-1, 10), \dots\}$
- b) The graph shows the relationship between the amount, A , of a radioactive isotope present and the age of a rock sample over time, t , in years.
- c) the relation described by the equation $m - 17 = 0.8n$



Solution

a) Method 1: Compare Changes in the Independent and Dependent Variables

Check to see if the independent variable increases or decreases at a constant rate and if, at the same time, the dependent variable increases or decreases at a constant rate.



The relation is linear. With each increase of 2 in the independent variable, the dependent variable increases by 5.

Method 2: Use a Table of Values to Compare Changes in Each Variable

	x	y	
+2	-9	-10	+5
+2	-7	-5	+5
+2	-5	0	+5
+2	-3	5	+5
+2	-1	10	+5

The relation is linear. Values of x (the independent variable) increase each time by 2. Values of y (the dependent variable) increase each time by 5.

b) The relation is not linear. The graph is not a straight line.

c) The degree is 1. The relation is linear.

How do you know that the degree is 1?

Your Turn

Determine whether each relation is linear. Explain why or why not.

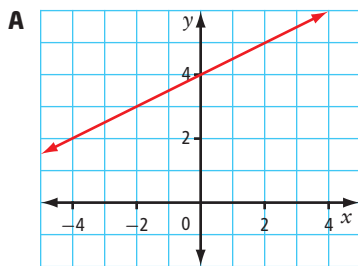
- a) the relationship between the cost to rent a dance hall and the number of people attending the dance, if the hall charges \$200 plus \$5 for each person who attends
- b) the relation described by the equation $x^2 + y^2 = 25$
- c) the relation described by the set of ordered pairs $\{(10, 12), (15, 4), (20, -4), (25, -12), (30, -20)\}$

Example 3 Match Representations of a Linear Relation

Match each linear relation with possible representations in the selections that are given. Justify your choices.

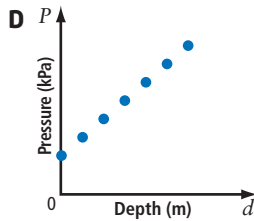
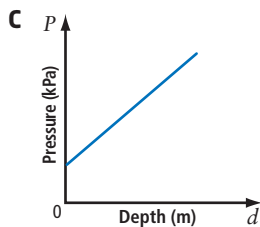
- a) The pressure, P , that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, d , below the surface.

b) $y = \frac{1}{2}x + 4$



B

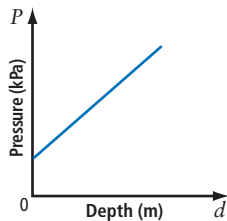
x	y
0	4
1	8
2	12
3	16



- E** One number is half another number increased by four.
F $(0, 101)$, $(25, 176)$, $(50, 251)$, $(75, 326)$, $(100, 401)$, $(125, 476)$

Solution

- a) The pressure, P , that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, d , below the surface. The graph in choice D is a possible representation.



Increases in pressure are constant as depth changes. Therefore, the relation is linear. Also, since the values for the independent variable are not restricted to whole numbers of metres, the data are continuous.



Did You Know?

The kilopascal is an SI unit for measuring pressure. Its abbreviation is kPa. This unit represents a force of 1000 N per square metre. The pascal is named after the French mathematician Blaise Pascal. In the 1600s Pascal experimented with barometers, which are instruments used to measure air pressure.

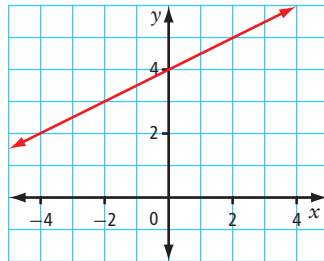


Blaise Pascal

b) $y = \frac{1}{2}x + 4$

The graph in choice A shows this relation.

The representation in choice E also expresses this relation in words.



The equation represents linear data and continuous data.

The points on this graph satisfy the relation.

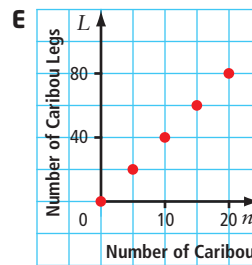
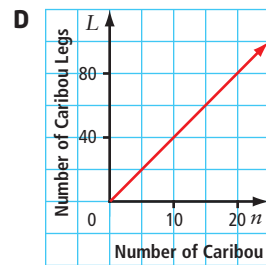
For example, $(-3, 2.5)$, $(-2, 3)$, and $(0, 4)$ are all solutions to the equation.

Your Turn

There is a linear relationship between the number of caribou, n , in a herd and the number of caribou legs, L . Which representations model this relation?



- A $L = 4n$
- B $(0, 0), (3, 12), (8, 32), (15, 60), (50, 200)$
- C $L = n + 4$

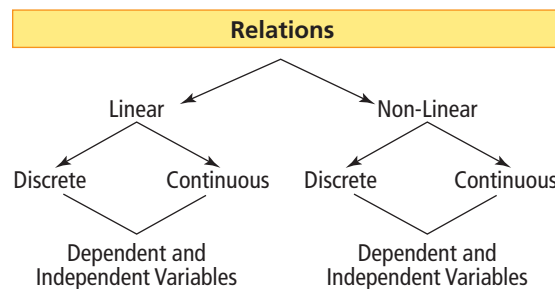


F

n	L
3	6
6	12
9	18
12	24

Key Ideas

- Relations can be represented in a variety of ways. You can use words, equations, tables of values, ordered pairs, or graphs.



These are different characteristics of relations, used to describe them in more detail.

Check Your Understanding

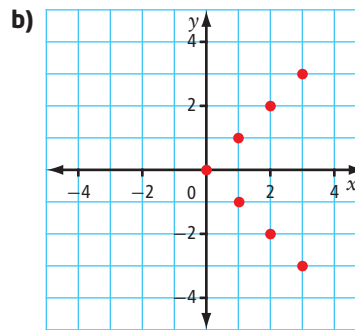
Practise

1. Convert each relation from its current representation to the one suggested.

a)

m	n
-2	5
-1	6
0	7
1	8
2	9
3	10
4	22

to ordered pairs

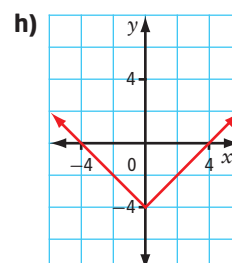
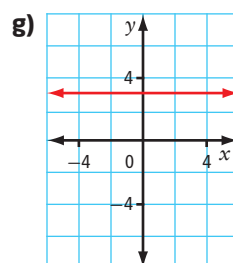
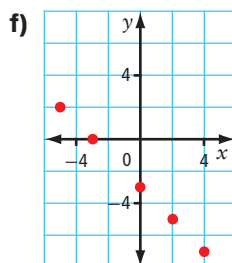


to a table of values

- c) $P = 2d + 5$ to a graph
 d) (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) to words, if the independent variable represents the number of children and the dependent variable represents the number of oranges eaten

2. Determine whether each relation is linear or non-linear. Explain your decision.

- a) $C = 2\pi r$
 b) $A = s^2$
 c) $y = 5x - 3$
 d) (0, 0), (1, 1), (4, 2), (9, 3), (16, 4)
 e) (5, 10), (10, 20), (15, 30), (20, 40), (25, 50)



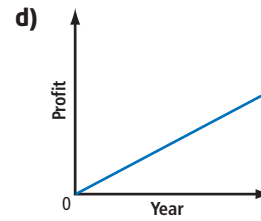
3. For each relation, state the dependent variable and the independent variable.

a) $A = \pi r^2$

b) $V = 7t + 2$

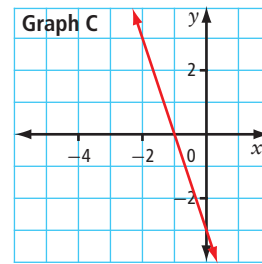
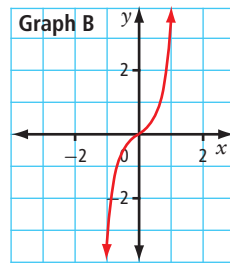
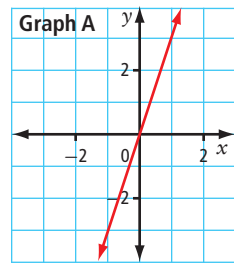
c)

n	A
0	1
1	3
2	9
3	27
4	81



e) Five eggs, e , per chicken, c , are laid each week.

4. Consider the relation described by the equation $y = x^3 + 2x$ and the three graphs shown. Without performing any calculations, predict which graph matches this relation. Check your prediction against a partner's. Discuss your answers.



Did You Know?

A bushel is a unit used for measuring the volume of grain, fruit, vegetables, or other dry things. One bushel is equal to about 0.036 m^3 .

Apply

5. More than half of all the wheat grown in western Canada is grown in Saskatchewan. One year, Saskatchewan farmers received \$6 per bushel for wheat. Consider the relationship between the total amount of money each farmer received and the number of bushels the farmer sold.



- Is this a linear or non-linear relationship? Explain how you know.
- Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable? Explain how you know.
- Assume that Saskatchewan wheat farms each produce up to 50 000 bushels of wheat. Create a table of values for the relationship.
- Are the data discrete or continuous? Explain how you know.
- Graph the relationship.

6. Give an example of a relation with each characteristic listed.
- a table of values that shows a non-linear relationship
 - a graph that shows a non-linear discrete relationship
 - an equation that shows a linear relation with independent variable m and dependent variable n
7. The refraction of light causes an object lying under water to appear closer to the surface than it actually is. The relationship between how deep a coin appears to be in a fountain and the coin's actual depth is given by the equation $D = 0.75d$, where D is the apparent depth and d is the actual depth.
- Is this relationship linear or non-linear? Explain.
 - Identify the dependent variable and the independent variable for this relation.
 - If the water is 2 m deep, how deep does the coin appear to be?
 - Can you use the formula to determine the apparent depth of the coin if the water is 2.8 m deep? Does this indicate that the relation is continuous or discrete? Explain.
8. A killer whale is swimming at a speed of 6 km/h. Consider the relationship between the total distance, in kilometres, travelled by the whale and time, in hours.
- Assign variables to represent each quantity in the relation. Identify the dependent variable and the independent variable.
 - Assume that the whale swims for 5 h without stopping. Create a set of ordered pairs for the relation.
 - Is the relation continuous or discrete? Explain.
 - Graph the relation.
 - Is the relation linear or non-linear? Explain.



Did You Know?

Killer whales have long played a role in the legends and beliefs of coastal First Nations cultures. The Haida, for example, used the killer whale as a symbol for family. They did this because the whales stayed in families, travelling in large pods.



Killer Whale (Sgáan) by Don Yeomans, Haida artist

9. An action plan is in place to clean up a lake that has high levels of mercury. Tests show 9 mg of mercury per 1000 L of water. It is estimated that mercury levels can be reduced each year by 0.8 mg per 1000 L of water.
- Create a table of values to represent the relationship between the amount, A , of mercury present and the number of years, t , since the clean-up plan started. A is the dependent variable and t is the independent variable in the relation.
 - According to this relation, how long will it take to rid the lake of mercury?

10. The Pacific Coast is the most earthquake-prone region in Canada, while Saskatchewan and Manitoba have the least probability of earthquakes.



Seismometer measuring seismic waves.

Did You Know?

A seismometer is an instrument used to measure seismic waves. Information about the waves is then used to determine the magnitude of an earthquake.

- A magnitude scale describes the relative size of earthquakes. An earthquake with a magnitude value that is one higher than another earthquake is about 10 times more intense. For example, an earthquake measuring 7 is about 10 times as great as one with a magnitude of 6. Is the relationship between an earthquake's magnitude and its size linear or non-linear? Explain.
- From northern Vancouver Island to the Queen Charlotte Islands, the oceanic Pacific plate is sliding to the northwest at about 6 cm per year. Starting with this year as 0, create a set of ordered pairs to show the relationship between the number of years from now and the amount of plate sliding for the next 8 years.
- Graph the ordered pairs. Should the points be connected? Should you continue the pattern in both directions? Explain your answers.

Extend

11. Does each table of values represent a linear or a non-linear relation? Explain.

a)

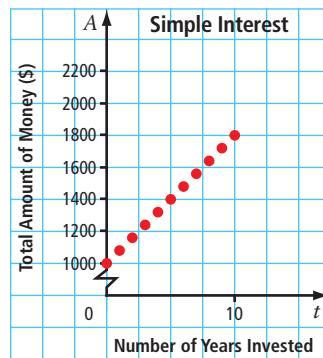
x	y
1	k
3	$2k$
5	$3k$
7	$4k$

b)

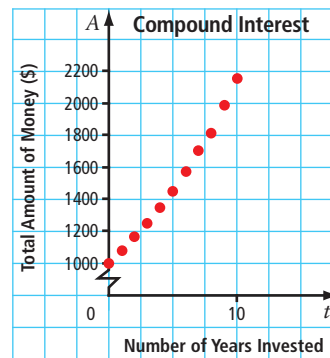
x	y
m	0
$m + 1$	$3n$
$m + 2$	$6n$
$m + 3$	$9n$

12. The graphs represent different investments based on investing \$1000 today. The first graph shows simple interest of 8% per year. The second graph shows compound interest at 8% per year.

Graph A



Graph B



- a) Which graph is linear? Which is non-linear? Explain.
- b) In one interest calculation, you receive a set percent of your original investment each year. In the other interest calculation, you receive a set percent of the amount that you have in your account. Identify which graph represents each of these scenarios. Explain your reasoning.

Create Connections

13. Which method do you prefer to use when representing a relation? Explain.
14. Think about a relationship between two things that you are interested in learning more about. Describe two ways you could collect the information.

6.3

Domain and Range



Scientists study our world to piece together the story of our past and make predictions about our future. Their writings are often accompanied by graphs to illustrate the data they are presenting. The more detail given with the graphs, the easier it is for the reader to interpret the information. One important aspect of a graph is the span of possible values for each quantity being compared.

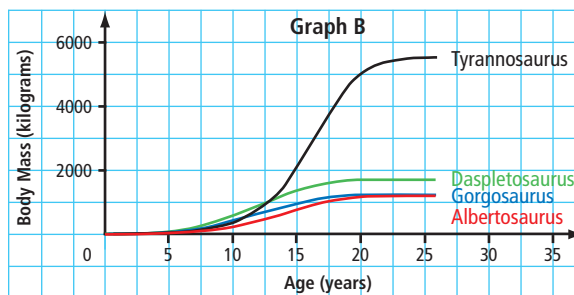
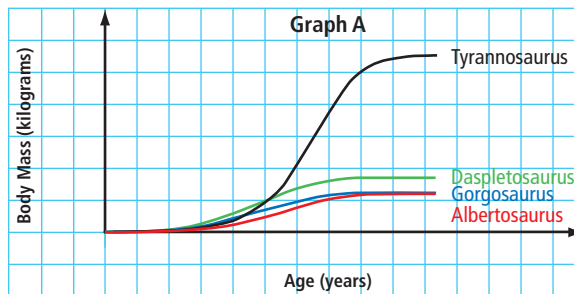
Focus on ...

- understanding the meaning of domain and range
- expressing domain and range in a variety of ways



Investigate Appropriate Values for the Dependent and Independent Variables

1. By studying changes in several specimens of tyrannosaurs, paleontologists can estimate the lifespan and growth rates of the species. The graphs show the relationship between body mass and age for four types of tyrannosaur.



- a) If you are given only Graph A, what conclusions can you make about the relationship between each tyrannosaur's body mass and its age?
- b) What additional information can you tell if you are given Graph B instead?

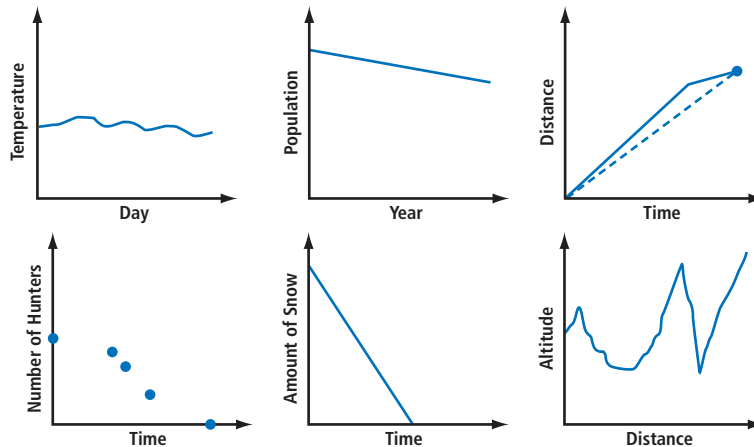
2. Read the following story inspired by the Dene legend for the creation of the seasons.

The first people of Earth had to endure **winter for twelve months** of the year. Most of the land was covered by massive, moving layers of ice and deep snow. No vegetation survived in the harsh gripping cold. All water was frozen in a land of endless cold.

One day, when the first people were hunting, they came upon a bear with a sack around his neck. The curious hunters asked what was in the sack, and the bear growled that it was filled with the abundance of summer's warmth and light. The hunters coveted the sack, but the bear would not part with it. They decided to lure the bear to a great feast of moose and **caribou**, fill him with food, and, when he slept, steal the sack. The bear readily accepted their invitation for a feast, but arrived without the sack around his neck. The bear ate his fill and fell asleep. The frustrated chief ordered **four of the village's skilled hunters to follow the bear home** and steal the sack by any means. Peering inside the bear's large cave next morning, the hunters spotted the sack on the cave floor with two black bears guarding it. The courageous hunters sprang into the cave to demand the sack. **A fierce fight killed three of the hunters and mortally wounded the fourth**, but before he died, he grabbed the sack and unleashed the abundance of warmth and light. Instantly, the air became warm and the sky filled with bright sunlight. **The snow melted** into rivers and lakes, and the **hills and valleys** became covered with trees, flowers, and bushes. Strange birds flew in great numbers and built nests, and streams filled with fish. Every year since that time, summer has come to the Dene.

Author unknown

The general graphs are related to the **bold text** in the story. In small groups, discuss how each graph depicts the information from the story.



3. a) Work with a partner. Choose any four of the graphs. Agree upon appropriate values for the dependent and independent variables. Clearly describe these possible values using words, lists, or number lines.
- b) Compare your results with those of another group. Share your reasoning with them.
4. **Reflect and Respond**
- a) Why did you not choose the two graphs that you did not use?
- b) Did the other group choose any graphs different from yours? If so, do you agree with their choice of possible values for each axis? Explain.
- c) How does adding values to each axis aid in the understanding of the graphs?

Link the Ideas

domain

- the set of all possible values for the independent variable in a relation

range

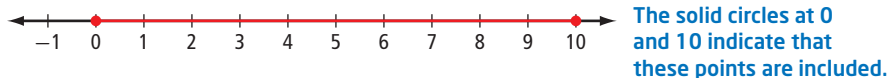
- the set of all possible values for the dependent variable as the independent variable takes on all possible values of the domain

When comparing two quantities, the words **domain** and **range** are used to describe the values that are appropriate.

In a set of ordered pairs, values for the domain are the first element of each pair. Values for the range are the second element. On a graph, values of the domain are plotted along the horizontal axis. Values of the range are plotted along the vertical axis.

There are a variety of ways to express the domain and range of a relation.

- Words** can be used to describe the values that are allowed. For example, the domain is the set of all real numbers between 0 and 10, inclusive. The range is the set of all real numbers greater than 20.
- Number lines** give a picture of the values that are allowed. For example, this number line represents all numbers between 0 and 10, inclusive.



This number line represents all numbers greater than 20.



This number line represents the discrete list of numbers -2 , 0 , 4 , 8 , and 10 .



- **A list** is a useful way to give the domain and range for discrete data when there are not many numbers in the set.
For the relation $(0, 0), (1, 5), (3, 7), (5, 7)$, the domain is $\{0, 1, 3, 5\}$ and the range is $\{0, 5, 7\}$.
- **Set notation** is a formal mathematical way to give the values of the domain and range.

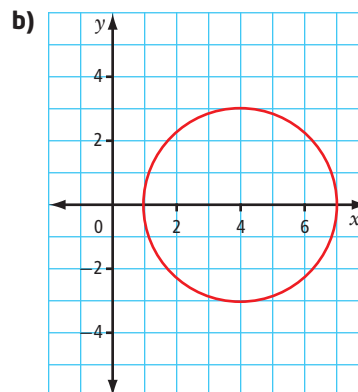
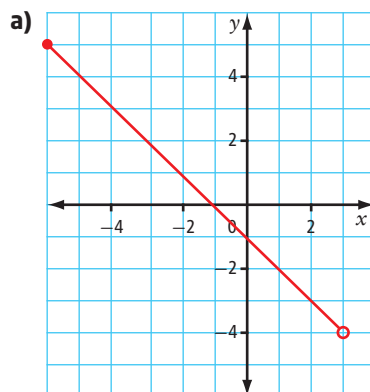
Set Notation	What It Means
The domain: $\{x \mid x \leq 10, x \in \mathbb{R}\}$	$\{\}$ is the type of brackets used for a set. \in means "is an element of". \mid means "such that". The statement is read as follows: x is an element of the real numbers such that x is less than or equal to 10.
The range: $\{y \mid y > 20, y \in \mathbb{R}\}$	The statement is read as follows: y is an element of the real numbers such that y is greater than 20.

Recall that the symbols used for the number sets are
 \mathbb{R} for real numbers
 \mathbb{Q} for rational numbers
 \mathbb{I} for integers
 \mathbb{N} for natural numbers
 \mathbb{W} for whole numbers

- **Interval notation** uses different brackets to indicate an interval.
This style of bracket, $]$, is used if the end number is included.
This style of bracket, $)$, is used if the end number is not included.
The infinity symbol, ∞ , is used if there is no end point.
A domain of all numbers between 0 and 10, inclusive, would be given as $[0, 10]$.
A range of all numbers greater than 20 would be given as $(20, \infty)$.



Example 1 Determine the Domain and Range From a Graph

For each graph, give the domain and range. Use words, a number line, interval notation, and set notation.





Solution

- a) From looking at the graph, you can see that the smallest value for x is -6 . The largest value for x is up to, but not including, 3 . The smallest value for y is down to, but not including, -4 . The largest value for y is 5 .

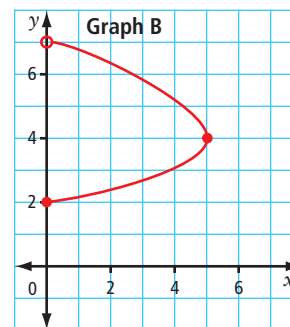
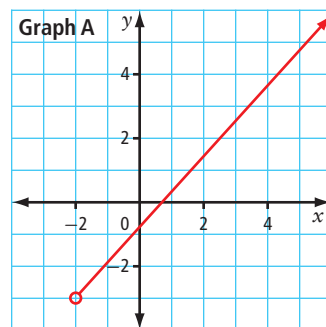
Domain	Range
<p><i>Words</i> All real numbers between -6 and 3, including -6 but not including 3</p> <p><i>Number Line</i> </p> <p><i>Interval Notation</i> $[-6, 3)$</p> <p><i>Set Notation</i> $\{x \mid -6 \leq x < 3, x \in \mathbb{R}\}$</p>	<p><i>Words</i> All real numbers between -4 and 5, not including -4 but including 5</p> <p><i>Number Line</i> </p> <p><i>Interval Notation</i> $(-4, 5]$</p> <p><i>Set Notation</i> $\{y \mid -4 < y \leq 5, y \in \mathbb{R}\}$</p>

- b) From looking at the graph, you can see that the smallest value for x is 1 . The largest value for x is 7 . The smallest value for y is -3 . The largest value for y is 3 .

Domain	Range
<p><i>Words</i> All real numbers between 1 and 7, inclusive</p> <p><i>Number Line</i> </p> <p><i>Interval Notation</i> $[1, 7]$</p> <p><i>Set Notation</i> $\{x \mid 1 \leq x \leq 7, x \in \mathbb{R}\}$</p>	<p><i>Words</i> All real numbers between -3 and 3, inclusive</p> <p><i>Number Line</i> </p> <p><i>Interval Notation</i> $[-3, 3]$</p> <p><i>Set Notation</i> $\{y \mid -3 \leq y \leq 3, y \in \mathbb{R}\}$</p>

Your Turn

For each graph, give the domain and range using words, a number line, interval notation, and set notation.

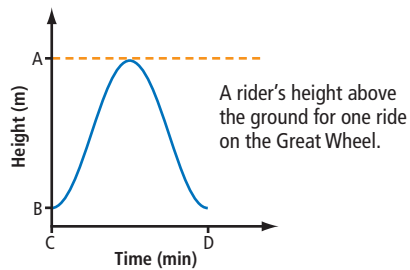


Example 2 Domain and Range for a Situation

The Great Wheel is being built in Beijing in the People's Republic of China. When finished, it will be the largest Ferris wheel in the world. The wheel will have a diameter of 193 m and will reach a maximum height of 208 m.

The graph shows a rider's height relative to the ground for a 20-min ride through one rotation.

- a) What are the values of points A, B, C, and D, and what do they represent?
b) What are the domain and the range of the graph?



Solution

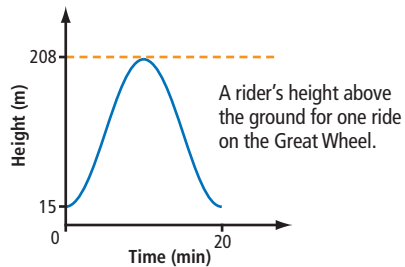
- a) Point A is the highest point on the graph, so it must represent the highest point on the ride. Its value is 208 m.

Point B is the lowest point on the graph. It represents the location where a rider boards the Ferris wheel. Since the diameter of the wheel is 193 m, this point is 193 m below point A. By subtraction, the value of B is 15 m.

Point C is the origin, $(0, 0)$. It represents ground level on the vertical axis and the starting time on the horizontal axis.

Why does the graph not begin at $(0, 0)$?

Point D represents the time it takes to complete one revolution. Its value therefore is 20 min.



Why is the graph in quadrant I?



- b) The domain and range can be described in several ways.

Words:

Domain: All times are between 0 min and 20 min, inclusive.

Range: A rider's height above the ground is between 15 m and 208 m, inclusive.

Number Line:

Domain: Ride time, in minutes



Range: Height above the ground, in metres



Interval Notation:

Domain: Ride time, in minutes: $[0, 20]$

Range: Height above the ground, in metres: $[15, 208]$

Set Notation:

Domain: Ride time, in minutes: $\{t \mid 0 \leq t \leq 20, t \in \mathbb{R}\}$

Range: Height above the ground, in metres: $\{h \mid 15 \leq h \leq 208, h \in \mathbb{R}\}$

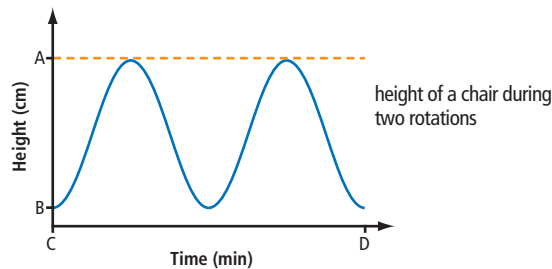
WWW Web Link

To view a graph of a rider's height above the ground when on a Ferris wheel, go to www.mhrmath10.ca and follow the links.

Your Turn

A motorized model Ferris wheel has a radius of 22 cm. The support structure keeps the bottom of the wheel 3 cm above the base. It takes 10 s to complete one revolution. The graph shows the height of one of the chairs during two rotations of the wheel, starting at the lowest point.

- a) What are the values of A, B, C, and D? What do they represent?
 b) What are the domain and range of the graph? Express each in words, as a number line, in interval notation, and in set notation.



Example 3 Domain and Range for Discrete Data

Caitlin is marking time for some music by clapping on the first beat of every bar. The table of values and the set of ordered pairs show the relationship between the total number of beats and her total number of claps. Give the domain and range of the relation using words and a list.

Number of Beats	Number of Claps
1	1
5	2
9	3
13	4
17	5
21	6
25	7
29	8

Ordered Pairs (Beats, Claps)

$\{(1, 1), (5, 2), (9, 3), (13, 4), (17, 5), (21, 6), (25, 7), (29, 8)\}$



Solution

Words:

Domain: The total number of beats is given by the numbers 1, 5, 9, 13, 17, 21, 25, and 29.

Range: The total number of claps is given by the whole numbers between 1 and 8, inclusive.

List:

Domain: $\{1, 5, 9, 13, 17, 21, 25, 29\}$

Range: $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Your Turn

Data for a relation are recorded in the table of values.

Give the domain and range using set notation and lists.

a	b
-3	5
-2	6
-1	7
0	8
1	9
2	10

Example 4 Use Technology to Graph a Relation With a Restricted Domain

A variety of corn plant grows at an average rate of 4.5 cm per day from the start of the third week of growth to the end of the sixth week. The plant's growth can be modelled using the formula $h = 4.5a + 25$, where h is the height of the plant, in centimetres, and a is the age of the plant, in days. Use a graphing calculator to create a graph of a cornstalk's height from the beginning of week 3 to the end of week 6.

Solution

To get a view of the plant's height for the required time frame, you need to restrict the window settings of the graphing calculator displaying the relation $h = 4.5a + 25$.

Which variable is the independent variable? How do you know?

In the domain you want only the values for the age of the plant during the period from week 3 to week 6, inclusive. At the beginning of week 3, the plant is 14 days old. At the end of week 6, the plant is 42 days old. Therefore, the domain is $[14, 42]$.

The range is the set of all possible plant heights during this growth period.

The model is valid only for these ages.

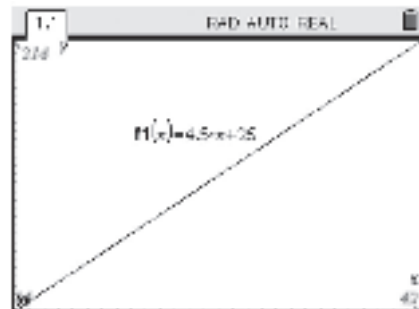
If $a = 14$, then $h = 4.5(14) + 25$ or $h = 88$.

If $a = 42$, then $h = 4.5(42) + 25$ or $h = 214$.

Therefore, the range is $[88, 214]$.

Graph the relation on a graphing calculator.

- Enter the equation of the relation.
- Set the values for the window settings.
- Graph the relation.

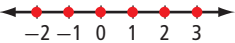


Your Turn

The same species of corn grows at an average rate of 5 cm per day from the start of week 7 until the end of week 9. The plant's growth in this period is modelled using the formula $h = 5a + 214$, where h is the height of the plant, in centimetres, and a is the age of the plant, in days. Use a graphing calculator to show a graph of the plant's height for these three weeks.

Key Ideas

- The domain of a relation is the set of all numbers for which the independent variable is defined.
- The domain of a relation may also be described as:
 - the set of first coordinates in a set of ordered pairs
 - the possible values in the first column of a table of values
 - the possible values on the horizontal axis of a graph
- The range of a relation is the set of all numbers for which the dependent variable is defined.
- The range of a relation may also be described as:
 - the set of second coordinates in a set of ordered pairs
 - the possible values in the second column of a table of values
 - the possible values on the vertical axis of a graph
- The domain and range can be expressed in different ways.

Words	All integers equal to or greater than -2 and less than or equal to 3
Number Line	
Interval Notation	$[-2, 3]$
Set Notation	$\{n \mid -2 \leq n \leq 3, n \in \mathbb{I}\}$
A List	$\{-2, -1, 0, 1, 2, 3\}$

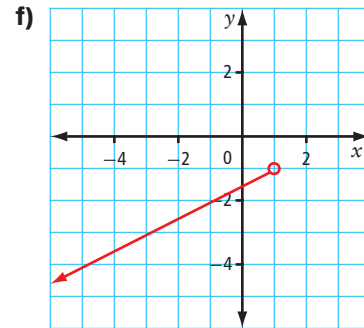
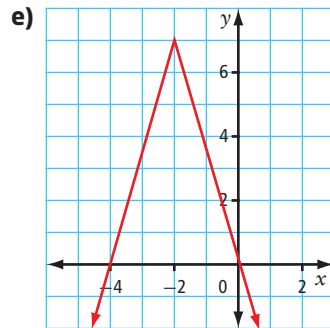
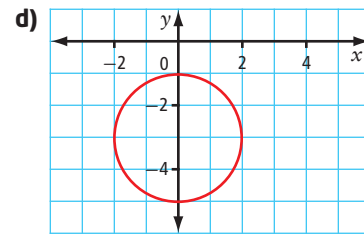
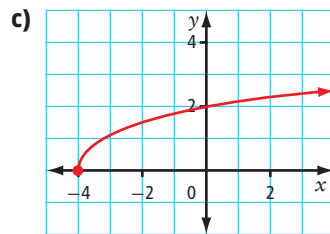
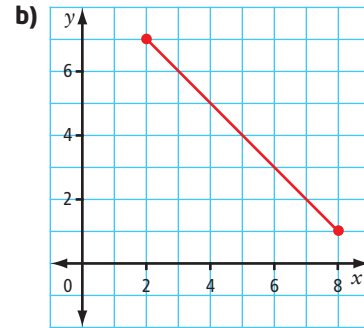
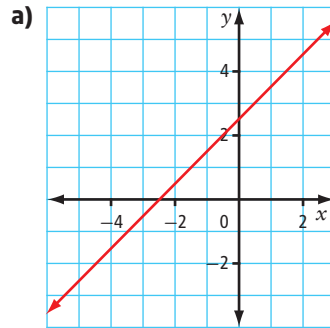
Check Your Understanding

Practise

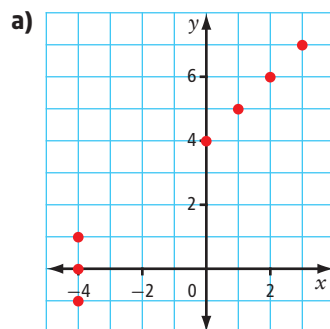
1. Describe the set of numbers indicated by each number line. Use words, interval notation, and set notation.



2. Give the domain and range of each graph. Use words, a number line, interval notation, and set notation.



3. Write the domain and the range of each relation as lists.



b)

s	v
-4	5
-2	5
0	7
2	7
4	9
6	9

c) (50, 10), (100, 20), (150, 30), (200, 40)

4. A relation is given by the formula $k = 2.8m - 3.5$.

- If the domain of the relation is $[0, 25]$, what is the range?
- Graph the relation on a graphing calculator. Record the window values you chose. Sketch the relation.

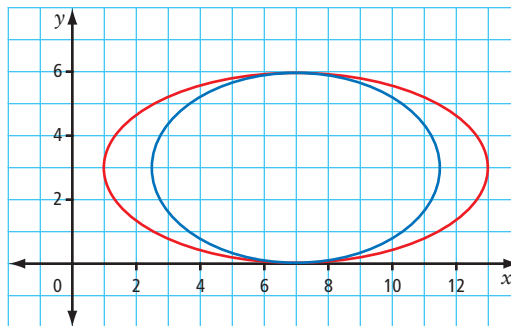
Apply

5. The table gives the average annual high temperature for a number of western and northern Canadian cities.

City	Average Annual High Temperature (°C)
Winnipeg	8.3
Regina	9.1
Edmonton	8.5
Calgary	10.5
Vancouver	13.7
Victoria	14.1
Whitehorse	4.5
Yellowknife	-0.2

- a) Give the domain and the range for the relation.
 b) Graph the relation.

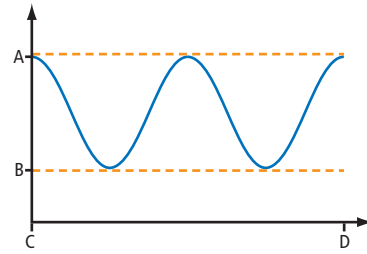
6. A company offers two models of above-ground oval swimming pools. The ovals in the graph are drawn using the dimensions, in metres, of the pools.



- a) Using interval notation, what are the domain and range of the blue oval?
 b) Using set notation, what are the domain and range of the red oval?
 c) What are the actual dimensions of each pool?
7. An electric car can travel 193 km before its battery needs to be recharged.
- a) Create a graph to show the distance travelled over time if the car is driven at an average speed of 60 km/h.
 b) Describe the domain and the range of your graph in two ways.



8. The graph shows the changes in tide levels for Bella Coola on the central coast of British Columbia for a 24-h period starting at 12:00 a.m. The greatest water depth, at high tide, is 15.9 ft. The least depth, at low tide, is 4.5 ft.



- Sketch a copy of the graph. Label each axis with appropriate dependent and independent variables.
 - What are the values of A, B, C, and D? What do they represent?
 - What are the domain and range of the graph? Express these in words, as a number line, in interval notation, and in set notation.
9. A hot-air balloon is flying at an altitude of 1236 m. It begins to descend at a rate of 10 m per minute.
- How long does it take the balloon to reach the ground?
 - Assign variables to represent each quantity in the relation. Identify the independent variable and the dependent variable. Graph the balloon's progress from the moment it begins its descent until it reaches the ground.
 - Does the graph continue in both directions? Explain.
 - What do the domain and range represent in this context?
 - Give the domain and the range using all forms appropriate for this situation.

Extend

- The domain of a linear relation is given by $\{x \mid -2 \leq x \leq m, x \in \mathbb{I}\}$. The range, in order, is given by $\{2, 6, 10, k, 18\}$. Use a graphing approach to determine the smallest possible values of m and k .
- James can store 8 GB of music on his phone. If a 3-min digital song requires 3.5 MB of storage space, what are the domain and range for the relationship between memory used and number of songs that are each 3 min in length?

Create Connections

- Using everyday language, how do you describe domain and range to someone not familiar with the concepts?
- For a topic that interests you, such as sports, music, or entertainment, give an example of a relation that has a restricted domain and range. Why do these restrictions exist?

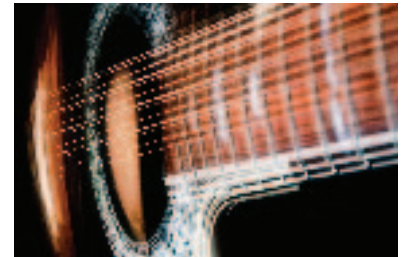
6.4

Functions

Focus on ...

- sorting relations into functions and non-functions
- using notation specifically designed for functions
- graphing linear functions

There is a special class of relations, called functions, where two quantities depend on each other in a particular way. The amount of tension on a guitar string determines the musical note played. The channel displayed on your television screen depends on the number you enter into the remote. Describe other examples from your daily life where two quantities depend on each other in a particular way.



Investigate Functions

Study the following relations. They are categorized as functions and non-functions.

These 8 relations are functions.

x	y	x	y
5	10	11	3
6	15	21	3
7	20	31	3

$\{(-2, -5), (0, 4), (2, 13), (4, 22)\}$

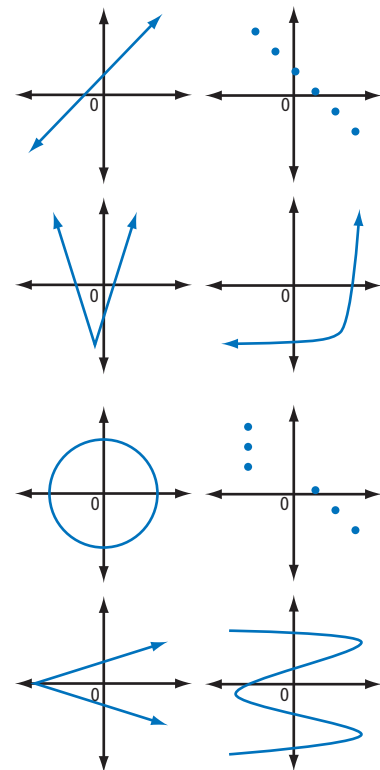
$\{(10, 10), (12, 10), (14, 12), (16, 12)\}$

These 8 relations are not functions.

x	y	x	y
6	10	3	11
6	15	3	21
7	20	3	31

$\{(10, 10), (12, 10), (12, 14), (12, 16)\}$

$\{(7, 5), (7, 8), (9, 11), (11, 14)\}$



1. What is similar about the functions? What is similar about the non-functions? Discuss your findings with a partner.

2. Reflect and Respond

- How can you tell whether or not a relation is a function?
- Share your explanation with your classmates. Once you have heard other explanations, revise yours to be more precise.

function

- a relation in which each value of the independent variable is associated with exactly one value of the dependent variable
- for every value in the domain there is a unique value in the range

function notation

- a symbolic notation used for writing a function
- $f(x)$ is read as “ f of x ” or “ f at x ”.

Did You Know?

The term *function* was developed and used by German mathematician Gottfried Wilhelm Leibniz (1646–1716). The notation we use today was created by Swiss mathematician Leonhard Euler (1707–1783). Euler contributed to many branches of mathematics.

Link the Ideas

Functions can be written using **function notation**. The function $y = 4x + 1$ is written as $f(x) = 4x + 1$. The name of the function is f , with a variable name of x . In this example, $4x + 1$ is the rule that assigns a unique value for y for each value of x . Any letter may be used to name a function. Two examples are $v(t) = 9.8t^2$ and $A(r) = \pi r^2$.

Function notation highlights the input/output aspect of a function. The function $f(x) = 4x + 1$ takes any input value for x , multiplies it by 4, and adds 1 to give the result. For example, if $x = 2$ is the input, then $f(2) = 9$ is the output.

$$f(2) = 4(2) + 1$$

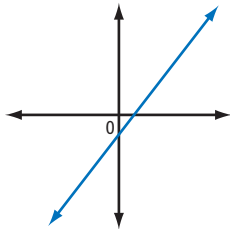
$$f(2) = 9$$

When $x = 2$, the value of the function is 9.
The point (2, 9) is on the graph of the function.

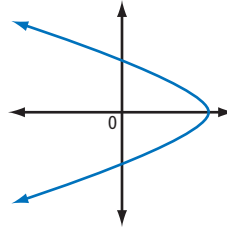
Example 1 Determine Whether a Relation Is a Function

For each pair of relations, decide which relation is a function and which is not a function. Explain your choice.

a) A



B



b) C

x	y
2	5
2	7
4	9
6	11

D

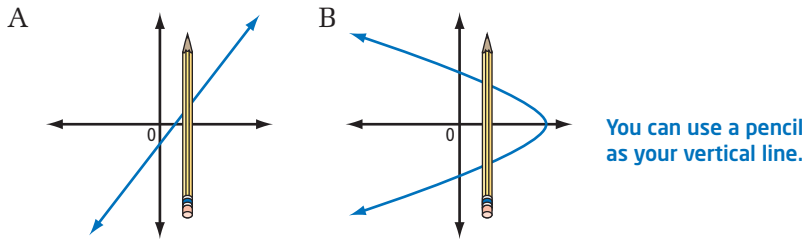
x	y
-3	3
-2	4
-1	3
0	4

- c) E $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
 F $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$

Solution

For each pair, check to see which relation has a domain value associated with more than one range value. These are not functions.

- a) When a relation is given as a graph, look for any points on the graph that line up vertically. If points line up vertically, a value of x has more than one corresponding value of y . One method to check this is called the **vertical line test**.



Relation A is a function, because it passes the vertical line test. Each domain value is used only once. Relation B does not pass the vertical line test. Many values of x have two values of y .

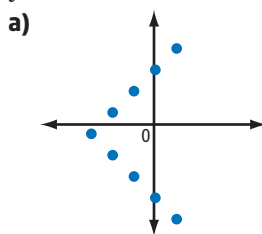
- b) In relation C, the value of the relation when x equals 2 is associated with y -values of 5 and 7, so this is not a function. Relation D is a function, because each domain value is used only once.
- c) In relation E, each domain value is used only once, so it is a function. In relation F, the domain value 1 is used repeatedly. Therefore, relation F is not a function.

vertical line test

- a test to see if a graph represents a function
- if any vertical line intersects the graph at more than one point, the relation is not a function

Your Turn

Which of the following relations are functions? Explain your choices.

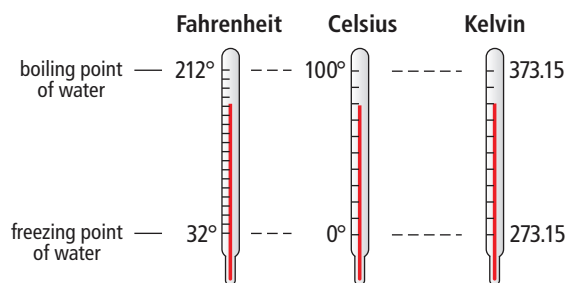


b) $\{(-2, 1), (0, 0), (2, 1), (5, 1)\}$

c)

x	y
1	3
2	3
3	4
4	4
5	4

Example 2 Work With Function Notation



The function $F(C) = 1.8C + 32$ is used to convert a temperature in degrees Celsius ($^{\circ}\text{C}$) to a temperature in degrees Fahrenheit ($^{\circ}\text{F}$).

- Determine $F(25)$. Explain your answer.
- Determine C so that $F(C) = 100$. Explain your answer.

Solution

a) $F(C) = 1.8C + 32$
 $F(25) = 1.8(25) + 32$
 $F(25) = 45 + 32$
 $F(25) = 77$

This means that 25°C is the same as 77°F .

b) $F(C) = 1.8C + 32$
 $100 = 1.8C + 32$
 $100 - 32 = 1.8C + 32 - 32$
 $68 = 1.8C$
 $\frac{68}{1.8} = \frac{1.8C}{1.8}$
 $37.8 = C$

This means that 100°F is the same as 37.8°C .

Your Turn

- Determine $F(86)$. Explain your answer.
- Determine C so that $F(C) = 98.6$. Explain your answer.
- Another measurement scale for temperature that is used in science is the Kelvin scale. The function $K(C) = C + 273.15$ can be used to convert from degrees Celsius to kelvins. Determine $K(80)$ and explain your answer.

Did You Know?

The Kelvin temperature scale was introduced by William Thomson, 1st Baron Kelvin (1824–1907), a Scottish physicist. Units on the scale are called kelvins. The Kelvin scale does not use the word *degrees* or the symbol $^{\circ}$.

Example 3 Graphing Linear Functions

Skye has a cell phone plan for a monthly fee of \$20 plus 15¢ for each text message to or from a number not on a list of favourites. Skye's monthly bill can be modelled by the relation $C = 0.15n + 20$, where C is the total charge, in dollars, and n is the number of additional text messages.

- Write the relation in function notation.
- Make a table of values. Graph the function if Skye sends up to four additional text messages.
- If Skye's cell phone bill for a certain month is \$22.25, how many additional text message charges are there?

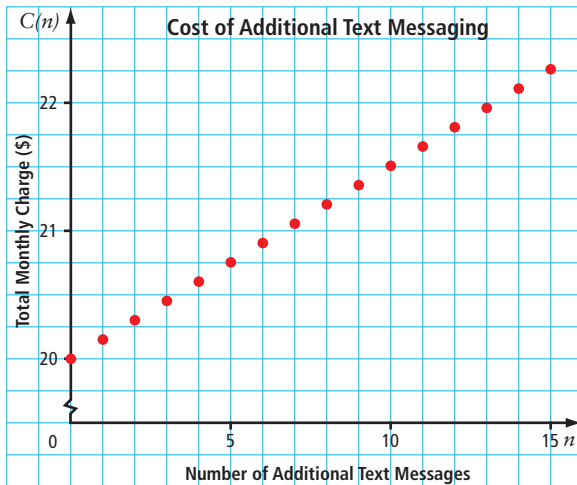
Solution

- The independent variable is n , the number of additional text messages. The function is $C(n) = 0.15n + 20$.
- Choose values for the independent variable, n . Use the formula to determine the total charge.

A table of values for the first five values of this function is shown.

n	$C(n)$
0	20.00
1	20.15
2	20.30
3	20.45
4	20.60

Continuing the pattern, the graph shows the first 16 values for the function.



Why are the points in this graph not connected?

c) Method 1: Use the Graph

Use the graph on page 309. Locate the value of \$22.25 on the total monthly charge axis. Read the corresponding number of text messages. From the graph, it can be seen that Skye's cell phone bill includes 15 additional text message charges.

Method 2: Use the Formula

The total charge is \$22.25.

Substitute $C = 22.25$ into the relation.

$$C(n) = 0.15n + 20$$

$$22.25 = 0.15n + 20 \quad \text{Solve for } n.$$

$$22.25 - 20 = 0.15n + 20 - 20$$

$$2.25 = 0.15n$$

$$\frac{2.25}{0.15} = \frac{0.15n}{0.15}$$

$$15 = n$$

Skye's phone bill includes 15 additional text message charges.

Your Turn

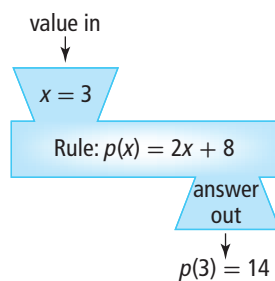
Use the relation $y = 3x - 1$.

- Write the relation in function notation using f for the name of the function.
- Make a table of values. Graph the function.
- Determine the value of x if $f(x) = 53$.

Key Ideas

- All functions are relations but not all relations are functions.
- A relation is classified as a function if each value in the domain corresponds to exactly one value in the range.
- Each function has its own formula, or rule, that is often given using a special notation, called function notation.

For example, $p(x) = 2x + 8$ shows that the function p takes an input value, multiplies it by 2, adds 8, and outputs the answer.



Check Your Understanding

Practise

1. Determine whether each relation is a function or is not a function. Give a reason for your answer.

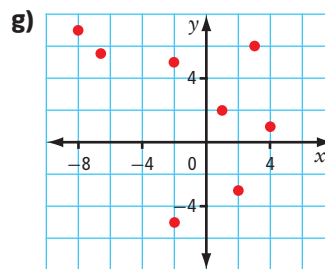
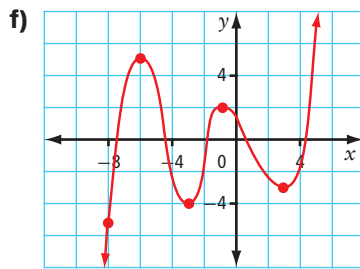
- a) $(-1, 2), (0, 1), (1, 2), (2, 5)$
 b) $(3, 12), (4, 12), (5, 14), (6, 14)$
 c) $(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)$

d)

x	y
0	0
1	-1
1	1
4	-2
4	2

e)

Name	Age
Naomi	14
Wam	15
Brigid	14
Sharon	16
Arvind	15

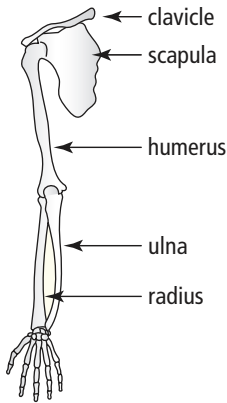


2. The formula for the surface area, A , of a sphere with radius r is $A = 4\pi r^2$. Write this formula using function notation.
3. The cost to have artwork printed on T-shirts is given by the function $C(n) = 3n + 50$, where n is the number of shirts and C is the cost, in dollars. Write this function as a formula in two variables.
4. If $f(x) = 10x - 8$, determine
 a) $f(2)$ b) $f(-3)$ c) x if $f(x) = 42$
5. If $h(x) = \frac{2}{3}x + 1$, determine
 a) $h(9)$ b) $h(-3)$ c) x if $h(x) = -7$
6. Consider the function $p(x) = -4x + 2$.
 a) What is the value of $p(0)$?
 b) Determine x so that $p(x) = -2$.

7. Make a table of values and graph each function.
- $g(x) = -3x + 5$ for the domain $\{-3, -2, -1, 0, 1, 2, 3\}$
 - $h(x) = \frac{x}{2}$ for the domain $\{x \mid -10 \leq x \leq 10, x \in \mathbb{R}\}$

Apply

8. Mike currently has \$200 and saves \$20 each week. The function $M(w) = 20w + 200$ describes his saving pattern. Ali currently has \$200 and spends \$20 each week. The function $A(w) = 200 - 20w$ describes her spending pattern.
- What does the variable w represent in each function?
 - Explain the meaning of $M(w)$ and $A(w)$.
 - What is the value of each function when $w = 4$? Explain your answer.
 - Determine the value of w when $A(w) = 0$. Explain your answer.



9. **Unit Project** Using skeletal remains, a forensic anthropologist can accurately determine the sex, race, age, and height of a person.
- The height, h , in centimetres, of a male can be determined using the function $h(L) = 2.9L + 70.6$, where L is the length, in centimetres, of his humerus. Suppose you find a humerus of a male and measure the bone to be 36.87 cm in length. How tall was the man?
 - The function $h(L) = 2.8L + 71.4$ can be used to determine the height, h , in centimetres, of a female, where L is the length, in centimetres, of her humerus. Determine $h(36.87)$. What does $h(36.87)$ represent?
 - Height, h , in centimetres, can also be determined using the functions $h(L) = 3.3L + 86.4$ for a male and $h(L) = 3.3L + 81.3$ for a female. In these functions, L represents the length of the radius bone, in centimetres. What is an appropriate range for each of these functions? Explain.
 - Based on the range you determine in part c), what is the span of values (the domain) for the radius bone in males and in females?
 - Measure the length of your radius bone. Use the appropriate function to determine your height. How accurate is the prediction?

10. Weight on the moon is not the same as it is on Earth because of differences in the force of gravity. The function $m(E) = \frac{E}{6}$ can be used to approximate your weight, m , on the moon, where E represents your weight on Earth.

- Does the function indicate that you would be heavier or lighter on the moon than on Earth? Explain.
- If a person weighs 80 kg on Earth, how much would the person weigh on the moon?
- How much would you weigh on the moon?
- What is an appropriate domain for this function? Using this domain, make a table of values and graph the function.

11. In air, the speed of sound depends on the air temperature. The speed of sound, S , in kilometres per hour, can be estimated using the function $S(t) = 1192 + 2.2t$, where t is the temperature of the air, in degrees Celsius.

- What is the speed of sound in air at a temperature of 20 °C?
- At an altitude of 10 km, the air temperature is about -55 °C. What is the speed of sound at this altitude?
- A jet “breaks the sound barrier” when it flies faster than the speed of sound. The aircraft’s speed is then typically referred to in Mach numbers. A jet’s Mach number can be determined using the function $M(V) = \frac{V}{S}$, where V is the speed of the jet and S is the speed of sound, both in kilometres per hour. Use your results for S from parts a) and b) to write the Mach number function when the temperature is 20 °C, and when the temperature is -55 °C.
- A jet is flying at 1800 km/h at an altitude of 10 km. What is the jet’s Mach number?



Water vapour created as jet breaks sound barrier

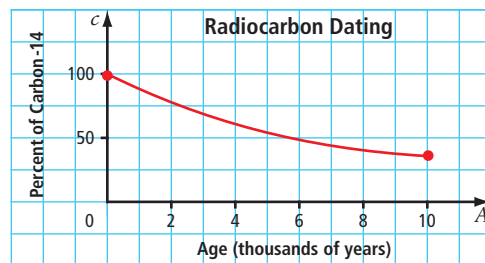
Extend

- Sharon creates a function in the form $f(x) = \blacksquare x + \blacksquare$ for her classmates to solve. To figure out the actual equation, students give Sharon input values. She gives them the output from the function. The values are $f(1) = 5$, $f(2) = 8$, $f(-1) = -1$, and $f(-2) = -4$. What is the equation for Sharon’s function?
- Create a function of your own. Have someone determine the equation of your function by giving you input values and studying the pattern in your responses.

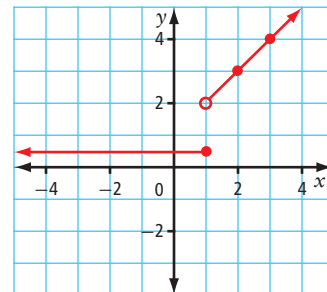
Did You Know?

First Nations hunters used a variety of approaches to hunt bison. Of all the methods the hunters devised, the most effective was the bison jump. At Head-Smashed-In Buffalo Jump, the cliff is about 10 m high. The oldest bones and stone tools at the jump are buried 10 m below this apron. This indicates that the cliff at one time was twice its current height.

13. **(Unit Project)** After an animal dies, the amount of radioactive carbon-14 in its bones declines. Archaeologists use this fact to determine the age of a bone based on the percent of carbon-14 remaining in the fossilized bones. The relation shows the age, A , in years, of an animal based on the percent, c , of carbon-14 remaining.



- a) Is this relation a function? Why?
- b) At Head-Smashed-In Buffalo Jump, in southwestern Alberta, the most recent bison bones found had 98% of the carbon-14 still remaining. From the graph, estimate the age of these bones.
- c) The oldest bison bones found at Head-Smashed-In Buffalo Jump were about 5800 years old. Estimate the percent of carbon-14 still remaining in these bones.
14. a) Does the graph represent a function? Explain.
- b) What is the value of $f(-4)$? $f(1)$? $f(3)$? $f(5)$?



15. The input for a function can be another function. If $h(x) = 2x - 5$, determine a simplified expression for each of the following.
- a) $h(4x)$ b) $h(2x + 3)$ c) $h\left(\frac{x}{2} - 1\right)$

Create Connections

16. Explain, using examples, why some relations are not functions but all functions are relations. Draw a Venn diagram to illustrate the relationship between functions and relations.
17. Explain the difference between $f(2)$ and $f(x) = 2$.
18. Jean-Marie has never seen function notation. When he sees a question that asks him to determine the value of $f(x + 2)$, he gives his answer as $fx + 2f$.
- a) How does Jean-Marie interpret the question?
- b) Explain the meaning of this question to Jean-Marie in the context of functions.

6.5

Slope



Focus on ...

- determining the slope of a line
- using slope to draw lines
- understanding slope as a rate of change
- solving problems involving slope

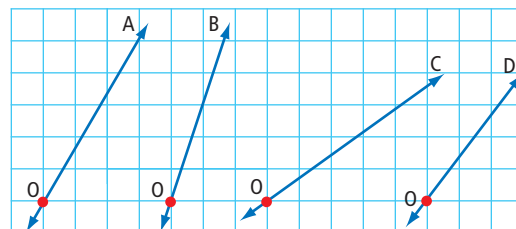
The national, provincial, and territorial parks of western and northern Canada feature some of the most beautiful backcountry in the world. To safely enjoy mountain adventures, specialized skills and knowledge, such as avalanche awareness, are essential. Though avalanches occur mostly in winter, they can happen at any time of the year. It is important to understand the many conditions that cause avalanches. The steepness, or slope, of a mountainside is one of them.

Materials

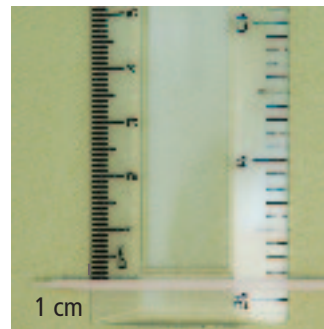
- grid paper
- plastic transparent ruler
- toothpick
- tape

Investigate Slope

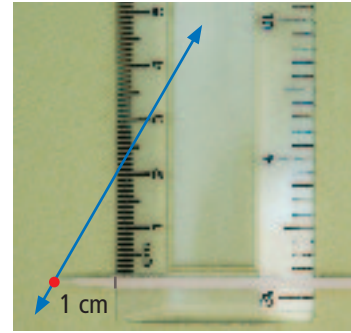
1. By observation, arrange the lines shown in order of steepness, from least steep to steepest. Explain your reasoning.



2. Convert a regular ruler into a “slope ruler” by taping a toothpick to the end of the ruler as follows:
 - Make a pencil mark 1 cm from the end of the toothpick.
 - Tape the toothpick so that its edge is aligned with the 0-cm mark of the ruler and the pencil mark is aligned with the edge of the ruler.



3. a) Use your slope ruler to measure and record the slope of each line. Place your ruler so that it is vertical and the end of the toothpick is on point O. Record the slope as the point where the line intersects the ruler.
- b) How do the slope values that you measure relate to the steepness of each line?



4. On grid paper, plot a point and label it A. Measure 5 cm straight up (vertical) from point A and plot another point. Label this point B. Label the distance from A to B.
5. Measure 5 cm to the left (horizontal) of point A and 5 cm to the right (horizontal) of point A. Plot a point after each measurement. Label these points C and D. Label the distance from A to C.
6. a) Using a ruler, connect points C, B, and D. This triangle is a model mountain. Determine the ratio of the height at the centre of the mountain (A to B) to the horizontal distance from the centre to the base (A to C). That is, determine $\frac{AB}{AC}$.
- b) Use your slope ruler to measure the slope of the mountain. How does the ratio compare to the slope given by the ruler?
7. a) Construct two other model mountains. Make one three times as high as the first one ($AB = 15$ cm) but with the same horizontal width ($AC = 5$ cm). Make the other the same height as the first one ($AB = 5$ cm) but with twice the horizontal width ($AC = 10$ cm).
- b) Is each mountain steeper or less steep than the one in step 6?
- c) Do you expect the slope value of each to be greater or less than the slope value in step 6?
- d) Compare the ratio $\frac{AB}{AC}$ of each mountain to the slope value given by using the ruler.

8. Reflect and Respond

- How do you draw a mountain with dimensions different from the model in step 6 but with the same slope? Draw this mountain. Check the slope by using your ruler.
- A student uses a slope ruler and measures the slope of a model to be 8. If the model has a height of 32 cm at the centre, what is the distance from the centre to the base of the model?
- Can a slope ruler give a measurement of $\frac{1}{4}$? If so, explain how.
- Can a slope ruler show a negative value? If so, what does that value look like?
- Suppose the horizontal distance for a slope ruler was 2 cm instead of 1 cm. What slope would a height reading of 8 cm represent on this slope ruler?

The horizontal distance for this slope ruler is 1 cm. When the slope ruler measures a slope of 8, this represents the ratio of $\frac{8}{1}$.

Link the Ideas

The **slope** of a line or line segment tells you how steep the line is. The sign of the slope value indicates the direction of the line.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$\text{or } m = \frac{\text{rise}}{\text{run}} \text{ or } m = \frac{\Delta y}{\Delta x}$$

m is the variable used for slope and Δ is a symbol used to indicate change. The expression Δy is read as "delta y."

slope

- the ratio of the vertical change, or rise, to the horizontal change, or run, of a line or line segment
- not expressed with units

A line or line segment that rises from left to right has a positive slope.

Move from point A to point B:

$\frac{\text{positive vertical change}}{\text{positive horizontal change}}$ results in a positive slope

Move from point B to point A:

$\frac{\text{negative vertical change}}{\text{negative horizontal change}}$ results in a positive slope

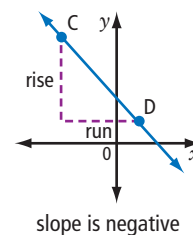
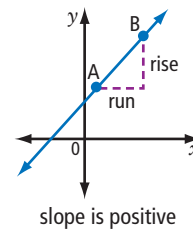
A line or line segment that falls from left to right has a negative slope.

Move from point C to point D:

$\frac{\text{negative vertical change}}{\text{positive horizontal change}}$ results in a negative slope

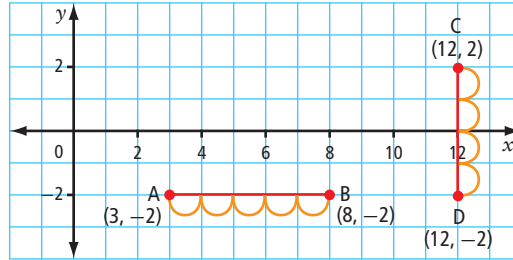
Move from point D to point C:

$\frac{\text{positive vertical change}}{\text{negative horizontal change}}$ results in a negative slope



Determining the Slope Using Points on a Line

When the distance from one point to another is along a horizontal line or a vertical line, you can find the distance by simply counting the spaces on the grid or by using subtraction. For example,



By counting, the distance AB is 5 units.

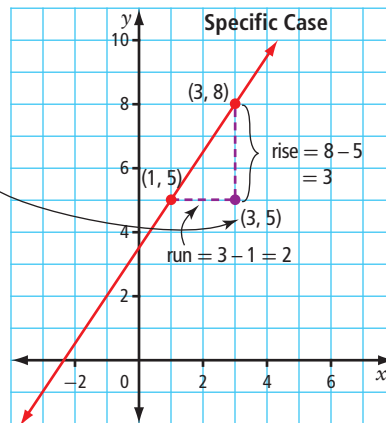
By subtraction, the distance AB is $8 - 3 = 5$ units.

By counting, the distance CD is 4 units.

By subtraction, the distance CD is $2 - (-2) = 4$ units.

Applying this idea, you can develop a formula to find the slope of any line.

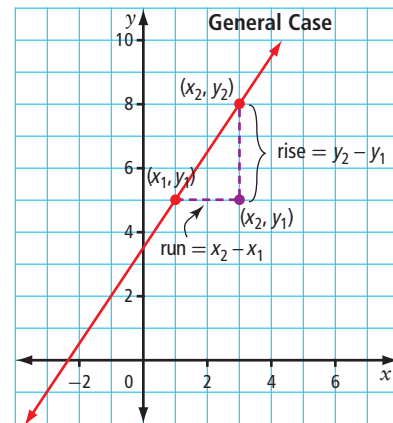
Notice how the coordinates of this point are related to the coordinates of the two points on the line.



Slope formula

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3}{2}$$



Slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

Applying the slope formula to line AB above shows that the slope of a horizontal line is 0.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{0}{8}$$

$$m = 0$$

Applying the slope formula to line CD above shows that the slope of a vertical line is undefined.

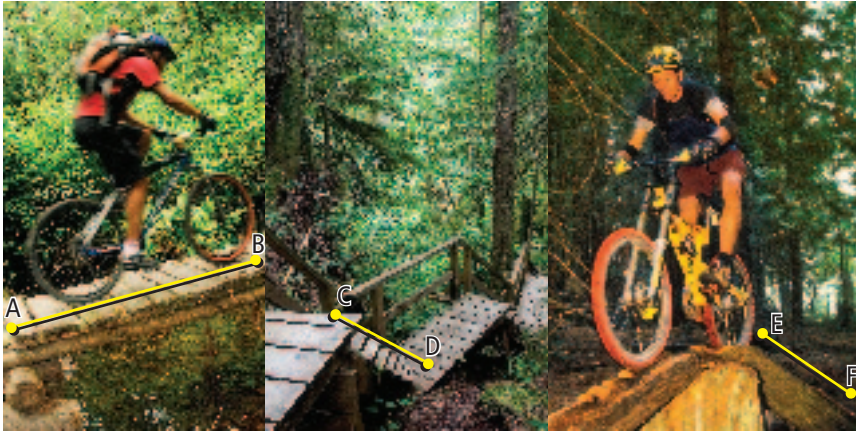
$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{6}{0}$$

$$m = \text{undefined}$$

Example 1 Classify the Slope of a Line

The North Shore in Vancouver is popular for hiking and biking. Bridges and stunt structures on trails are complex and often extremely challenging. They have a huge variety of slopes. Classify each slope marked on the photographs as either positive or negative.

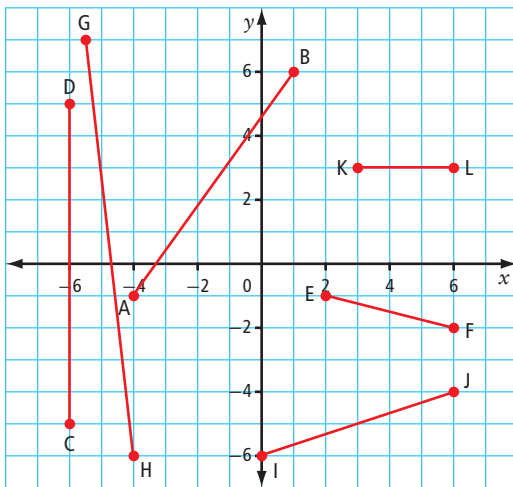


Solution

Lines and line segments that rise from left to right have positive slopes. Therefore, line segment AB has a positive slope. Lines and line segments that fall from left to right have negative slopes. Therefore, line segments CD and EF have negative slopes.

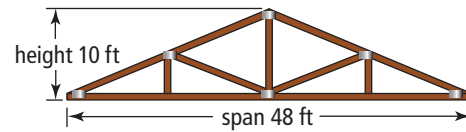
Your Turn

Classify the slope of each line segment as positive, negative, or neither.



Example 2 Determine the Value of a Slope

When discussing a roof truss, carpenters refer to the *span* instead of the *width*. They talk about the *pitch* rather than the *slope*. Determine the pitch of the roof supported by the truss shown. Explain the meaning of your answer.



Solution

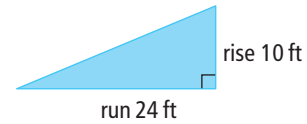
The pitch of the roof is its slope.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{10}{24}$$

$$m = \frac{5}{12}$$

How is the run determined?



The pitch of the roof is $\frac{5}{12}$. This means that the roof rises 5 units for every 12 units of horizontal distance.

Your Turn

Suppose that the roof truss in Example 2 has a height of 1 m and a span of 8 m. Determine the pitch and explain your answer.

Example 3 Determine the Slope of a Line Segment

What is the slope, m , of each line segment with the given end points?

- a) S(-3, 6) and T(5, 2)
- b) H(4, 3) and K(4, 8)
- c) M(-9, -7) and N(-1, -7)

Solution

Method 1: Use a Graph

Plot the points on grid paper. Count the rise and run.

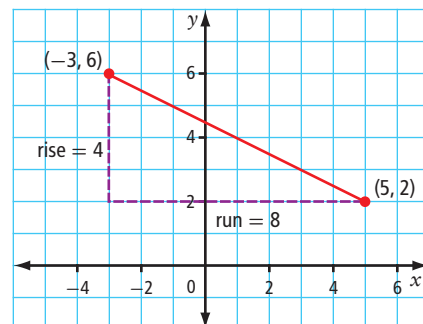
- a) Plot the points (-3, 6) and (5, 2).

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = -\frac{4}{8}$$

$$m = -\frac{1}{2}$$

Recall that a line that falls from left to right has a negative slope.



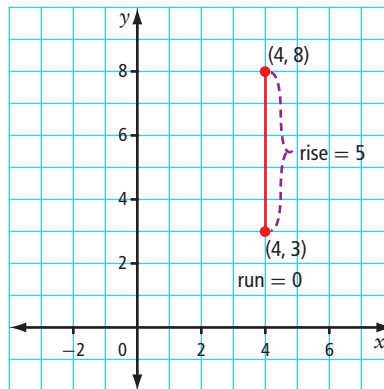
- b) Plot the points (4, 3) and (4, 8).

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{5}{0}$$

m is undefined

Division by zero is not defined in the real number system.

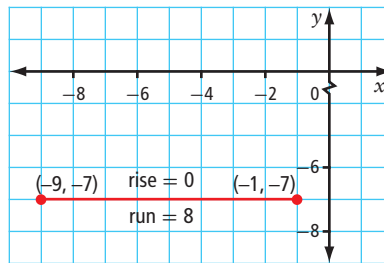


- c) Plot the points (-9, -7) and (-1, -7).

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{0}{8}$$

$m = 0$



Method 2: Use the Slope Formula

Label the points and substitute into the formula.

- a) S(-3, 6) T(5, 2) or T(5, 2) S(-3, 6)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - 6}{5 - (-3)}$$

$$m = \frac{-4}{8}$$

$$m = -\frac{1}{2}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 - 2}{-3 - 5}$$

$$m = \frac{4}{-8}$$

$$m = -\frac{1}{2}$$

- b) H(4, 3) K(4, 8)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8 - 3}{4 - 4}$$

$$m = \frac{5}{0}$$

m is undefined

It does not matter which point is selected as (x_1, y_1) ; the value of the slope is unaffected.

- c) N(-1, -7) M(-9, -7)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-7 - (-7)}{-9 - (-1)}$$

$$m = \frac{0}{-8}$$

$m = 0$

Your Turn

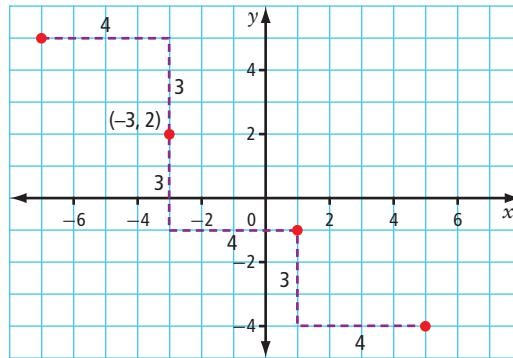
- Use a graph to determine the slope of the line segment with endpoints $P(-5, 6)$ and $Q(1, 10)$.
- Use the slope formula to determine the slope of the line segment with endpoints $W(2, -2)$ and $X(-5, 5)$.

Example 4 Use Slope to Graph a Line

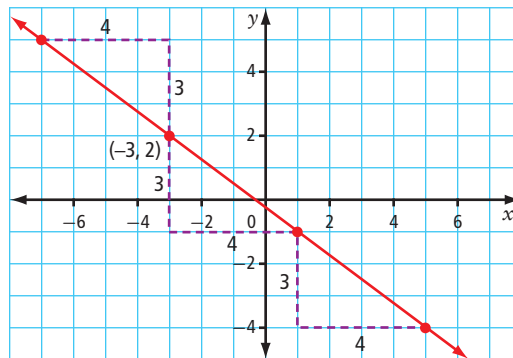
The point $(-3, 2)$ is on a line that has a slope of $-\frac{3}{4}$. List three other points on the line. Graph the line.

Solution

The slope of the line gives the rise and run from one point to another. Plot the point $(-3, 2)$. From this point, use the slope to locate other points on the line. Since the slope is negative, move down 3 units and right 4 units, or move up 3 units and left 4 units.



Three other points on the line are $(-7, 5)$, $(1, -1)$, and $(5, -4)$. Now draw the line through the points.



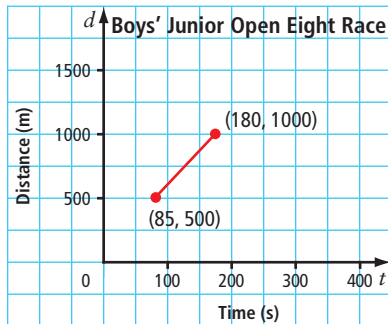
Move down 6 units and right 8 units from the point $(-3, 2)$. What do you notice? Explain.

Your Turn

The point $(-6, 1)$ is on a line that has a slope of $\frac{1}{3}$. List three other points on the line and graph the line.

Example 5 Slope as a Rate of Change

The Brentwood Regatta in Mill Bay, BC, is the largest junior rowing regatta hosted by a single school in North America. The races are all 1500 m in length. The graph shows the approximate times at the 500-m mark and the 1000-m mark for one of the boys' races. Determine the average rate of change for this portion of the race.



Solution

The slope of the line segment gives the ratio of the change in distance to the change in time. For this portion of the race,

$$\text{Rate of change} = \frac{\Delta d}{\Delta t}$$

$$\text{Rate of change} = \frac{(1000 - 500)}{(180 - 85)}$$

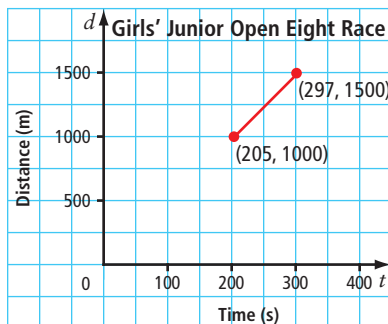
$$= \frac{500}{95}$$

To help you interpret the meaning of the rate of change, look at the units that are used. This rate of change represents the rowers' average speed.

The average rate of change is approximately 5.26 m/s.

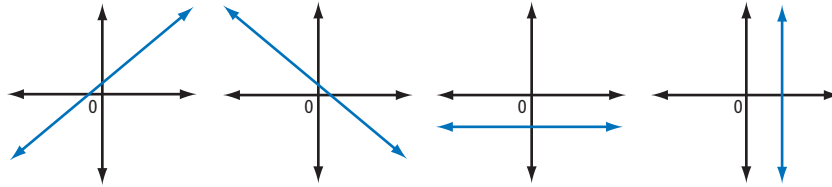
Your Turn

The graph shows the approximate times at the 1000-m mark and at the 1500-m mark for a rowing crew of the girls' junior open eight race at the Brentwood Regatta. Determine the average rate of change for this portion of the race.

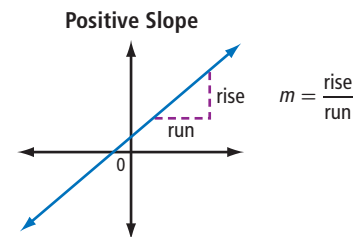


Key Ideas

- **Positive Slope** **Negative Slope** **Slope is 0.** **Slope is undefined.**



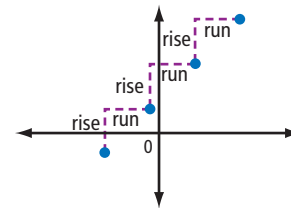
- The slope of a line is the ratio of the rise to the run.



- The slope of a line can be determined using two points on the line, (x_1, y_1) and (x_2, y_2) .

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

- If you know one point on the line, you can use the slope to find other points on the line.



- The slope gives the average rate of change.

Time (s)	Distance (m)
1	4
2	7
3	10
4	13
5	16
6	19
7	22

$$\text{Rate of change} = \frac{\Delta d}{\Delta t}$$

$$\text{Rate of change} = \frac{3}{1}$$

The average rate of change is 3 m/s.

Time (s)	Distance (m)
1	4
2	7
3	10
4	13
5	16
6	19
7	22

$$\text{Rate of change} = \frac{\Delta d}{\Delta t}$$

$$\text{Rate of change} = \frac{6}{2}$$

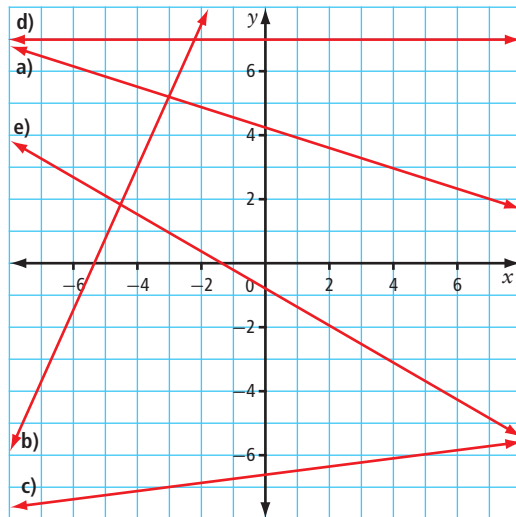
$$\text{Rate of change} = \frac{3}{1}$$

The average rate of change is 3 m/s.

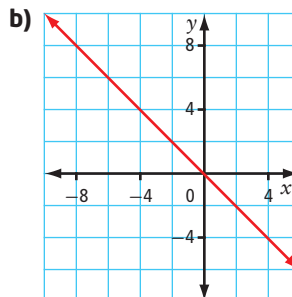
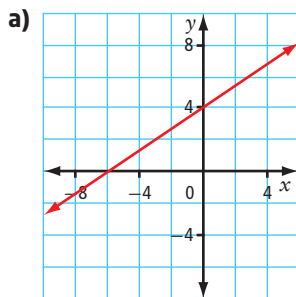
Check Your Understanding

Practise

1. For each line, identify the slope as positive, negative, or zero.



2. Determine the slope of each line.



3. Use the slope formula to determine the slope of the line passing through each pair of points.

a) $(2, 4), (9, 8)$

b) $(1, 12), (6, 12)$

c) $(-2, -5), (1, -7)$

d) $(3, 6), (-3, -12)$

e) $(-9, 16), (-9, 25)$

f) $(3.9, 10.6), (10.3, 13.8)$

4. Graph each line, given a point on the line and its slope.

a) $(2, 3), m = -\frac{2}{3}$

b) $(-4, -4), m = \frac{3}{5}$

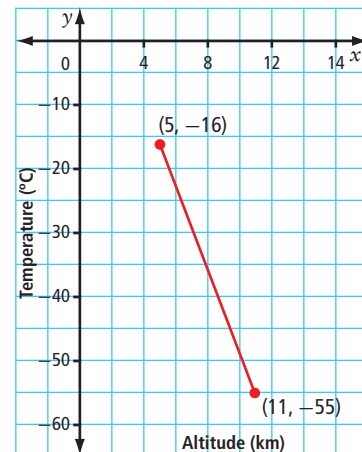
c) $(5, -3), m = -4$

d) origin, $m = \frac{1}{4}$

e) $(-1, 6), m = 0$

f) $(4, 0), m = \text{undefined}$

5. The graph shows the air temperature at different altitudes above Earth's surface. Determine the average rate of change.



Apply

6. Time and height values (seconds, metres) are given for the Free Fall Slide in Kenosee, SK. Determine your average rate of change if you go down this slide.



7. a) Create a graph showing the melting of a 75-cm-high snow bank in spring. Plot the height, in centimetres, of the snow bank on the vertical axis and time, in days, on the horizontal axis. Draw a segment with a slope of -3 , with one endpoint at $(0, 75)$ and the other endpoint along the horizontal axis.
- b) What does each point on the graph represent?
- c) What does the endpoint along the horizontal axis represent?
- d) Explain the meaning of the slope in this situation.
8. Marjorie is having a wheelchair ramp built at the front entrance of her house.
- a) The rise to Marjorie's front door is 18 in. What is the shortest run, x , allowed for the ramp if the building code in her town sets a maximum slope of $\frac{1}{12}$?
- b) How long is the ramp?
- c) How long would the ramp be if Marjorie decides to have a gentler slope of $\frac{1}{16}$?

9. This sign on the Trans-Canada Highway indicates that a steep hill is ahead.

- a) Written as a ratio, what is the slope of the hill, as indicated by the sign?
- b) Describe this slope in words.



10. The Penny Ice Cap glacier in Auyuittuq National Park on Baffin Island, NU is melting. In 2009, some areas of the glacier were about 1000 ft thick. It is estimated that if the glacier continues to melt at its current rate, the ice cap could be 967 ft thick by 2020. What is the estimated rate of change in thickness?



Did You Know?

Auyuittuq National Park was established in 1976. To honour the Penny Ice Cap, the people of Pangnirtung gave the park its name, Auyuittuq. This means The Land That Never Melts.

11. In 1800, the wood bison population in North America was estimated at 168 000. The population declined to only about 250 animals in 1893. That year, Wood Buffalo National Park was established on the Alberta/Northwest Territories border. In 2006, there were about 5600 bison in the park.

- a) What was the average rate of change in the bison population from 1800 to 1893? Describe the meaning of this rate.
- b) What was the average rate of change in the bison population from 1893 to 2006? Describe the meaning of this rate.



12. The mountain pine beetle is infesting many forests in British Columbia and Alberta. In 2004, about 1000 infested trees were counted in Alberta. In 2007, the number of infested trees in the province was about 2.8 million.

- a) Determine the average rate of change per year.
- b) What does this rate of change represent?
- c) What assumptions did you make? Predict the number of infested trees in Alberta in 2012.

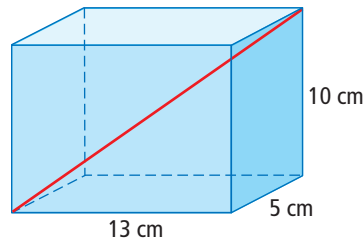
Did You Know?

The roof of the speed skating oval for the Vancouver 2010 Winter Olympics is made almost entirely of wood. The wood is from pine trees that had been infested by the mountain pine beetle.

13. Since the speed of light is faster than the speed of sound, you see lightning before you hear the sound of the thunderclap. If a thunderstorm is 1100 m away, the sound of thunder is heard in 3.2 s. If the storm is 4950 m away, the sound reaches you in 14.5 s.
- Determine the average rate of change, to the nearest metre per second.
 - What does this rate of change represent?
 - If you hear thunder 30 s after you see lightning, how far away is the storm?

Extend

14. What is the slope from the bottom front corner of the box to the top back corner of the box shown?

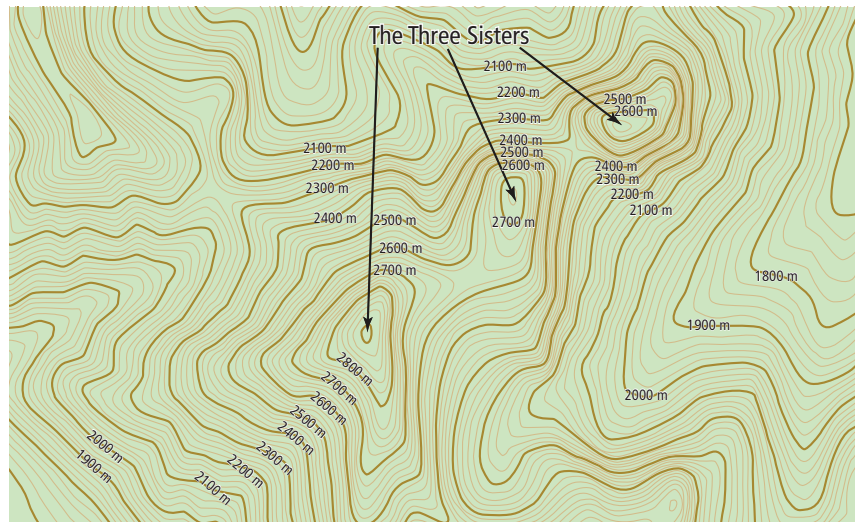


15. A metal cube has a side length of 5 cm. The cube is heated, causing it to expand uniformly to a side length of 5.01 cm.
- Determine the volume of the cube before and after heating.
 - Determine the average rate of change of the cube's volume with respect to its side length.
16. The points that a line passes through are given as algebraic expressions, $(-4x, 7x^2)$ and $(8x, 15x^2)$. Determine a simplified algebraic expression for the slope of the line.

Create Connections

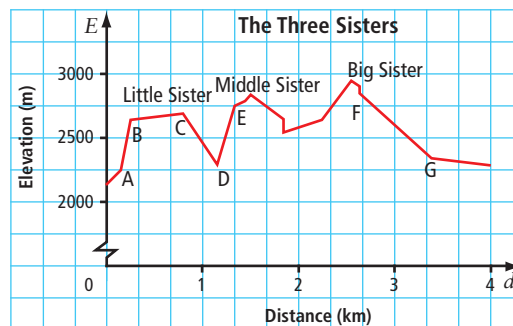
17. Explain why the slope of a line is *constant*. Use the terms *rise* and *run* in your explanation.
18. Matthew measured the slope of a ramp to be $\frac{1}{16}$. He then used trigonometry to determine the angle that the ramp made with the ground.
- Describe how he did this.
 - Determine the angle.

19. **MINI LAB** Topographic maps show hills and valleys using contour lines. Contour lines connect points of equal elevation. Contour lines are usually labelled with the elevation above sea level, as shown in this sample map.



- Step 1** The map shows contour lines and selected elevations for The Three Sisters mountains in southern Alberta. If the change in elevation between two adjacent lines is 20 m, what is the approximate height of each peak?

- Step 2** The diagram is a simplified cross profile, or side elevation, of the Three Sisters. Compare the diagram and the map.



Which peak on the map represents Big Sister?

- Step 3** The slope of a mountain will vary from the bottom to the top. Explain how to estimate the average slope of Middle Sister.
- Step 4** Suppose the greatest risk for an avalanche occurs when the slope is between 0.5 and 1.7. From the diagram, determine the approximate slope of the following sections of the Three Sisters: AB, BC, CD, DE, and FG. Which section(s) pose the greatest avalanche risk?
- Step 5** The risk of an avalanche is reduced if the slope of the mountainside is less than 0.5 or greater than 1.7. Explain why this statement is true.

WWW Web Link

To learn more about avalanche awareness and safety, go to www.mhrmath10.ca and follow the links.

6 Review

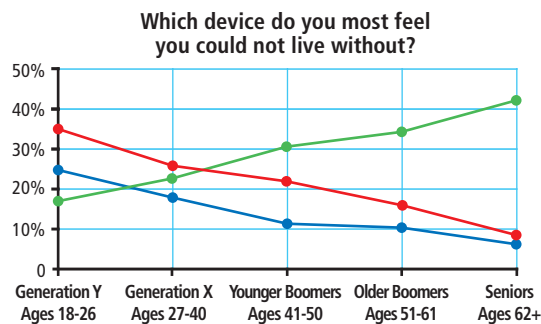
6.1 Graphs of Relations, pages 268–278

1. Consider the following scenario.

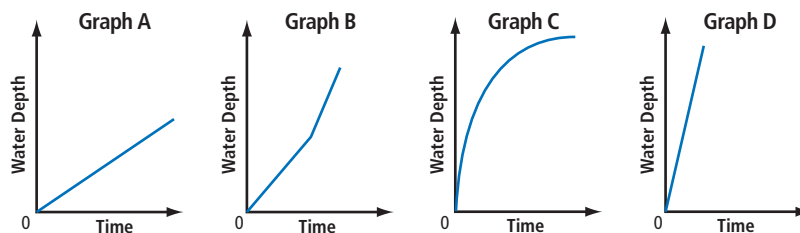
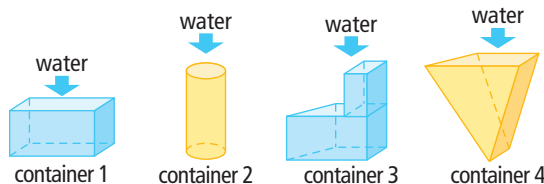
Your car's gas tank is about half full. You drive to meet Ahmed and Jackie at the movie theatre which is on the way to Ahmed's house. There is a gas station located on the edge of town just past Ahmed's house. After the movie, you drive your friends home. First you drop off Ahmed. On your way to Jackie's house, you realize that you have only one quarter of a tank of gas left, so you stop at the gas station and fill up. You drive Jackie home and then you drive to your home.

- Draw a map to represent this scenario. Then, based on your map, sketch a graph showing your distance from home versus time.
- Sketch a graph showing the amount of gas in the car's gas tank versus time.

2. The lines on the graph represent three devices: televisions, computers, and cell phones. Which line do you think represents each device? Explain your answer.



3. The graphs show the depth of water as various containers are being filled. Match each container to its graph. Explain your choice.

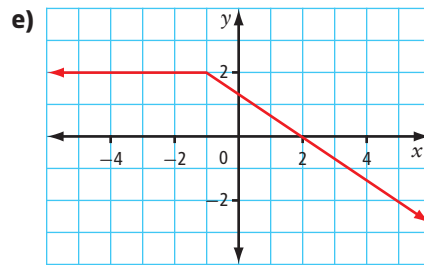


6.2 Linear Relations, pages 279–291

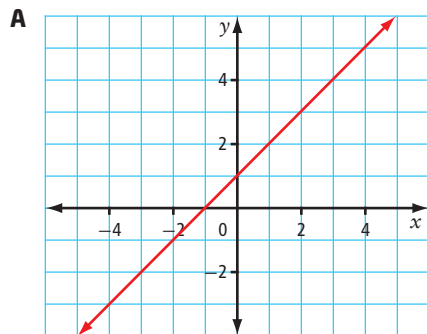
4. Is each relation linear or non-linear? Explain your choice.
- the orbit of a planet around the sun on an elliptical path, in terms of time and distance of the planet from the sun
 - $(-10, -5), (-5, 0), (0, 5), (5, 10)$
 - $y = 2x^2 + 3x - 1$

d)

x	y
82	16
91	20
100	25
109	31
118	38



5. Which of the following represent the same relation?



B

x	y
-2	0
0	2
2	4
4	6

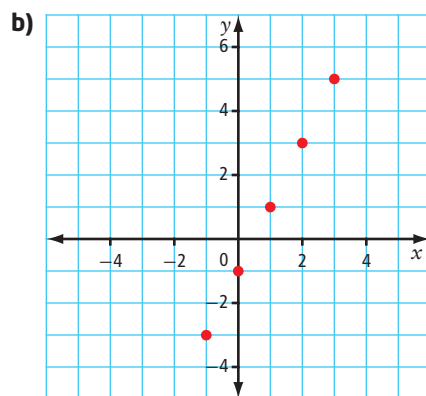
- One number is double another.
 - $(-2, 0), (0, 2), (2, 4), (4, 6)$
6. Katya wants to sell a camper trailer.
- The cost to place an advertisement in a newspaper is \$37.95 for three lines of text and a picture, plus \$7 for each additional line of text. Consider the relation of total cost versus number of lines of text.
- Is this relation linear? Explain.
 - Identify the dependent variable and the independent variable.
 - Create a set of ordered pairs to represent 3, 4, 5, 6, and 7 lines of text in Katya's advertisement.
 - Is this relation discrete or continuous?
 - Graph the relation.



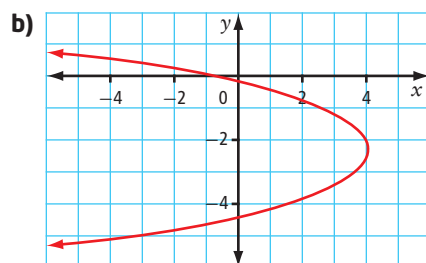
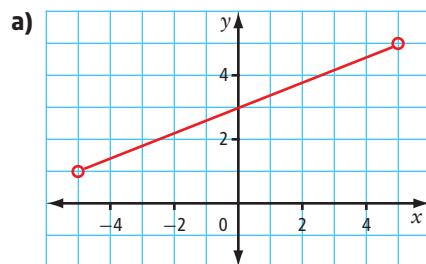
6.3 Domain and Range, pages 292–304

7. State the domain and range of each relation.

a) $(-9, 5), (-5, 5), (0, 5), (0, 8), (2, 8)$



8. Express the domain and the range of each relation as a number line, in interval notation, and in set notation.



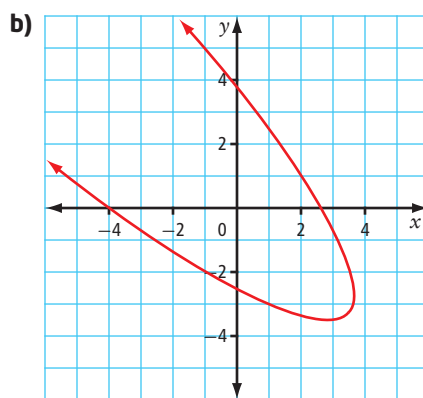
6.4 Functions, pages 305–314

9. The circumference of a circle is given by the function $C(d) = \pi d$, where d is the diameter of the circle. Write this function as a linear formula in two variables.
10. The formula for volume of a cube with radius r is $V = r^3$. Write this formula using function notation.

11. Which relations are functions? Explain how you know.

a)

x	y
1	1
2	2
3	3
4	4
5	5



c)

Hair Colour	Gender
brown	male
black	male
black	female
blonde	female
red	female

d) (2.3, 5.1), (8.6, 9.4), (8.6, 9.2), (9.5, 10.0)

12. The function $M(E) = \frac{E}{2.7}$ can be used to approximate your weight, M , in kilograms, on Mars, where E , in kilograms, is your weight on Earth.

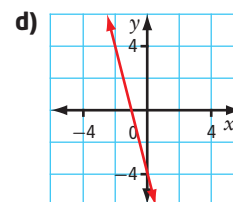
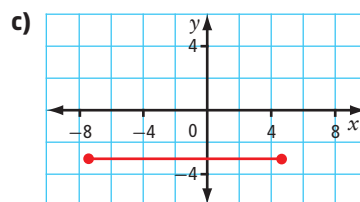
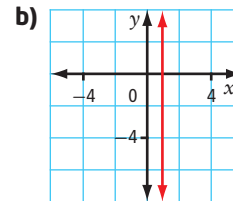
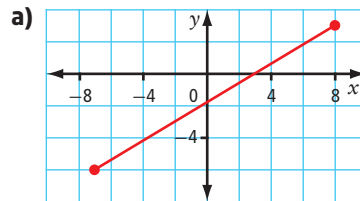
- Suppose you weigh 66 kg on Earth. How much would you weigh on Mars?
- How much would a Martian who weighs 14 kg on Mars weigh on Earth?

13. For her local Run for the Cure, Amber donates \$50 of her own money. She also collects \$25 pledges. The function $P(n) = 25n + 50$ represents the total funds she contributes.

- Determine an appropriate domain and range. Then, use a table of values to graph the function.
- Determine the value of $P(8)$. Explain the meaning of your answer.
- Prizes are awarded to students who collect more than \$675. How many pledges must Amber collect to receive a prize?
- Explain why this situation depicts a function.

6.5 Slope, pages 315–329

14. Determine the slope of each line segment or line.



Did You Know?

Blackstrap Mountain was made in 1969–1970 for the 1971 Canada Winter Games in Saskatoon.

15. Blackstrap Mountain in Blackstrap Provincial Park, SK, is an artificial hill. The hill is approximately 90 m high with an average rate of change from top to bottom of $\frac{1}{5}$. If the coordinates $(0, 90)$ represent the top of Blackstrap Mountain, what coordinates represent its bottom?
16. Use the slope formula to determine the slope of the line passing through each pair of points.
- a) $(-3, 8)$ and $(13, 12)$ b) $(5.6, -8.2)$ and $(-0.4, 3.8)$
17. Carl is running in a 10-km race.
- Create a graph showing Carl's progress during the race. Plot distance, in kilometres, as the dependent variable and time, in minutes, as the independent variable. Start your graph at the point $(0, 0)$ and draw a line with a slope of $\frac{1}{4}$.
 - What does each point on the graph represent?
 - At what point does your line end? What does this point represent?
 - Explain the meaning of slope in this situation.
18. In Manitoba, the number of people aged 12 and older who have asthma was 63 028 in 1996 and 73 427 in 2005.
- What was the average rate of change per year?
 - What does this rate of change represent?
 - If the number of people in Manitoba living with asthma continues to increase at this rate, how many people in Manitoba will have asthma in 2017?

6 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

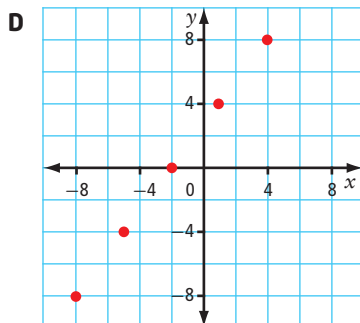
1. Which of these relations is *not* linear?

A $y + 3 = \frac{2}{3}x$

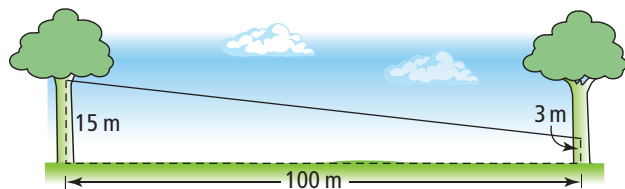
B (2, 3), (4, 6), (6, 9), (8, 12)

C

<i>a</i>	<i>d</i>
-2	3
-1	4
0	6
1	8
2	11



2. A zip line is attached to two trees as shown. What is the slope of the line?



A $\frac{10}{12}$

B $\frac{3}{10}$

C $-\frac{6}{5}$

D $-\frac{3}{25}$

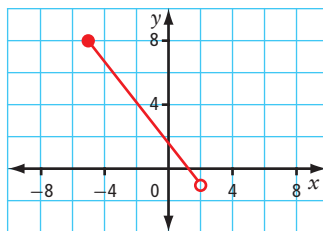
3. Determine the domain of the relation in the graph.

A $\{-5, -4, -3, -2, -1, 0, 1, 2\}$

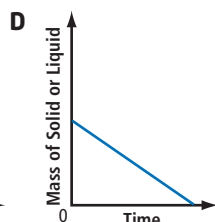
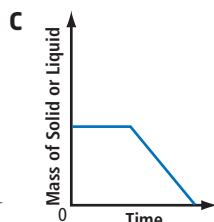
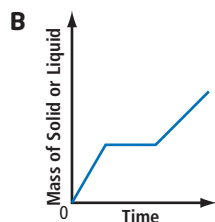
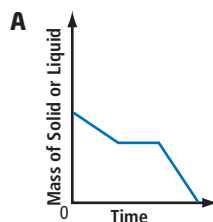
B $[-5, 2)$

C $\{x \mid -5 < x \leq 2, x \in \mathbb{R}\}$

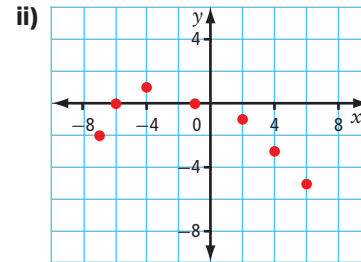
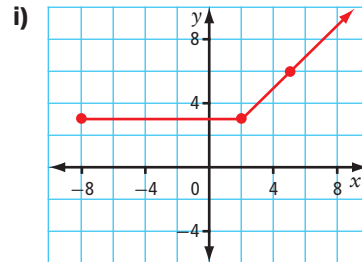
D all numbers between -5 and 2 , inclusive



4. As ice is heated, it changes state from solid to liquid and then to gas. Which graph could represent the amount of solid or liquid remaining as heat is applied to a block of ice?



5. How many of these relations are functions?



iii) $(1, 8), (2, 12), (5, 12), (7, 18)$

iv) $(-9, -9), (-3, -3), (-1, 0), (9, 9)$

A 1

B 2

C 3

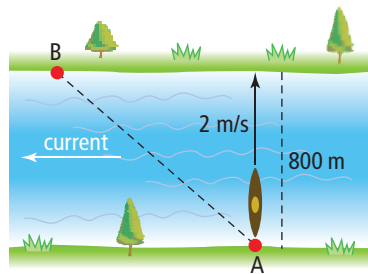
D 4

Short Answer

6. a) Determine the slope of the line that passes through the points $(-2, -3)$ and $(5, 4)$.

b) Give the coordinates of another point on this line.

7. The Slave River rapids between Fort Fitzgerald, AB, and Fort Smith, NT, are popular for experienced whitewater kayakers. To cross an 800-m-wide section of the river, a kayaker points his kayak perpendicular to the current. He maintains this position as he paddles at a rate of 2 m/s. However, the current affects the kayaker's actual course. He reaches the other side in 6 min 40 s (400 s) at a point 2000 m downstream from his original position.



a) What is the kayaker's average rate of change from point A to point B?

b) In the context of this question, what does this rate represent?



8. A boat travelling at 5 m/s begins to accelerate. Its new speed, S , in metres per second, is modelled by the function $S(t) = 5 + 2.5t$, where t is the length of time, in seconds, that it accelerates.

- What does $S(6)$ represent?
- Determine the value of $S(6)$.
- How long does the boat accelerate in order to reach a speed of 23 m/s?

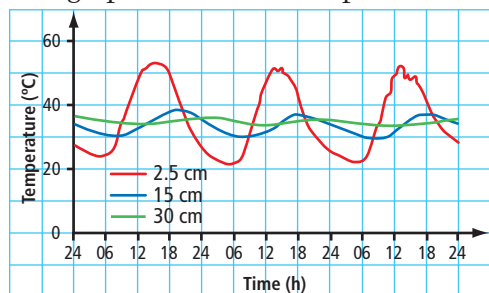
9. Select and graph the function that represents continuous data.

Function A: $f(x) = 2x - 5$, where the domain is $\{x \in \mathbb{R}\}$

Function B: $h(x) = 3x + 4$, where the domain is $\{-2, -1, 0, 1, 2\}$

Extended Response

10. The graph shows soil temperature over 3 days at different depths.



- All three curves have the same domain. Express this domain in words.
 - Express the range for the curve representing a depth of 2.5 cm using a number line.
 - Express the range for the curve representing a depth of 15 cm using interval notation.
 - Express the range for the curve representing a depth of 30 cm using set notation.
 - What conclusions can you make from these relations?
11. Dan is training for the Canadian Orienteering Championships in Spruce Woods Provincial Park, MB. Create a distance-time graph showing Dan's distance from the start as he completes the practice course shown.

