CHAPTER 5

Polynomials

Historically, mathematics has involved simplifying complex concepts and calculations so that they can be done more quickly and easily. Take, for example, the Chinese work, *Arithmetic in Nine Sections*, written around 200 B.C.E. It begins, "three sheafs of good crop, two sheafs of mediocre crop, and one sheaf of bad crop are sold for 29 dou." Today, we would represent this situation with the polynomial equation $3g + 2m + b = 29$, where the variables g, m, and b represent the prices for the *good*, *mediocre*, and *bad* crops. Problem situations that involve factoring polynomials were originally represented by the ancient Greeks using geometric constructions. Today, those problems are still represented using diagrams. They can also be represented using algebraic expressions.

Big Ideas

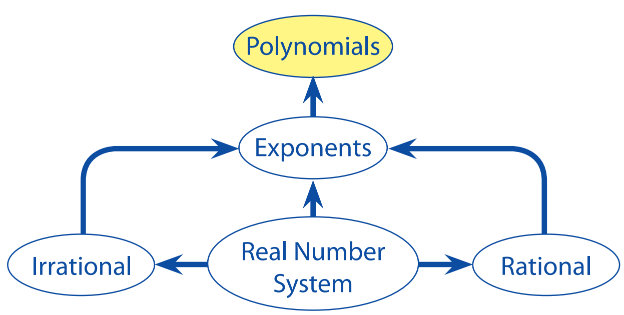
When you have completed this chapter, you will be able to …

- identify the factors of whole numbers and algebraic expressions
- multiply polynomials concretely, pictorially, and algebraically
- factor polynomials concretely, pictorially, and algebraically

Key Terms

polynomial
binomial
distributive property
trinomial
greatest common factor
least common multiple
difference of squares

Your Algebra and Number Organizer



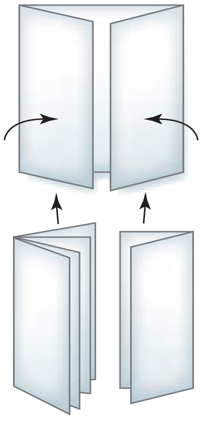
Recreation Facilities Manager

Recreation Facilities Managers are involved in their communities through maintaining and often improving recreation facilities such as pools, arenas, and fitness and dance studios. They schedule the use of the facilities by individuals and community groups. They also oversee the financial aspect of the facilities, as well as the staff. They may also make suggestions for any renovations or facility improvements.

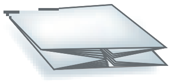
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 5.

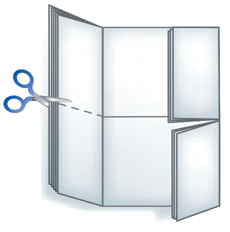
1. Fold a sheet of 11×17 paper as shown. Then, fold three sheets of 8.5×11 paper. Attach two pages inside the left flap and one page inside the right flap.



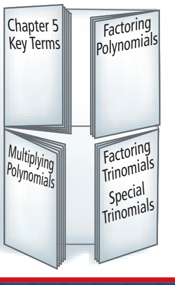
2. Fold in half.



3. Cut tabs along the fold lines.



4. Label as shown. Label the back What I Need to Work On, and Project Ideas and Questions. Label the inside centre Algebra Tile Models.



5.1 Multiplying Polynomials

Focus on ...
- multiplying polynomials
- explaining how multiplication of binomials is related to area and to the multiplication of two-digit numbers

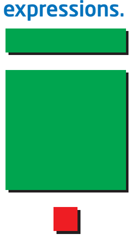
Geometric abstraction is a form of abstract art based on the use of geometric shapes. Piet Mondrian is one of many artists who used this style of painting. Mondrian's art has influenced designers of everything from cups to buildings.

In what ways can you relate **polynomial** multiplication to Mondrian's painting?

polynomial
- a sum of monomials
- for example, $x + 5$, $2a^2 - 6ab + 18b^2$

Materials
- algebra tiles

You can use algebra tiles to model algebraic expressions.

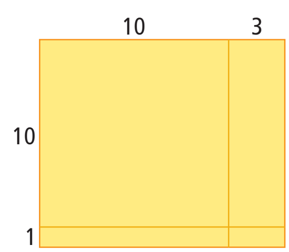


The same tiles in white represent negative quantities.

Investigate Multiplying Polynomials

1. Complete the multiplication (13)(11).

2. a) You can express 13 as the sum 10 + 3, and 11 as the sum 10 + 1. Complete an area model with the dimensions 10 + 3 and 10 + 1.

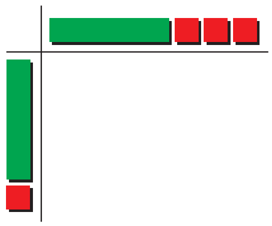


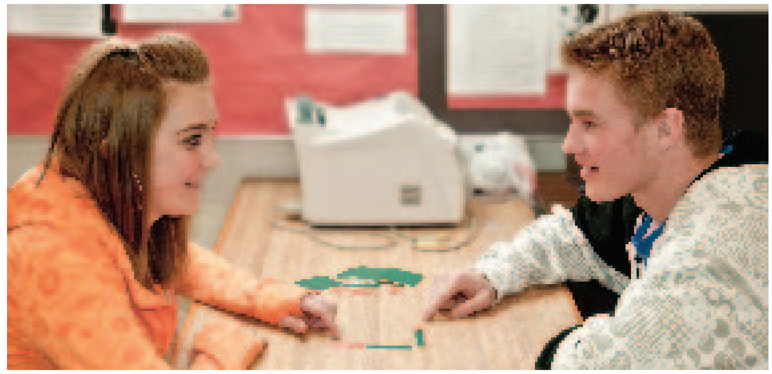
b) How does your model show the factors 10 + 3 and 10 + 1?

c) What is the product?

3. Compare the methods in step 1 and step 2. How are they similar? How are they different?

4. a) Use algebra tiles to determine the product $(x + 3)(x + 1)$. Sketch your model.





b) How does your model show the factors $x + 3$ and $x + 1$?

c) What is the product?

d) How is your model similar to and different from the model you used in step 2?

5. Use algebra tiles to determine each product.

a) $(x + 5)(x - 2)$

b) $(p + 4)(p + 4)$

c) $(2x + 4)(3x - 1)$

6. **Reflect and Respond** Discuss the following questions with a partner.

a) Use your answers from step 5 to look for patterns that relate the factors to the products.

b) Use the patterns you found to explain how you could multiply **binomials** without using algebra tiles.

binomial

- a polynomial with two terms
- for example, $x + 3$, $2x - 5y$

Link the Ideas

You can make connections between multiplying whole numbers and multiplying polynomials. Multiply 42 by 26 using the **distributive property**.

$$\begin{aligned}(42)(26) &= (40 + 2)(20 + 6)\\ &= 40(20 + 6) + 2(20 + 6)\\ &= (40)(20) + (40)(6) + (2)(20) + (2)(6)\\ &= 800 + 240 + 40 + 12\\ &= 1092\end{aligned}$$

distributive property

- the rule that states $a(b + c) = ab + ac$
- for example, $40(20 + 6) = (40)(20) + (40)(6)$

Example 1 Multiply Binomials

Multiply.

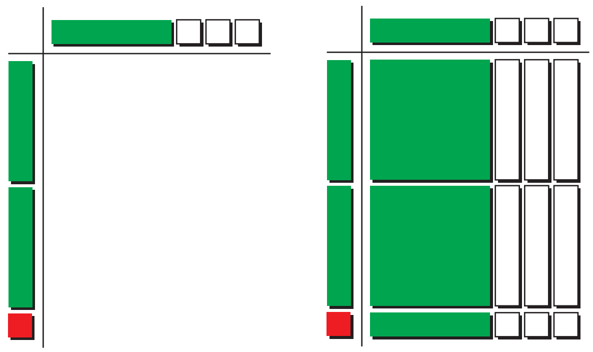
a) $(x - 3)(2x + 1)$ **b)** $(x - 2y)(x - 4y)$

Solution

a) Method 1: Use Algebra Tiles

Use algebra tiles to show the dimensions $x - 3$ and $2x + 1$. Then, complete a rectangle that has these dimensions.

To create an area model, you place the negative tiles on top of the positive tiles. Explain why.



There are two x^2-tiles, six negative x-tiles, one positive x-tile, and three negative 1-tiles in the rectangle.
Therefore, $(x - 3)(2x + 1) = 2x^2 - 5x - 3$.

Method 2: Use the Distributive Property

Multiply each term in the first binomial by each term in the second binomial. Then, combine like terms.

$$\begin{aligned}(x - 3)(2x + 1) &= x(2x + 1) - 3(2x + 1)\\ &= (x)(2x) + (x)(1) + (-3)(2x) + (-3)(1)\\ &= 2x^2 + 1x - 6x - 3\\ &= 2x^2 - 5x - 3\end{aligned}$$

b) $$\begin{aligned}(x - 2y)(x - 4y) &= x(x - 4y) - 2y(x - 4y)\\ &= x^2 - 4xy - 2xy + 8y^2\\ &= x^2 - 6xy + 8y^2\end{aligned}$$

Check:
You can verify your work by substituting values for the variables x and y. For example, substitute $x = 5$ and $y = 1$.

Left Side	Right Side
$(x - 2y)(x - 4y)$	$x^2 - 6xy + 8y^2$
$= [5 - 2(1)][5 - 4(1)]$	$= (5)^2 - 6(5)(1) + 8(1)^2$
$= (5 - 2)(5 - 4)$	$= 25 - 30 + 8$
$= (3)(1)$	$= 25 - 22$
$= 3$	$= 3$

Left Side = Right Side

Your Turn

Determine each product.

a) $(x - 3)(x - 5)$ **b)** $(5m - 1)(2m - 6)$

Example 2 Multiply a Binomial and a Trinomial

Multiply the following binomial and **trinomial**.
$(x + 2)(2x^2 - 5x + 1)$

trinomial
- a polynomial with three terms
- for example, $x^2 + 3x - 1$, $2x^2 - 5xy + 10y^2$

Solution

$$
\begin{aligned}
&(x + 2)(2x^2 - 5x + 1)\\
&= x(2x^2 - 5x + 1) + 2(2x^2 - 5x + 1)\\
&= 2x^3 - 5x^2 + x + 4x^2 - 10x + 2\\
&= 2x^3 - x^2 - 9x + 2
\end{aligned}
$$

Multiply each term in the binomial by each term in the trinomial. Then, combine like terms.

Your Turn

Determine each product.

a) $(r - 4)(3r^2 + 8r - 6)$
b) $(5x - 3)(2x^2 - 6x + 12)$

Example 3 Perform Operations on Products of Polynomials

Simplify.

a) $(x + 1)(5x + 3) + 3(2x + 4)(6x - 2)$
b) $(3w - 2)(4w + 5) - (w - 7)(2w + 3)$

Solution

a)
$$
\begin{aligned}
&(x + 1)(5x + 3) + 3(2x + 4)(6x - 2)\\
&= x(5x + 3) + 1(5x + 3) + 3[2x(6x - 2) + 4(6x - 2)]\\
&= 5x^2 + 3x + 5x + 3 + 3(12x^2 - 4x + 24x - 8)\\
&= 5x^2 + 3x + 5x + 3 + 36x^2 - 12x + 72x - 24\\
&= 41x^2 + 68x - 21
\end{aligned}
$$

When you have three factors, you can multiply in any order. What are some other ways you could multiply $3(2x + 4)(6x - 2)$?

b)
$$
\begin{aligned}
&(3w - 2)(4w + 5) - (w - 7)(2w + 3)\\
&= (3w)(4w + 5) + (-2)(4w + 5) - [w(2w + 3) - 7(2w + 3)]\\
&= 12w^2 + 15w - 8w - 10 - (2w^2 + 3w - 14w - 21)\\
&= 12w^2 + 15w - 8w - 10 - 2w^2 - 3w + 14w + 21\\
&= 10w^2 + 18w + 11
\end{aligned}
$$

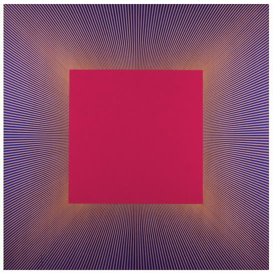
Your Turn

Multiply and then combine like terms.

a) $(x + 3)(5x - 2) + 4(x - 1)(2x + 5)$
b) $2(3x - 2) - (4x + 7)(2x - 5)$

Example 4 Apply Binomial Multiplication

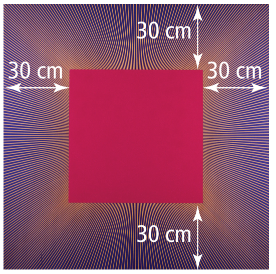
The painting shown is *Deep Magenta Square* by Richard Anuszkiewicz. It can be used to represent binomial multiplication. The length of the red square in the painting is unknown. The width of the border around the square is 30 cm.



a) What polynomial expression represents the total area of the painting?

b) What is the total area of the painting if the red square has an area of 3600 cm^2?

Solution



a) Let x represent the length of the red square. The length of the painting can be represented by $x + 30 + 30 = x + 60$.
The area of the painting can be represented by the polynomial expression
$(x + 60)(x + 60) = x^2 + 120x + 3600$.

b) If the red square has an area of 3600 cm^2, the side length of the red square is $\sqrt{3600} = 60$. Substitute this value into the expression $(x + 60)(x + 60)$ or the expression $x^2 + 120x + 3600$.

$$\begin{aligned}(60 + 60)(60 + 60) &= (120)(120)\\ &= 14\,400\end{aligned}$$

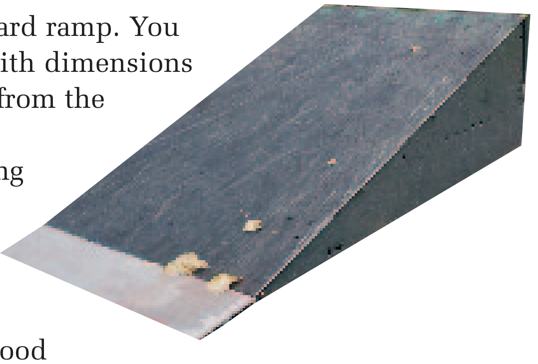
or

$$\begin{aligned}(60)^2 + 120(60) + 3600 &= 3600 + 7200 + 3600\\ &= 14\,400\end{aligned}$$

The area of the painting is 14 400 cm^2.

Your Turn

You are building a skateboard ramp. You have a piece of plywood with dimensions of 4 ft by 8 ft. You cut x ft from the length and width.



a) Sketch a diagram showing the cuts made to the piece of plywood. Label the dimensions.

b) What is the area of the remaining piece of plywood that will be used for the ramp?

Key Ideas

- You can use the distributive property to multiply polynomials. Multiply each term in the first polynomial by each term in the second polynomial.

$$\begin{aligned}(3x - 2)(4x + 5) &= (3x)(4x + 5) - 2(4x + 5)\\ &= 12x^2 + 15x - 8x - 10\\ &= 12x^2 + 7x - 10\end{aligned}$$

$$\begin{aligned}(c - 3)(4c^2 - c + 6) &= c(4c^2 - c + 6) - 3(4c^2 - c + 6)\\ &= 4c^3 - c^2 + 6c - 12c^2 + 3c - 18\\ &= 4c^3 - 13c^2 + 9c - 18\end{aligned}$$

Check Your Understanding

Practise

1. Multiply using algebra tiles.

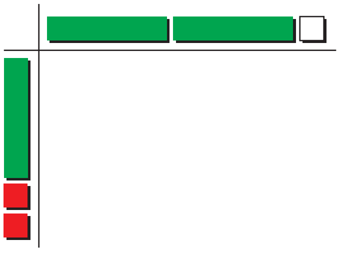
a) $(x - 2)(x + 3)$ **b)** $(3x - 4)(2x - 1)$

c) $(x - 5)(x - 2)$ **d)** $(x + 3)^2$

$(x + 3)^2$ means $(x + 3)(x + 3)$.

e) $(x + 4)(x + 7)$ **f)** $(2x - 5)(x - 3)$

2. a) What product does the algebra tile model show?



b) What are the dimensions of the model?

3. Multiply using the distributive property.

a) $(x + 5)(x - 2)$ **b)** $(x - 3)^2$

c) $(c - d)(c + d)$ **d)** $(4x + y)(x + y)$

e) $(y + 3)^2$ **f)** $(4j + 2k)(6j - 3k)$

4. Use the distributive property to determine each product.

a) $x(3x^2 - 5x + 8)$ **b)** $a(7b^2 + b - 1)$

c) $(x - 3)(6x^2 - 4x - 12)$ **d)** $(2x - 1)(5x^2 + 4x - 5)$

e) $(4s^2 + s)(3s^2 - 2s + 6)$ **f)** $(2y^2 + 3y - 1)(y^2 + 4y + 5)$

5. Match each binomial multiplication on the left with a trinomial on the right.

a) $(x + 1)(x - 2)$	A $x^2 + 13x + 36$
b) $(x - 3)(x - 4)$	B $x^2 - x - 2$
c) $(x - 1)^2$	C $x^2 - 2x - 1$
d) $(x + 4)(x - 3)$	D $x^2 + x - 12$
e) $(x - 3)(x - 5)$	E $x^2 + 6x + 9$
f) $(x + 3)^2$	F $x^2 - 2x + 1$
g) $(x + 9)(x + 4)$	G $x^2 - 9x + 18$
h) $(x - 6)(x - 3)$	H $x^2 - 7x + 12$
	I $x^2 - 7x - 12$
	J $x^2 - 8x + 15$

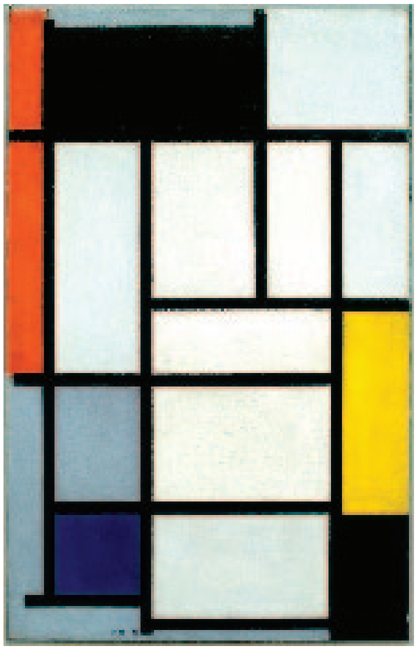
6. Multiply. Then, combine like terms.
   a) $(4n + 2) + (2n - 3)(3n - 2)$
   b) $(f + 7)(2f - 4) - (3f + 1)^2$
   c) $(b - 2d)(5b - 3d) + (b + d)(4b + d)$
   d) $(4x - 2)(3x - 5) + 2(7x + 5)(2x - 6)$
   e) $3(5a + 3c)(2a - 3c) - (4a + c)^2$
   f) $(y^2 - 5y - 6)(4y^2 + 6y + 1)$

Apply

7. The painting shown is by Métis artist Leah Marie Dorion from Prince Albert, Saskatchewan. It is called *Hawk Woman* (2006). The frame is 2 in. wide on each side of the square painting. Write an expression to represent the dimensions and area of the painting. Multiply, and then combine like terms.

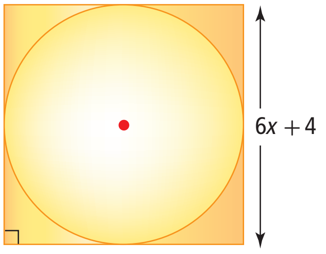
8. **Unit Project** Sketch an area model or an algebraic model to represent each multiplication. Use specific polynomials for each multiplication. Label your diagrams. Then, write the result of each multiplication as an equation.
   a) (monomial)(binomial)
   b) (binomial)(binomial)
   c) (binomial)(trinomial)

9. **Unit Project** Use an arrangement of algebra tiles to show combining like terms of polynomials. Arrange them artistically. Use the style of Piet Mondrian's paintings, shown here and on page 204. Write the corresponding algebraic equation that summarizes your result.



10. Darien trimmed a square photo to fit into a rectangular frame. He cut 7 cm from one side and 4 cm from the other. Let x represent the side length of the original square photo. Write an expression for the area of the trimmed photo. Multiply, and then combine like terms.

11. A circle is inset into a square with a side length of $6x + 4$, as shown. Write an expression to represent the area of the circle. Multiply, and then combine like terms.



12. Bryan was asked to multiply two binomials. He completed the following work.

$(p + 3)(p + 7)$	$= p^2 + 7p + 3p + 21$	Step 1
	$= p^2 + 10p + 21$	Step 2
	$= 11p^2 + 21$	Step 3

a) Is Bryan's work correct? If not, which step is incorrect?

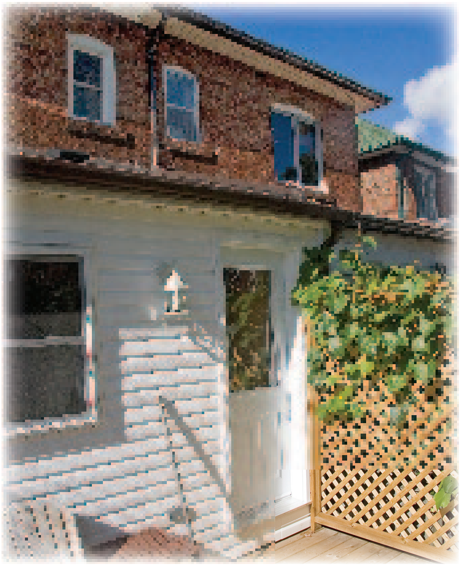
b) Choose any number for p. Determine whether the following equation is true.
$(2p - 3)(p + 4) = 2p^2 - 5p - 12$

13. The Li family has a house with a length of 13 m and a width of 9 m. Due to lot restrictions, they can make an addition of only y metres to the width and x metres to the length.

a) Sketch a diagram of the area of the house. Label the dimensions.

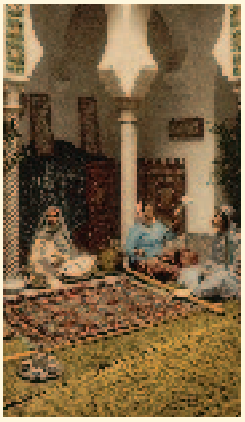
b) Write an expression for the area of the house, including the addition.

c) Calculate the area if $x = 1$ m and $y = 2$ m.



Did You Know?

Authentic oriental rugs are hand woven and knotted. They are produced primarily in Asia, the Middle East, and India. The rugs are made for decorative, practical, and sometimes spiritual purposes.



14. a) Susan is making a rectangular area rug with a similar design to the square rug she made earlier. What are the dimensions if the new rug is 2 ft longer and 1 ft narrower than the square rug?

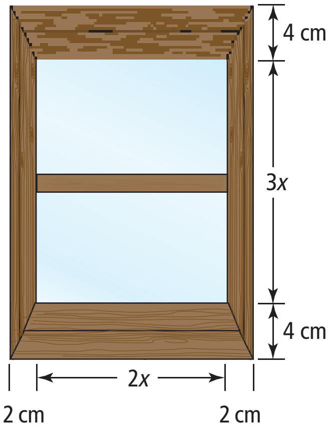
b) Write an expression for the area of the new rug.

c) If the square rug is 3 ft by 3 ft, which rug has the greater area? Show your work.

15. Vera is installing a kitchen window that has a height-to-width ratio of 3 : 2. The window frame adds 4 cm to the width and 8 cm to the height.

a) Write a polynomial expression that represents the total area of the window, including the frame. Multiply and combine like terms.

b) Calculate the area when $x = 12$ cm.



16. André multiplied the expression $(2x - 4)(3x + 5)$. When he checked his answer, he discovered an error.

$$\begin{aligned}(2x - 4)(3x + 5) &= 2x(3x + 5) - 4(3x + 5)\\ &= 6x^2 + 10x - 12x + 20\\ &= 6x^2 - 2x + 20\end{aligned}$$

Check:
Let $x = 4$.

Left Side	Right Side
$(2x - 4)(3x + 5)$	$6x^2 - 2x + 20$
$= [2(4) - 4][3(4) + 5]$	$= 6(4)^2 - 2(4) + 20$
$= (8 - 4)(12 + 5)$	$= 96 - 8 + 20$
$= (4)(17)$	$= 108$
$= 68$	

a) Explain how André knew that he had made an error.

b) Explain the error and how to correct it.

Extend

17. The average number of burgers, b, sold at The Burger Barn daily can be represented by $b = 550 - 100p$, where p is the price of a burger, in dollars.

a) How does the average number of burgers sold change as the price of a burger increases?

b) Solve the equation for p.

c) The revenue from burger sales can be represented by $R = np$, where R is the total revenue, in dollars, and n is the number of burgers sold. Substitute your expression for p from part b). Then, multiply to get an expression for the daily burger revenue.

Create Connections

18. a) Choose four consecutive whole numbers. Multiply the first and last numbers. Multiply the second and third numbers. Repeat this multiplication with several different groups of four consecutive whole numbers. What pattern to you notice?

b) Let n represent your first number. What algebraic expressions represent your second, third, and fourth numbers?

c) Use algebraic multiplication to show that your pattern in part a) is always true.

19. The product of 45 and 34 can be thought of as $(40 + 5)(30 + 4)$. You can represent $40 + 5$ as $4t + 5$, where t represents 10.

a) What expression could represent $30 + 4$?

b) Use binomial multiplication of the algebraic expression. Substitute to find the product of 45 and 34.

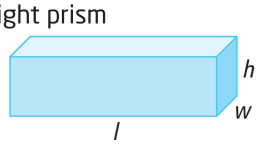
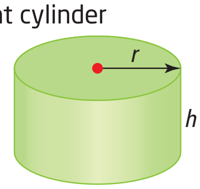
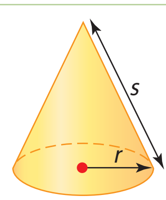
5.2 Common Factors

Cubism is an early 20th-century art style. It was pioneered by artists Pablo Picasso and Georges Braque. In cubist artworks, natural forms are broken up. The pieces are reassembled into simplified 3-D shapes. The idea is to portray an object from multiple points of view at the same time. The painting shown is Picasso's *Factory, Horta de Ebbo* (1909).

Focus on …

- determining prime factors, greatest common factors, and least common multiples of whole numbers
- writing polynomials in factored form
- applying your understanding of factors and multiples to solve problems

When calculating the surface area of a 3-D shape, the same formula can often be used in different ways. For example, the formula for the surface area of a right prism, a right cylinder, or a right cone can be written in two forms:

Shape	Formula #1	Formula #2
Right prism 	$SA = 2lw + 2lh + 2wh$	$SA = 2(lw + lh + wh)$
Right cylinder 	$SA = 2\pi r^2 + 2\pi rh$	$SA = 2\pi r(r + h)$
Right cone 	$SA = \pi r^2 + \pi rs$	$SA = \pi r(r + s)$

Compare the two surface area formulas for each shape. What is similar about the two formulas? What is different about the two formulas? Is there one surface area formula for each shape that you prefer to use? Explain.

Investigate Common Factors

1. a) Write the number 30 as a product of prime factors. How do you know the factors are prime?
   b) Can you write the number 1 as a product of prime factors? Explain why or why not.
   c) Can you write the number 0 as a product of prime factors? Explain why or why not.

2. Write each of the following pairs of numbers as a product of prime factors. Identify the **greatest common factor (GCF)** of each pair.
   a) 60 and 48 b) 25 and 40
   c) 16, 24, and 36

3. Identify the **least common multiple (LCM)** of each pair of numbers.
   a) 12 and 15 b) 20 and 25
   c) 18 and 32

4. a) What is the GCF of 72 and 48?
   b) Write each number as the product of two factors, where one factor is the GCF.
   c) Explain how you determined the second factor.

5. a) Identify the GCF of each pair of terms.
   6^2 and 6^3 8^4 and 8^7 x^5 and x^2
   b) Compare the methods you used to identify the GCF of whole numbers and the GCF of variable terms. What are the similarities and differences between the methods?

6. a) Identify the GCF of x^5 and x^7.
   b) Write each term as the product of two factors, where one factor is the GCF.
   c) Explain how you determined the second factor.

7. a) Identify the GCF of the polynomial $12x^4 + 8x^3$.
   b) Rewrite the polynomial as the sum of products. Express each term as a product of two factors, where the first factor is the GCF.
   c) Explain how you determined the second factor.

How would writing each term as a product help?

8. **Reflect and Respond** Explain how to factor a polynomial using the GCF.

greatest common factor (GCF)
- the largest factor shared by two or more terms
- for example, the GCF of 12 and 28 is 4

least common multiple (LCM)
- the smallest multiple shared by two or more terms
- Multiples of 6 and 8 are 24, 48, 72, … . The LCM is 24.

Link the Ideas

Factor out the GCF from a polynomial by dividing each term by the GCF. Then, the polynomial can be written in a simpler form to solve more complex problems.

$15x^2 + 10x = 5x(3x + 2)$

Example 1 Determine the Greatest Common Factor

Determine the GCF of $16x^2y$ and $24x^2y^3$.

Solution

Method 1: Use Prime Factorization

List the prime factorization of the numerical coefficients.

$16 = (2)(2)(2)(2)$

$24 = (2)(2)(2)(3)$

$$\begin{aligned}\text{Numerical GCF} &= (2)(2)(2)\\ &= 8\end{aligned}$$

List the prime factorization of the variables.

$x^2y = (x)(x)(y)$

$x^2y^3 = (x)(x)(y)(y)(y)$

$$\begin{aligned}\text{Variable GCF} &= (x)(x)(y)\\ &= x^2y\end{aligned}$$

Therefore, the GCF of $16x^2y$ and $24x^2y^3$ is $8x^2y$.

Method 2: List the Factors

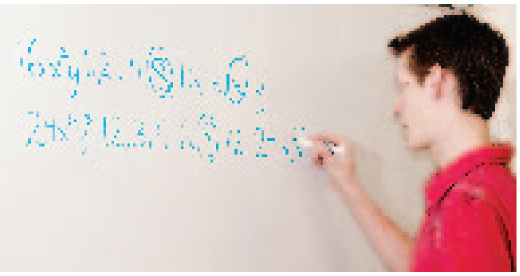
Write the factors of each term.

$16x^2y$: 1, 2, 4, 8, 16, x, x^2, y

$24x^2y^3$: 1, 2, 3, 4, 6, 8, 12, 24, x, x^2, y, y^2, y^3

The greatest common factors are 8, x^2, and y.

Therefore, the GCF of $16x^2y$ and $24x^2y^3$ is $8x^2y$.



Did You Know?

For *prime factorization*, write a number as the product of prime numbers. For example, the prime factorization of 18 is $2 \times 3 \times 3$.

Your Turn

Determine the GCF of each pair of terms.

a) $5m^2n$ and $15mn^2$

b) $48ab^3c$ and $36a^2b^2c^2$

Example 2 Write a Polynomial in Factored Form

Write $7a^2b - 28ab + 14ab^2$ in factored form.

Solution

Identify the GCF of the numerical coefficients by listing the prime factorization for each coefficient.

$7 = 7$

$28 = (2)(2)(7)$

$14 = (2)(7)$

The GCF is 7.

Identify the GCF of the variables.

$a^2b = (a)(a)(b)$

$ab = (a)(b)$

$ab^2 = (a)(b)(b)$

The GCF is ab.

Therefore, the GCF of $7a^2b - 28ab + 14ab^2$ is $7ab$.

Divide each term by the GCF.

$7a^2b - 28ab + 14ab^2 = 7ab(a - 4 + 2b)$

Check:

Multiply.

$$\begin{aligned} 7ab(a - 4 + 2b) &= (7ab)(a) + (7ab)(-4) + (7ab)(2b) \\ &= 7a^2b - 28ab + 14ab^2 \end{aligned}$$

Multiplying is the reverse of factoring.

Your Turn

Write each polynomial in factored form.

a) $27r^2s^2 - 18r^3s^2 - 36rs^3$

b) $4np^2 + 10n^4p - 12n^3p$

Example 3 Determine Binomial Factors

Write each expression in factored form.

a) $3x(x - 4) + 5(x - 4)$

b) $y^2 + 8xy + 2y + 16x$

Solution

a) The GCF can be a binomial expression. The GCF for the terms $3x(x - 4)$ and $5(x - 4)$ is $(x - 4)$.

$3x(x - 4) + 5(x - 4) = (x - 4)(3x + 5)$

b) A GCF may be found by grouping terms.

$$\begin{aligned} y^2 + 8xy + 2y + 16x &= (y^2 + 8xy) + (2y + 16x) \\ &= y(y + 8x) + 2(y + 8x) \\ &= (y + 2)(y + 8x) \end{aligned}$$

Check:

Multiply.

$$\begin{aligned} (y + 2)(y + 8x) &= y(y + 8x) + 2(y + 8x) \\ &= y^2 + 8xy + 2y + 16x \end{aligned}$$

Your Turn

Write each expression in factored form.

a) $4(x + 5) - 3x(x + 5)$

b) $a^2 + 8ab + 2a + 16b$

Example 4 Using the Greatest Common Factor to Solve a Problem

Paula has 18 toonies, 30 loonies, and 48 quarters. She wants to group her money so that each group has the same number of each coin and there are no coins left over.

a) What is the maximum number of groups she can make?

b) How many of each coin will be in each group?

c) How much money will each group be worth?

Solution

a) To find the maximum number of groups, identify the GCF of 18, 30, and 48.
The factors of 18 are 1, 2, 3, 6, 9, and 18.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.
The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.
The GCF is 6. Therefore, the maximum number of groups is 6.

b) Divide each number of coins by the GCF.
$\frac{18}{6} = 3 \qquad \frac{30}{6} = 5 \qquad \frac{48}{6} = 8$
There will be 3 toonies, 5 loonies, and 8 quarters in each group.

c) Multiply the number of each coin by its value.
Toonies: (3)($2) = $6
Loonies: (5)($1) = $5
Quarters: (8)($0.25) = $2
6 + 5 + 2 = 13
Each group will have a value of $13.

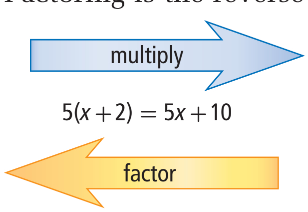
Your Turn

The students in Mr. Noyle's Construction class have decided they want to build dog houses for their class project. The class will be split up into groups. Each group will construct their dog house with the same type and amount of lumber. Mr. Noyle has 24 ten foot 1 by 4s, 32 eight foot 2 by 4s, and 8 sheets of plywood (4′ by 8′) available to use for this project.

a) What is the maximum number of groups of students that can build dog houses?
b) How much of each lumber type will each group have to work with?
c) What is the total length of 2 by 4s and 1 by 4s that each group will have to work with?

Key Ideas

• Factoring is the reverse of multiplying.



- To find the GCF of a polynomial find the GCF of the coefficients and variables.
- To factor a GCF from a polynomial divide each term by the GCF.
- Polynomials can be written as a product of the GCF and the sum or difference of the remaining factors.

  $2m^3n^2 - 8m^2n + 12mn^4 = 2mn(m^2n - 4m + 6n^3)$
- A common factor can be any polynomial, such as a binomial.

  $a(x + 2) - b(x + 2)$ has a common factor of $x + 2$.

Check Your Understanding

Practise

1. Copy the table. List all of the factors of each pair of numbers. Then, identify the greatest common factor (GCF).

a) 20: 30: GCF:	**b)** 28: 40: GCF:
c) 30: 48: GCF:	**d)** 36: 27: GCF:

2. Identify the GCF of the following sets of numbers.

a) 48 and 36
b) 144 and 96
c) 81 and 54
d) 256, 216, and 78
e) 50, 100, and 625

3. Identify the least common multiple (LCM) of the following sets of numbers.

a) 12 and 16
b) 15 and 20
c) 18 and 30
d) 10, 15, and 25
e) 22, 33, and 44

4. Identify the GCF of the following sets of terms.

a) $15a^2b$ and $18ab$
b) $27m^2n^3$ and $81m^3n$
c) $8x^2y^2$ and $24x^3y^3$
d) $12a^3bc^2$, $28a^2c$, and $36a^2b^2c^2$
e) $14p^4q^5$, $-24p^5q^4$, and $7p^3q^3$

5. Factor the following polynomials.

a) $5x + 15$

b) $3y^2 - 5y$

c) $w^2x + w^2y - w^2z$

d) $6a^3b - 18ab^2$

e) $9x^3 - 12x^2 + 6x$

6. State the missing factor.

a) $6a^2bc + 9ab^2 = (\blacksquare)(2ac + 3b)$

b) $3s^2 - 15 = 3(\blacksquare)$

c) $3d^2 - 21d = 3d(\blacksquare)$

d) $16x^2 - 2x = 2x(\blacksquare)$

e) $12x^2y^2 - 16xy = (\blacksquare)(3xy - 4)$

7. Factor the following polynomials.

a) $3y(y - 2) + 4(y - 2)$

b) $5a(a - 4) - 2(a - 4)$

c) $2cx - 8x + 7c - 28$

d) $3x^2 - 9x - 8x + 24$

e) $2y^4 + y^3 - 10y - 5$

Apply

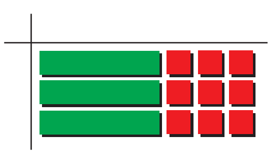
8. Mei is stacking toy blocks that are 12 cm tall next to blocks that are 18 cm tall. What is the shortest height at which the two stacks will be the same height?

9. Explain the difference between listing the factors of a number and listing the multiples of a number.

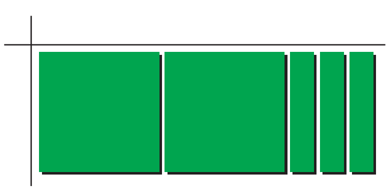
10. The models show rectangles of algebra tiles. Answer the following questions for each rectangle.

- What expression does each model represent?
- What are the possible dimensions that could produce each rectangle?
- Write an expression for each model, using your dimensions.

a)



b)



11. a) Write a polynomial with two terms that have a GCF of $6x$.

b) Write a polynomial with two terms that have a GCF of $4ab$.

c) Write a polynomial with three terms that have a GCF of $2m^3n^2$.

12. Each of the following factored polynomials has an error or is not fully factored. Describe what needs to be fixed and correct each one.

a) $15x^2 - 3x = 3x(5x - 0)$

b) $5x(x - 2) - (x - 2) = (x - 2)(5x)$

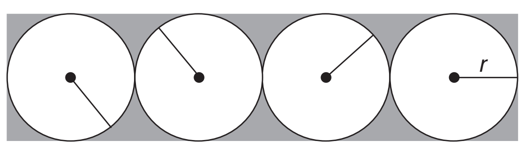
c) $9a^2b^3 - 27a^2b^2 + 81a^3b^3 = 9ab(ab^2 - 3ab + 9a^2b^2)$

d) $4fx + 16f + 2x + 8 = 2f(2x + 8) + 1(2x + 8)$

e) $2p^2 - 20p + 6p - 10 = 2p(p - 10 - 3) - 10$
$= 2p(p - 23)$

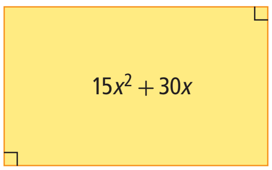
13. Nikolai has 30 pencils, 48 pens, and 36 erasers. He needs to package these items in containers for the participants of a workshop he is giving. He wants to divide them into identical containers, so that each container has the same number of each of the pencils, pens, and erasers. If he wants each container to have the greatest number of items possible, how many plastic containers does he need?

14. Some natural gas meters have four dials to show the gas use. Write a factored expression to represent the area of the metal plate around the dials, shaded in grey.



15. Mario wants to cut two pieces of material into equal-size squares with no material wasted. One piece measures 12 in. by 36 in. The other measures 6 in. by 42 in. What is the largest size square that he can cut?

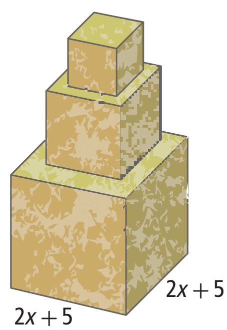
16. A rectangle has an area that can be represented by the expression $15x^2 + 30x$. The length and width can be found by factoring the expression. Write possible expressions for the length and the width.



Extend

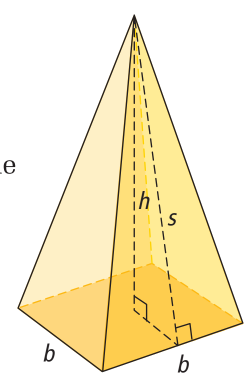
17. The greatest common factor of two numbers is 871. Both numbers are even. Neither is divisible by the other. What are the smallest two numbers they could be?

18. The pedestal design is made up of square-based prisms. The base length of each prism is 3 cm less than that of the layer immediately below.
   a) Write an algebraic expression for the total top surface area of the three prisms used to make the pedestal.
   b) Multiply and then simplify.
   c) Factor the expressions from part b).

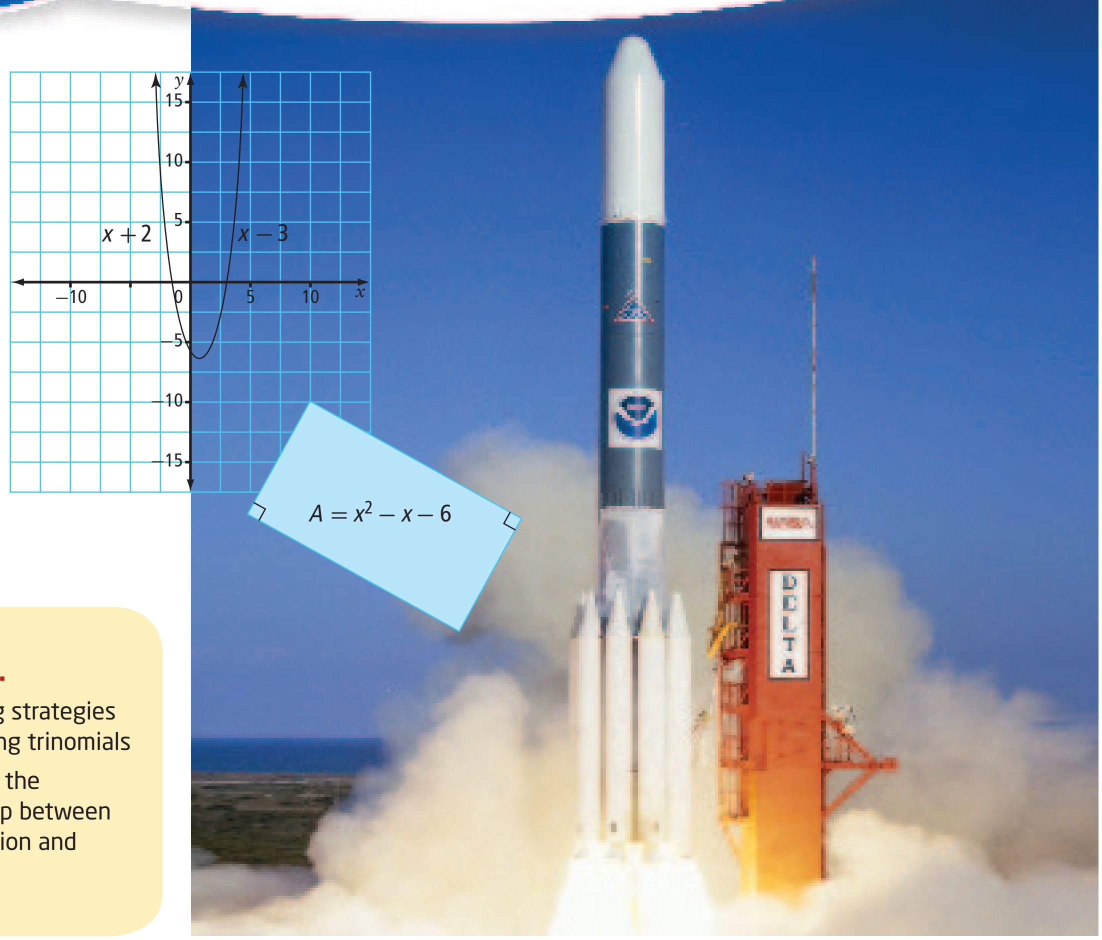


Create Connections

19. The surface area of a square-based pyramid is $SA = b^2 + 2bs$.
   a) Write the formula in factored form.
   b) Use both forms of the formula to calculate the surface area when $b = 5$ cm and $s = 4$ cm.
   c) What is the same about each surface area you calculated? What is different about each surface area you calculated?
   d) Is there one surface area formula you prefer to use? Explain.



5.3 Factoring Trinomials



Focus on …

- developing strategies for factoring trinomials
- explaining the relationship between multiplication and factoring

The trinomial, rectangle, and graph shown all have something in common. They each represent the same relationship. These kinds of relationships allow us to represent physical situations, such as the dimensions of a field or the height of a rocket, with mathematical expressions. Then, the expressions can be used to solve a variety of real-life problems.

Materials

- algebra tiles

Investigate Factoring Trinomials

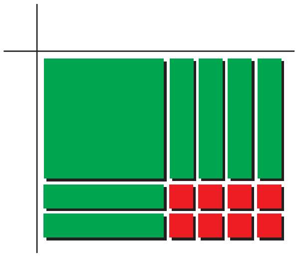
1. a) Use algebra tiles to model $(x + 4)(x + 1)$ as the dimensions of a rectangle.

b) Complete the rectangle. What is the product of $(x + 4)(x + 1)$?

c) How is the product represented by the algebra tiles?

d) How are the factors represented by the algebra tiles?

2. a) Use algebra tiles to factor the trinomial $x^2 + 6x + 8$. Create a rectangle so that the length and width represent the factors of the trinomial.



b) Place tiles along the top and left side of the rectangle to show the length and width of the rectangle. What are the two dimensions?

c) Record the dimensions as a product of binomials. What is this product equivalent to?

d) Multiply the two binomials. Compare the result to the original trinomial. Are they equivalent?

3. Repeat step 2 for each trinomial.

a) $x^2 + 5x + 6$

b) $x^2 + 8x + 12$

c) $x^2 + 3x + 2$

4. Each trinomial in step 2 and step 3 is of the form $x^2 + bx + c$. What do you notice about b and c and the binomial factors for each trinomial? Describe the relationship.

5. Test your observations from step 4 on each of the following trinomials. Use algebra tiles to check your answer.

a) $x^2 + 7x + 6$

b) $x^2 + 8x + 15$

6. **Reflect and Respond** Describe a process for finding the factors of a trinomial of the form $x^2 + bx + c$.

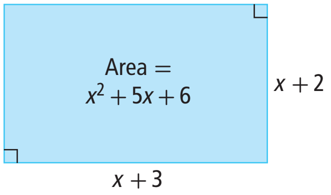
Link the Ideas

A rectangle can have an area that is a trinomial. By finding the dimensions of the rectangle, you are reversing the process of multiplying two binomials. This process is called *factoring*.

You can factor a trinomial of the form $x^2 + bx + c$ and the form $ax^2 + bx + c$ by studying patterns. Observe patterns that result from multiplying two binomials.

Factor Trinomials of the Form $ax^2 + bx + c$, $a = 1$

Multiply $x + 2$ and $x + 3$.



$$\begin{aligned}(x + 2)(x + 3) &= x^2 + 3x + 2x + (2)(3)\\ &= x^2 + 3x + 2x + (2)(3)\\ &= x^2 + (3 + 2)x + (2)(3)\end{aligned}$$

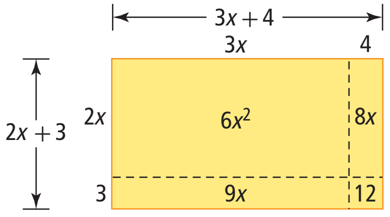
Since $(x + 2)(x + 3) = x^2 + 5x + 6$
and $(x + 2)(x + 3) = x^2 + (3 + 2)x + (2)(3)$,
then $x^2 + 5x + 6 = x^2 + (3 + 2)x + (2)(3)$.

Note that $3 + 2 = 5$ and $(2)(3) = 6$.

To factor trinomials of the form $x^2 + bx + c$, you can use patterns. Replace bx with two terms whose integer coefficients have a sum of b and a product of c.

Factor Trinomials of the Form $ax^2 + bx + c$, $a \neq 1$

Multiply $2x + 3$ and $3x + 4$.



$$\begin{aligned}(2x + 3)(3x + 4) &= 6x^2 + 8x + 9x + 12\\ &= 6x^2 + 17x + 12\end{aligned}$$

You can combine the two middle terms because they are like terms.

Notice the patterns:
- The sum of 8 and 9 is 17.
- The product of 8 and 9 is the same as (6)(12).

To factor trinomials of the form $ax^2 + bx + c$, you can use patterns. Replace bx with two terms whose integer coefficients have a sum of b and a product of $(a)(c)$.

Example 1 Factor Trinomials of the Form $ax^2 + bx + c$, $a = 1$

Factor, if possible.

a) $x^2 + 5x + 4$

b) $x^2 + 4x + 6$

c) $x^2 - 29x + 28$

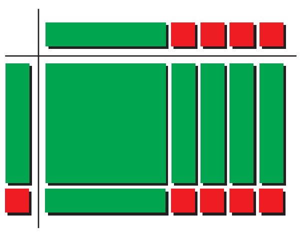
d) $x^2 + 3xy - 18y^2$

Solution

a) Factor $x^2 + 5x + 4$.

Method 1: Use Algebra Tiles

Arrange one x^2-tile, five x-tiles, and four 1-tiles into a rectangle. Then, add tiles to show the dimensions.



The dimensions of the rectangle are $x + 4$ and $x + 1$.
Therefore, the factors are $x + 4$ and $x + 1$.

How do you know that the dimensions are correct?

Method 2: Use a Table

Use a table to find two integers with
- a product of 4
- a sum of 5

In order to have a positive product and a positive sum, what signs do the two integers need to have?

Factors of 4	Product	Sum
1, 4	4	5
2, 2	4	4

Therefore, the factors are $x + 1$ and $x + 4$.

Check:
Multiply.

$$\begin{aligned}(x + 4)(x + 1) &= x(x + 1) + 4(x + 1)\\ &= x^2 + 1x + 4x + 4\\ &= x^2 + 5x + 4\end{aligned}$$

Did You Know?

When a polynomial cannot be factored such that the factors include only integer coefficients, we say that the polynomial cannot be factored *over the integers.*

b) Use a table to find two integers with
- a product of 6
- a sum of 4

In order to have a positive product and a positive sum, what signs do the two integers need to have?

Factors of 6	Product	Sum
1, 6	6	7
2, 3	6	5

No two integers have a product of 6 and sum of 4.
Therefore, you cannot factor $x^2 + 4x + 6$ over the integers.

c) Use a table to find two integers with
- a product of 28
- a sum of −29

In order to have a positive product and a negative sum, what signs do the two integers need to have?

Factors of 28	Product	Sum
−1, −28	28	−29
−2, −14	28	−16
−4, −7	28	−11

Therefore, the factors are $x - 1$ and $x - 28$.

Check:
Multiply.

$$\begin{aligned}(x - 1)(x - 28) &= x(x - 28) - 1(x - 28)\\ &= x^2 - 28x - 1x + 28\\ &= x^2 - 29x + 28\end{aligned}$$

d) Use a table to find two integers with
- a product of −18
- a sum of 3

In order to have a negative product and a positive sum, what signs do the two integers need to have?

Factors of −18	Product	Sum
1, −18	−18	−17
2, −9	−18	−7
3, −6	−18	−3
6, −3	−18	3
9, −2	−18	7
18, −1	−18	17

Therefore, the factors are $x + 6y$ and $x - 3y$.
Check:
Multiply.

$$\begin{aligned}(x + 6y)(x - 3y) &= x(x - 3y) + 6y(x - 3y)\\ &= x^2 - 3xy + 6xy - 18y^2\\ &= x^2 + 3xy - 18y^2\end{aligned}$$

Your Turn

Factor, if possible.

a) $x^2 + 7x + 10$

b) $r^2 - 10rs + 9s^2$

Example 2 Factor Trinomials of the Form $ax^2 + bx + c, a \neq 1$

Factor, if possible.

a) $3x^2 + 8x + 4$

b) $6x^2 - 5xy + y^2$

c) $3x^2 + 2x + 4$

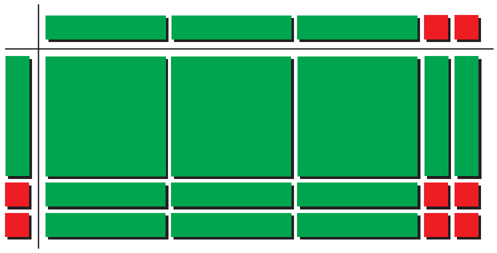
d) $24x^2 - 30x - 9$

Solution

a) First, check for a GCF. The GCF of the polynomial $3x^2 + 8x + 4$ is 1.

Method 1: Use Algebra Tiles

Arrange three x^2-tiles, eight x-tiles, and four 1-tiles into a rectangle. Then, add tiles to show the dimensions.



The dimensions of the resulting rectangle are $3x + 2$ and $x + 2$.

How do you know that the dimensions are correct?

Check:

Multiply.

$$\begin{aligned}(3x + 2)(x + 2) &= 3x(x + 2) + 2(x + 2)\\ &= 3x^2 + 6x + 2x + 4\\ &= 3x^2 + 8x + 4\end{aligned}$$

Method 2: Use a Table

Use a table to find two integers with
- a product of $(3)(4) = 12$
- a sum of 8

What signs do the two integers need to have?

Factors of 12	Product	Sum
1, 12	12	13
2, 6	12	8
3, 4	12	7

Write $8x$ as the sum $2x + 6x$. Then, factor by grouping.

$$\begin{aligned}3x^2 + 8x + 4 &= 3x^2 + (2x + 6x) + 4\\ &= (3x^2 + 2x) + (6x + 4)\\ &= x(3x + 2) + 2(3x + 2)\\ &= (3x + 2)(x + 2)\end{aligned}$$

Therefore, the factors are $3x + 2$ and $x + 2$.

Check:

Multiply.

$$\begin{aligned}(3x + 2)(x + 2) &= 3x(x + 2) + 2(x + 2)\\ &= 3x^2 + 6x + 2x + 4\\ &= 3x^2 + 8x + 4\end{aligned}$$

b) First, check for a GCF. The GCF of the polynomial $6x^2 - 5xy + y^2$ is 1. Use a table to find two integers with

- a product of 6
- a sum of -5

What signs do the two integers need to have?

Factors of $6y^2$	Product	Sum
−1, −6	6	−7
−2, −3	6	−5

Write $-5xy$ as $-2xy - 3xy$. Then, factor by grouping.

$$\begin{aligned} 6x^2 - 5xy + y^2 &= 6x^2 + (-2xy - 3xy) + y^2 \\ &= (6x^2 - 2xy) + (-3xy + y^2) \\ &= 2x(3x - y) - y(3x - y) \\ &= (3x - y)(2x - y) \end{aligned}$$

Therefore, the factors are $3x - y$ and $2x - y$.

Check:

Multiply.

$$\begin{aligned} (3x - y)(2x - y) &= 3x(2x - y) - y(2x - y) \\ &= 6x^2 - 3xy - 2xy + y^2 \\ &= 6x^2 - 5xy + y^2 \end{aligned}$$

c) First, check for a GCF. The GCF of the polynomial $3x^2 + 2x + 4$ is 1. Use a table to find two integers with

- a product of $(3)(4) = 12$
- a sum of 2

What signs do the two integers need to have?

Factors of 12	Product	Sum
1, 12	12	13
2, 6	12	8
3, 4	12	7

No two integers have a product of 12 and sum of 2. Therefore, you cannot factor $3x^2 + 2x + 4$ over the integers.

d) First, remove the greatest common factor (GCF). The GCF of the polynomial is 3. Therefore, $24x^2 - 30x - 9 = 3(8x^2 - 10x - 3)$. Use a table to find two integers with

- a product of $(8)(-3) = -24$
- a sum of -10

What signs do the two integers need to have?

Factors of −24	Product	Sum
−1, 24	−24	23
−2, 12	−24	10
−3, 8	−24	5
−4, 6	−24	2
−6, 4	−24	−2
−8, 3	−24	−5
−12, 2	−24	−10
−24, 1	−24	−23

Write $-10x$ as $-12x + 2x$. Then, factor by grouping.

$$\begin{aligned} 3(8x^2 - 10x - 3) &= 3(8x^2 - 12x + 2x - 3) \\ &= 3[(8x^2 - 12x) + (2x - 3)] \\ &= 3[4x(2x - 3) + 1(2x - 3)] \\ &= 3(4x + 1)(2x - 3) \end{aligned}$$

Therefore, the factors are 3, $4x + 1$, and $2x - 3$.

Check:
Multiply.

$$\begin{aligned} 3(4x + 1)(2x - 3) &= 3[4x(2x - 3) + 1(2x - 3)] \\ &= 3(8x^2 - 12x + 2x - 3) \\ &= 3(8x^2 - 10x - 3) \\ &= 24x^2 - 30x - 9 \end{aligned}$$

Your Turn

Factor, if possible.

a) $2x^2 + 7x - 4$

b) $-3s^2 - 51s - 30$

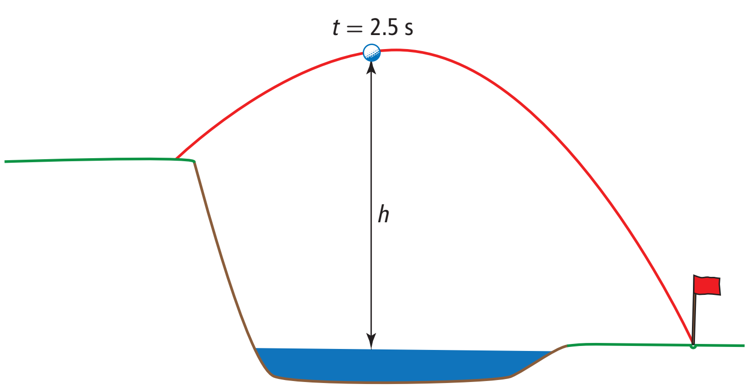
c) $2y^2 + 7xy + 3x^2$

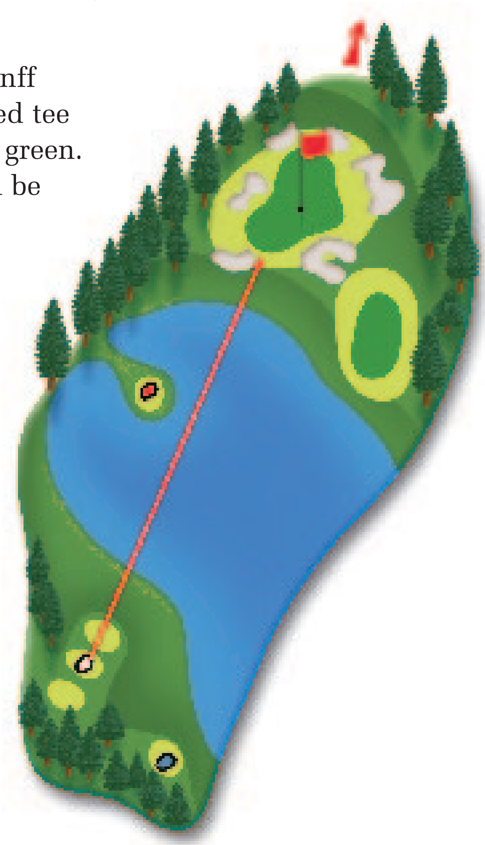
Example 3 Apply Factoring

The world famous *Devil's Cauldron* is the 4th hole at the Banff Springs Golf Course. This is a tough tee shot from an elevated tee that must carry the ball across a glacial lake to a small bowl green. The approximate height of the ball during a typical shot can be represented by the formula $h = -5t^2 + 25t + 30$, where t is the time, in seconds, and h is the height of the ball relative to the green, in metres.

a) Write the formula in factored form.

b) What is the height of the golf ball after 2.5 s?





Why is it easier if you remove a GCF of -5 instead of $+5$?

Solution

a) The expression for the height of the golf ball can be factored by first removing the GCF. The GCF of -5, 25, and 30 is -5.

$-5t^2 + 25t + 30 = -5(t^2 - 5t - 6)$

Use a table to find two integers with

- a product of -6
- a sum of -5

Factors of –6	Product	Sum
1, –6	–6	–5
2, –3	–6	–1
3, –2	–6	1
6, –1	–6	5

Therefore, the factors are $t + 1$ and $t - 6$.
The factored form is $h = -5(t + 1)(t - 6)$.

Check:
Multiply.

$$\begin{aligned}-5(t + 1)(t - 6) &= -5[t(t - 6) + 1(t - 6)]\\ &= -5(t^2 - 6t + t - 6)\\ &= -5(t^2 - 5t - 6)\\ &= -5t^2 + 25t + 30\end{aligned}$$

b) Substitute $t = 2.5$ into $h = -5t^2 + 25t + 30$ or $h = -5(t + 1)(t - 6)$.

$h = -5(2.5)^2 + 25(2.5) + 30$ or $h = -5(2.5 + 1)(2.5 - 6)$

$h = -5(6.25) + 62.5 + 30$ | $h = -5(3.5)(-3.5)$

$h = -31.25 + 62.5 + 30$ | $h = 61.25$

$h = 61.25$

After 2.5 s, the golf ball is 61.25 m above the green.

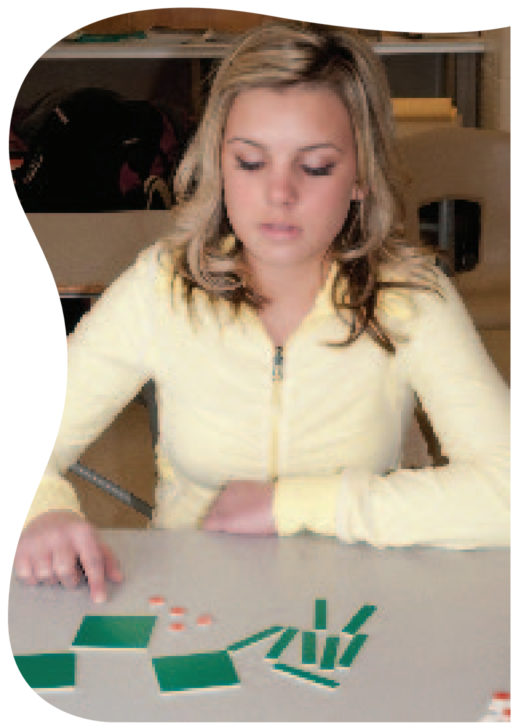
Your Turn

A rescue worker launches a signal flare into the air from the side of a mountain. The height of the flare can be represented by the formula $h = -16t^2 + 144t + 160$. In the formula, h is the height, in feet, above ground, and t is the time, in seconds.

a) What is the factored form of the formula?

b) What is the height of the flare after 5.6 s?

Key Ideas



- To factor a trinomial of the form $x^2 + bx + c$, first find two integers with
  - a product of c
  - a sum of b

  For $x^2 + 12x + 27$, find two integers with
  - a product of 27
  - a sum of 12

  The two integers are 3 and 9.
  Therefore, the factors are $x + 3$ and $x + 9$.
- To factor a trinomial of the form $ax^2 + bx + c$, first factor out the GCF, if possible. Then, find two integers with
  - a product of $(a)(c)$
  - a sum of b

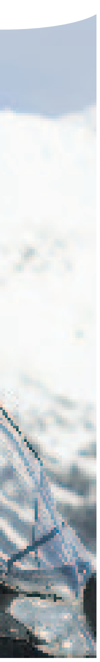
  Finally, write the middle term as a sum. Then, factor by grouping.

  For $8k^2 - 16k + 6$, the GCF is 2, so $8k^2 - 16k + 6 = 2(4k^2 - 8k + 3)$.

  Identify two integers with
  - a product of $(4)(3) = 12$
  - a sum of -8

  The two integers are -2 and -6. Use these two integers to write the middle term as a sum. Then, factor by grouping.

  $2(4k^2 - 2k - 6k + 3) = 2(2k - 3)(2k - 1)$
- You cannot factor some trinomials, such as $x^2 + 3x + 5$ and $3x^2 + 5x + 4$, over the integers.

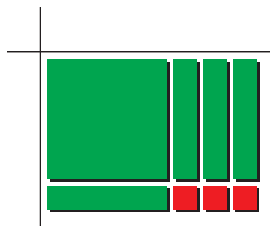


Check Your Understanding

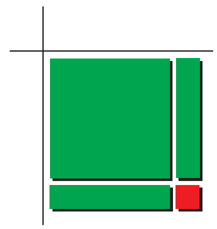
Practise

1. Write the trinomial represented by each rectangle of algebra tiles. Then, determine the dimensions of each rectangle.

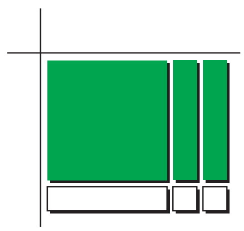
a)



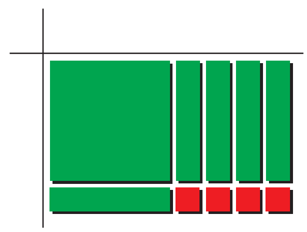
b)



c)



d)



2. Use algebra tiles or a diagram to factor each trinomial.

a) $2x^2 + 5x + 3$

b) $3x^2 + 7x + 4$

c) $3x^2 + 7x - 6$

d) $6x^2 + 11x + 4$

3. Identify two integers with the given product and sum.

a) product $= 45$, sum $= 14$

b) product $= 6$, sum $= -5$

c) product $= -10$, sum $= 3$

d) product $= -20$, sum $= -8$

4. Factor, if possible.

a) $x^2 + 7x + 10$

b) $j^2 + 12j + 27$

c) $k^2 + 5k + 4$

d) $p^2 + 9p + 12$

e) $d^2 + 10d + 24$

f) $c^2 + 4cd + 21d^2$

5. Factor each trinomial.

a) $m^2 - 7m + 10$

b) $s^2 + 3s - 10$

c) $f^2 - 7f + 6$

d) $g^2 - 5g - 14$

e) $b^2 - 3b - 4$

f) $2r^2 - 14rs + 24s^2$

6. Factor, if possible.

a) $2x^2 + 7x + 5$

b) $6y^2 + 19y + 8$

c) $3m^2 + 10m + 8$

d) $10w^2 + 15w + 3$

e) $12q^2 + 17q + 6$

f) $3x^2 + 7xy + 2y^2$

7. Factor, if possible.

a) $4x^2 - 11x + 6$

b) $w^2 + 11w + 25$

c) $x^2 - 5x + 6$

d) $2m^2 + 3m - 9$

e) $6x^2 - 3xy - 3y^2$

f) $12y^2 + y - 1$

g) $6c^2 + 7cd - 10d^2$

h) $4k^2 + 15k + 9$

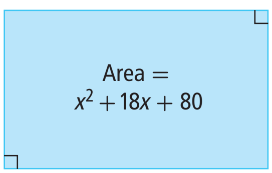
i) $a^2 + 11ab + 24b^2$

j) $6m^2 + 13mn + 2n^2$

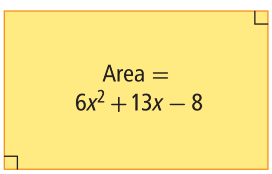
Apply

8. Identify binomials that represent the length and width of each rectangle. Then, calculate the dimensions of the rectangle if $x = 15$ cm.

a)



b)



9. Determine two values of b that allow each expression to be factored.

a) $x^2 + bx + 12$

b) $y^2 - by + 4$

c) $x^2 - bx - 8$

d) $p^2 + bp - 10$

10. Determine two values of c that allow each expression to be factored.

a) $x^2 + 6x + c$

b) $a^2 - 8a - c$

c) $x^2 - x + c$

d) $w^2 + 2w - c$

11. Find two values of n that allow each trinomial to be factored over the integers.

a) $x^2 + nx + 16$

b) $3y^2 + ny + 25$

c) $6a^2 + nab + 7b^2$

12. Determine one value of k that allows each trinomial to be factored over the integers.

a) $36m^2 + 18m + k$

b) $18x^2 - 15x + k$

c) $kp^2 - 18pq + 16q^2$

13. a) Make up an example of a trinomial expression that cannot be factored.

b) Explain why it cannot be factored.

14. **Unit Project** Use algebra tiles or area models to show the following relationships. Create a poster displaying your models.

a) the relationship between a monomial multiplied by a binomial and common factoring

b) the relationship between a binomial multiplied by a binomial and factoring a trinomial of the form $ax^2 + bx + c$, where a, b, and c are integers

15. You can estimate the height, h, in metres, of a toy rocket at any time, t, in seconds, during its flight. Use the formula $h = -5t^2 + 23t + 10$. Write the formula in factored form. Then, calculate the height of the rocket 3 s after it is launched.

16. The total revenue from sales of ski jackets can be modelled by the expression $720 + 4x - 2x^2$, where x represents the number of jackets sold above the minimum needed to break even. Revenue is also calculated as the product of the number of jackets sold and the price per jacket. Factor the given expression to determine the number sold and the price per jacket. The minimum price of a jacket is \$18. Hint: As the price increases, the number sold decreases.

Extend

17. Find three values of k such that the trinomial $3x^2 + kx + 5$ can be factored over the integers.

18. A square has an area of $9x^2 + 30xy + 25y^2$ square centimetres. What is the perimeter of the square? Explain how you determined your answer.

19. You have been asked to factor the expression $30x^2 - 39xy - 9y^2$ Explain how you would factor this expression. What are the factors?

20. The area of a certain shape can be represented by the expression $8x^2 + 10x - 7$.

a) Identify a possible shape.

b) Write expressions for the possible dimensions of the shape you identified in part a).

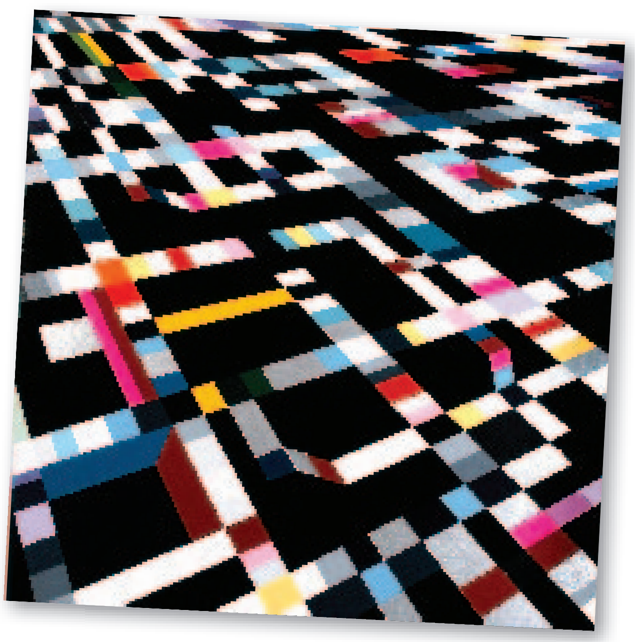
Create Connections

21. Describe, using examples, how multiplying binomials and factoring a trinomial are related.

22. **Unit Project**

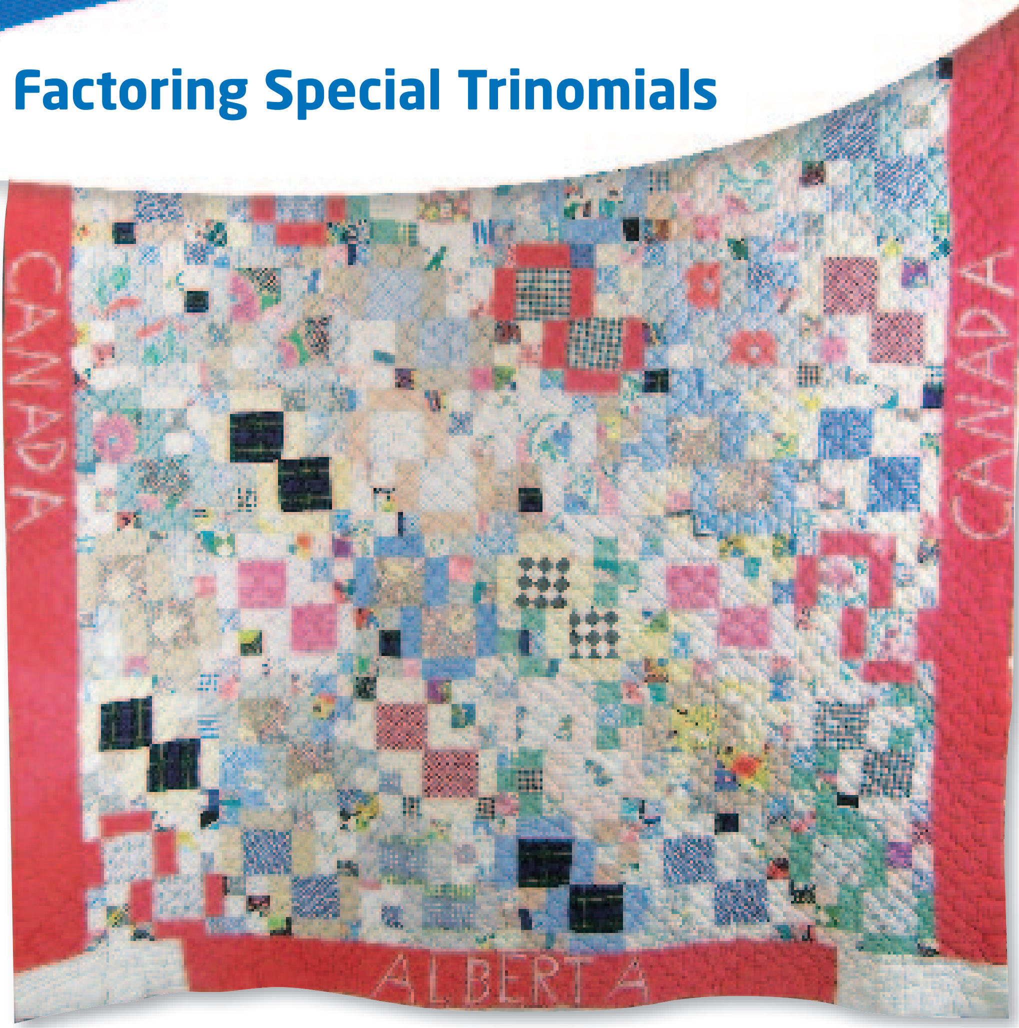
a) Use algebra tiles to create a model of a polynomial of your choice.

b) Create a piece of art that includes your polynomial in some way. Your artwork may be a drawing, painting, sculpture, or other form of your choice.



5.4 Factoring Special Trinomials

Focus on ...
- factoring the difference of squares
- factoring perfect squares



Did You Know?

Quilting has often been a way to unite people from different countries and cultures. The quilt shown here was part of a collection of quilts made by the Canadian Red Cross during WWII. These quilts were sent to families in Britain who had been displaced because of the war.

Patchwork quilts are made of square pieces of fabric sewed together to form interesting patterns. How could you relate these squares to polynomials and their factors?

Some polynomials, like perfect square trinomials and **differences of squares**, follow patterns that allow you to recognize a type of factoring method to use.

difference of squares
- an expression of the form $a^2 - b^2$ that involves the subtraction of two squares
- for example, $x^2 - 4$, $y^2 - 25$

Investigate Factoring Differences of Squares

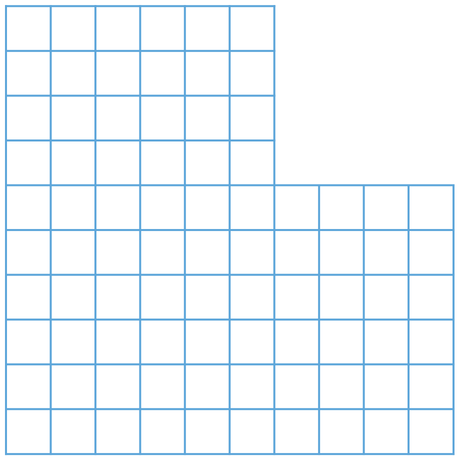
Materials
- centimetre grid paper
- scissors

1. Cut a 10-cm by 10-cm square out of a piece of centimetre grid paper.
   a) What is the area of the square?
   b) How did you calculate this area?

2. Cut a 4-cm by 4-cm piece from the corner of the square.

a) What is the area of this cutout piece?

b) How did you calculate this area?



3. a) Calculate the area of the remaining paper.

b) How did you calculate this area?

c) Are there other methods you could use to calculate this area? Explain.

4. Make one cut to the irregular shape that remains, so that you can rearrange it to form a rectangle.

a) What are the dimensions of the rectangle?

b) What is the area of the rectangle?

c) How is the area of the shape in step 3 related to the area of this rectangle?

5. Repeat step 1 to step 4 for two additional squares of different sizes.

a) Can each irregular shape always be rearranged into a rectangle? Compare your answer with a partner's.

b) List the dimensions of each rectangle.

c) Explain how the area of the cutout shape relates to the area of the rectangle.

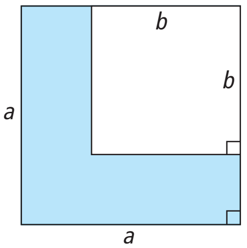
6. a) Write an algebraic expression to represent the area remaining when a square of area 25 cm^2 is removed from a square of area x^2 square centimetres.

b) If the resulting shape is rearranged into a rectangle, what are its dimensions?

c) Explain the relationship between your answers to parts a) and b).

d) Write an equation showing this relationship.

7. **Reflect and Respond** The diagram shows what remains when a square of dimensions b by b is removed from a square of dimensions a by a.



a) Write an expression to represent the area of the remaining shaded shape.

b) The shape is rearranged to form a rectangle. What are the dimensions of the rectangle?

c) Write an expression to represent the area of the rectangle.

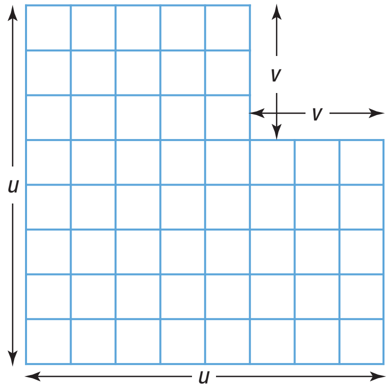
d) Write an equation to show the relationship between the area of the remaining shape and the area of the rectangle.

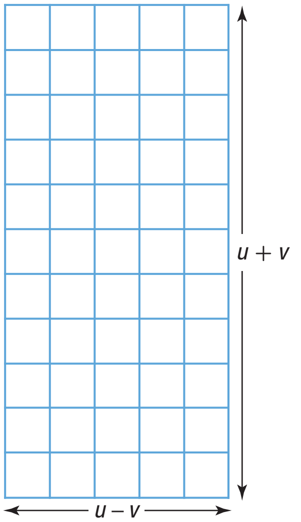
8. **a)** What are the patterns you observed from cutting out and rearranging the squares?

b) What conclusions can you make about subtracting the area of a smaller square from the area of a larger square?

Link the Ideas

When you cut a square out of a square, the area of the remaining shape is a difference of two squares. When you cut and rearrange this paper into a rectangle, you can write the area as a product of its dimensions.





You will find patterns helpful in factoring polynomials with special products. These include differences of squares and perfect square trinomials.

Difference of Squares

When you multiply the sum and the difference of two terms, the product will be a difference of squares.

$$\begin{aligned}(u + v)(u - v) &= (u)(u - v) + (v)(u - v)\\ &= (u)(u) - (u)(v) + (v)(u) + (v)(-v)\\ &= u^2 - uv + uv - v^2\\ &= u^2 - v^2\end{aligned}$$

In a difference of squares
- the expression is a binomial
- the first term is a perfect square: u^2
- the last term is a perfect square: v^2
- the operation between the two terms is subtraction

A difference of squares, $u^2 - v^2$, can be factored into $(u + v)(u - v)$.

Perfect Square Trinomial

When you square a binomial, the result is a perfect square trinomial.

$$\begin{aligned}(x + 5)^2 &= (x + 5)(x + 5)\\ &= x(x + 5) + 5(x + 5)\\ &= x^2 + 5x + 5x + 25\\ &= x^2 + 10x + 25\end{aligned}$$

In a perfect square trinomial
- the first term is a perfect square: x^2
- the last term is a perfect square: 5^2
- the middle term is twice the product of the square root of the first term and the square root of the last term: $(2)(x)(5) = 10x$

Example 1 Factor a Difference of Squares

Factor each binomial, if possible.

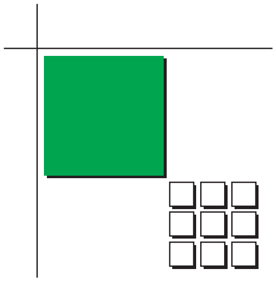
a) $x^2 - 9$ **b)** $16c^2 + 25a^2$

c) $m^2 + 16$ **d)** $7g^3h^2 - 28g^5$

Solution

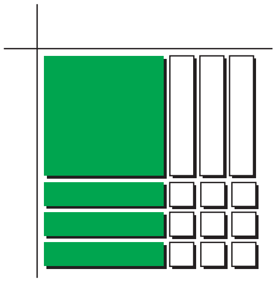
a) Method 1: Use Algebra Tiles

Create an algebra tile model to represent $x^2 - 9$.

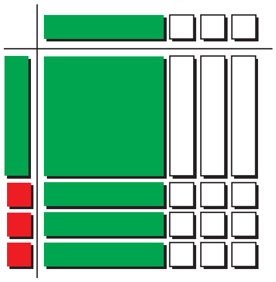


What is the result when you combine a positive x-tile and a negative x-tile?

Add three positive x-tiles and three negative x-tiles to represent the middle term.



The dimensions represent the factors of $x^2 - 9$.



The factors are $x - 3$ and $x + 3$.
Therefore, $x^2 - 9 = (x - 3)(x + 3)$.
Check:
Multiply.

$$\begin{aligned}(x - 3)(x + 3) &= x(x + 3) - 3(x + 3)\\ &= x^2 + 3x - 3x - 9\\ &= x^2 - 9\end{aligned}$$

Method 2: Factor by Grouping

$$\begin{aligned}x^2 - 9 &= x^2 - 3x + 3x - 9\\ &= (x^2 - 3x) + (3x - 9)\\ &= x(x - 3) + 3(x - 3)\\ &= (x - 3)(x + 3)\end{aligned}$$

The middle term must be included. Add in the zero pairs.

Method 3: Factor as a Difference of Squares

The binomial $x^2 - 9$ is a difference of squares.
The first term is a perfect square: x^2
The last term is a perfect square: 3^2
The operation is subtraction.

$$\begin{aligned} x^2 - 9 &= x^2 - 3^2 \\ &= (x - 3)(x + 3) \end{aligned}$$

b) You can write $-16c^2 + 25a^2$ as $25a^2 - 16c^2$.
The binomial $25a^2 - 16c^2$ is a difference of squares.
The first term is a perfect square: $(5a)^2$
The last term is a perfect square: $(4c)^2$
The operation is subtraction.

$$\begin{aligned} 25a^2 - 16c^2 &= (5a)^2 - (4c)^2 \\ &= (5a - 4c)(5a + 4c) \end{aligned}$$

Check:
Multiply.

$$\begin{aligned} (5a - 4c)(5a + 4c) &= (5a)(5a + 4c) + (-4c)(5a + 4c) \\ &= 25a^2 + 20ac - 20ac - 16c^2 \\ &= 25a^2 - 16c^2 \\ &= -16c^2 + 25a^2 \end{aligned}$$

c) The binomial $m^2 + 16$ can be written as a trinomial where the middle term is $0m$.
$m^2 + 0m + 16$
To factor this expression, you need to find two integers with
- a product of 16
- a sum of 0

Since the product is positive, both integers need to be either positive or negative.
If both integers are either positive or negative, a sum of 0 is not possible. Therefore, the binomial $m^2 + 16$ cannot be factored over the integers.

d) First, factor out the GCF from $7g^3h^2 - 28g^5$.
$7g^3h^2 - 28g^5 = 7g^3(h^2 - 4g^2)$
The binomial is a difference of squares.
The first term is a perfect square: h^2
The last term is a perfect square: $(2g)^2$
The operation is subtraction.

$$\begin{aligned} 7g^3h^2 - 28g^5 &= 7g^3(h^2 - 4g^2) \\ &= 7g^3[h^2 - (2g)^2] \\ &= 7g^3(h - 2g)(h + 2g) \end{aligned}$$

Check:
Multiply.

$$\begin{aligned} 7g^3(h - 2g)(h + 2g) &= 7g^3[h(h + 2g) - 2g(h + 2g)] \\ &= 7g^3(h^2 + 2gh - 2gh - 4g^2) \\ &= 7g^3(h^2 - 4g^2) \\ &= 7g^3h^2 - 28g^5 \end{aligned}$$

Your Turn

Factor each binomial, if possible.

a) $49a^2 - 25$

b) $125x^2 - 40y^2$

c) $9p^2q^2 - 25$

Example 2 Factor Perfect Square Trinomials

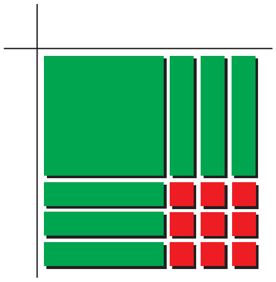
Factor each trinomial, if possible.

a) $x^2 + 6x + 9$ **b)** $2x^2 - 44x + 242$ **c)** $c^2 - 12x - 36$

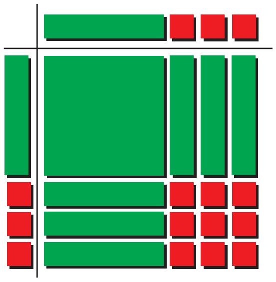
Solution

a) Method 1: Use Algebra Tiles

Create an algebra tile model to represent $x^2 + 6x + 9$.



The dimensions represent the factors of $x^2 + 6x + 9$.



The factors are $x + 3$ and $x + 3$.

Therefore, $x^2 + 6x + 9 = (x + 3)(x + 3)$

$= (x + 3)^2$

Check:

Multiply.

$$\begin{aligned}(x + 3)(x + 3) &= x(x + 3) + 3(x + 3)\\ &= x^2 + 3x + 3x + 9\\ &= x^2 + 6x + 9\end{aligned}$$

Method 2: Factor by Grouping

$$\begin{aligned}x^2 + 6x + 9 &= (x^2 + 3x) + (3x + 9)\\ &= x(x + 3) + 3(x + 3)\\ &= (x + 3)(x + 3)\\ &= (x + 3)^2\end{aligned}$$

Method 3: Factor as a Perfect Square Trinomial

The trinomial $x^2 + 6x + 9$ is a perfect square.
The first term is a perfect square: x^2
The last term is a perfect square: 3^2
The middle term is twice the product of the square root of the first term and the square root of the last term: $(2)(x)(3) = 6x$
The trinomial is of the form $(ax)^2 + 2abx + b^2$.

$$\begin{aligned} x^2 + 6x + 9 &= (x + 3)(x + 3) \\ &= (x + 3)^2 \end{aligned}$$

b) First, factor out the GCF from $2x^2 - 44x + 242$.
$2x^2 - 44x + 242 = 2(x^2 - 22x + 121)$
The first term in the brackets is a perfect square: x^2
The last term in the brackets is a perfect square: 11^2
The middle term is twice the product of the square root of the first term and the square root of the last term: $(2)(x)(11) = 22x$
The trinomial is of the form $(ax)^2 - 2abx + b^2$.

$$\begin{aligned} 2x^2 - 44x + 242 &= 2(x^2 - 22x + 121) \\ &= 2(x - 11)(x - 11) \\ &= 2(x - 11)^2 \end{aligned}$$

Check:
Multiply.

$$\begin{aligned} 2(x - 11)(x - 11) &= 2[x(x - 11) - 11(x - 11)] \\ &= 2(x^2 - 11x - 11x + 121) \\ &= 2(x^2 - 22x + 121) \\ &= 2x^2 - 44x + 242 \end{aligned}$$

c) The trinomial $c^2 - 12x - 36$ is not a perfect square.
The first and last terms are perfect squares.
The middle term is twice the product of the square root of the first term and the square root of the last term.
However, the trinomial is not of the form $(ax)^2 + 2abx + b^2$ or $(ax)^2 - 2abx + b^2$.
Therefore, the trinomial cannot be factored over the integers.

Your Turn

Factor each trinomial, if possible.

a) $x^2 - 24x + 144$
b) $y^2 - 18y - 81$
c) $3b^2 + 24b + 48$

Key Ideas

- Some polynomials are the result of special products. When factoring, you can use the pattern that formed these products.

  Difference of Squares:

  The expression is a binomial.
  The first term is a perfect square.
  The last term is a perfect square.
  The operation between the terms is subtraction.

  $$\begin{aligned} x^2 - 25 &= x^2 - 5^2 \\ &= (x - 5)(x + 5) \end{aligned}$$

  Perfect Square Trinomial:

  The first term is a perfect square.
  The last term is a perfect square.
  The middle term is twice the product of the square root of the first term and the square root of the last term.
  The trinomial is of the form $(ax)^2 + 2abx + b^2$ or $(ax)^2 - 2abx + b^2$.

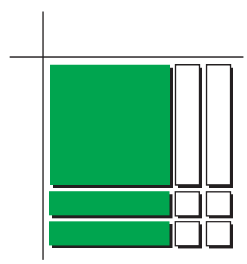
  $$\begin{aligned} x^2 + 16x + 64 &= x^2 + 8x + 8x + 64 \\ &= x(x + 8) + 8(x + 8) \\ &= (x + 8)(x + 8) \end{aligned}$$

Check Your Understanding

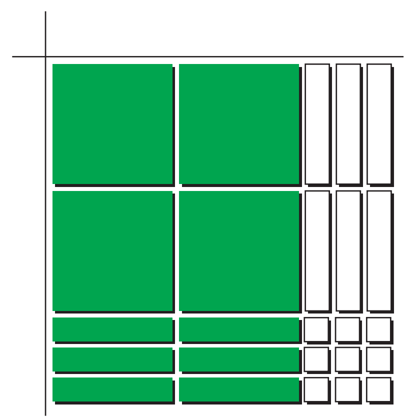
Practise

1. Identify the factors of the polynomial shown by each algebra tile model.

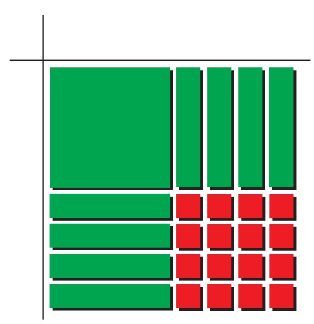
a)



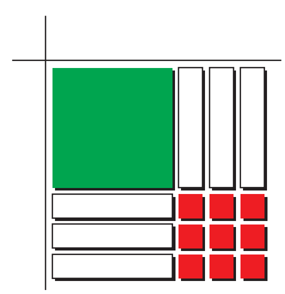
b)



c)



d)



2. Determine each product.

a) $(x - 8)(x + 8)$ **b)** $(2x + 5)(2x - 5)$

c) $(3a - 2b)(3a + 2b)$ **d)** $3(t - 5)(t + 5)$

3. What is each product?

a) $(x + 3)^2$ **b)** $(3b - 5a)^2$

c) $(2h + 3)^2$ **d)** $5(x - 2y)^2$

4. Identify the missing values for a difference of squares or a perfect square trinomial.

a) $\blacksquare - y^2 = (\blacksquare - y)(m + \blacksquare)$

b) $16r^6 - \blacksquare = (\blacksquare - \blacksquare)(\blacksquare + 9)$

c) $x^2 - 12x + \blacksquare = (\blacksquare - 6)^2$

d) $4x^2 + \blacksquare + \blacksquare = (\blacksquare + 5)^2$

e) $\blacksquare + \blacksquare + 49 = (5x + \blacksquare)(\blacksquare + \blacksquare)$

5. Factor each binomial, if possible.

a) $x^2 - 16$ **b)** $b^2 - 121$

c) $w^2 + 169$ **d)** $9a^2 - 16b^2$

e) $36c^2 - 49d^2$ **f)** $h^2 + 36f^2$

g) $121a^2 - 124b^2$ **h)** $100 - 9t^2$

6. Factor each trinomial, if possible.

a) $x^2 + 12x + 36$ **b)** $x^2 + 10x + 25$

c) $a^2 - 24a - 144$ **d)** $m^2 - 26m + 169$

e) $16k^2 - 8k + 1$ **f)** $49 - 14m + m^2$

g) $81u^2 + 34u + 4$ **h)** $36a^2 + 84a + 49$

7. Factor completely.

a) $5t^2 - 100$ **b)** $10x^3y - 90xy$

c) $4x^2 - 48x + 36$ **d)** $18x^3 + 24x^2 + 8x$

e) $x^4 - 16$ **f)** $x^4 - 18x^2 + 81$

Apply

8. Determine two values of n that allow each polynomial to be a perfect square trinomial. Then, factor.

a) $x^2 + nx + 25$

b) $a^2 + na + 100$

c) $25b^2 + nb + 49$

d) $36t^2 + nt + 121$

9. Each of the following polynomials cannot be factored over the integers. Why not?

a) $25a^2 - 16b$

b) $x^2 - 7x - 12$

c) $4r^2 - 12r - 9$

d) $49t^2 + 100$

10. **Unit Project** Use models or diagrams to show what happens to the middle terms when you multiply two factors that result in a difference of squares. Include at least two specific examples.

11. Many number tricks can be explained using factoring. Use $a^2 - b^2 = (a - b)(a + b)$ to make the following calculations possible using mental math.

a) $19^2 - 9^2$

b) $28^2 - 18^2$

c) $35^2 - 25^2$

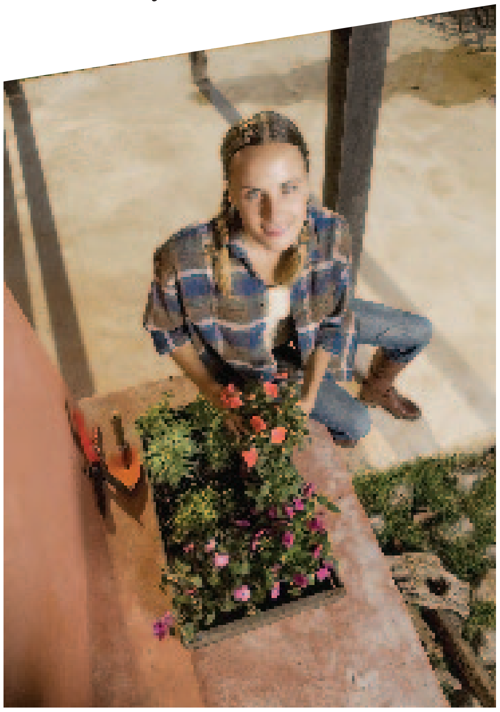
d) $5^2 - 25^2$

12. **Unit Project**

a) Use models or diagrams to show the squaring of a binomial. Include at least two specific examples.

b) Create a rule for squaring any binomial. Show how your rule relates to your models or diagrams.

13. Zoë wants to construct a patio in the corner of her property. The area of her square property has a side length represented by x metres. The patio will take up a square area with a side length represented by y metres. Write an expression, in factored form, to represent the remaining area of the property.

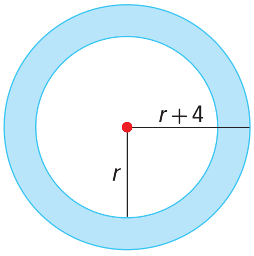


14. The diagram shows two concentric circles with radii r and $r + 4$.

a) Write an expression for the area of the shaded region.

b) Factor this expression completely.

c) If $r = 6$ cm, calculate the area of the shaded region. Give your answer to the nearest tenth of a square centimetre.

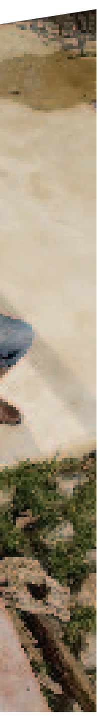


15. An object is reduced or enlarged uniformly in all dimensions. The print shown is a watercolour painting called *August Chinook* by Gena LaCoste of Medicine Hat, Alberta. This print is going to be enlarged by a factor of 3. The side length of the original can be represented by $(2x - 3)$ cm.

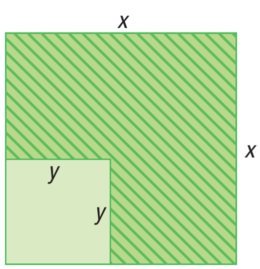
a) Use your understanding of differences of squares to write an expression that represents the difference in the areas of the original print and the enlargement.

b) Multiply this expression to write it in the form $ax^2 + bx + c$.

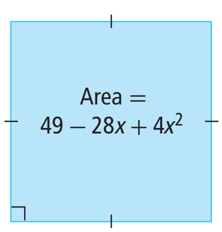
c) Verify that your expressions in parts a) and b) are correct by substituting a value for x.



16. Explain how the diagram shows a difference of squares.



17. The area of a square can be given by $49 - 28x + 4x^2$, where x represents a positive integer. Write a possible expression for the perimeter of the square.



Did You Know?

The painted butterfly drum, made by Odin Lonning, is a circular drum made from rawhide over a cedar frame.

18. The circular area of the painted butterfly drum can be represented by the expression $(9x^2 + 30x + 25)\pi$. Determine an expression for the smallest diameter the drum could have.

Traditional Tlingit hand drums are used in ceremony, cultural and social events, and as artwork. Traditional drums should always be handled with respect following appropriate protocol.

19. State whether the following equations are *sometimes*, *always*, or *never* true. Explain your reasoning.

a) $a^2 - 2ab - b^2 = (a - b)^2$, $b \neq 0$

b) $a^2 + b^2 = (a + b)(a + b)$

c) $a^2 - b^2 = a^2 - 2ab + b^2$

d) $(a + b)^2 = a^2 + 2ab + b^2$

20. Rahim and Kate are factoring $16x^2 + 4y^2$. Who is correct? Explain your reasoning.

Rahim

$16x^2 + 4y^2 = 4(4x^2 + y^2)$

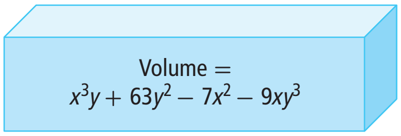
Kate

$16x^2 + 4y^2 = 4(4x^2 + y^2)$

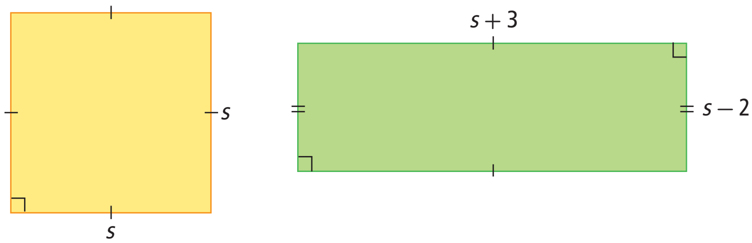
$= 4(2x + y)(2x - y)$

Extend

21. The volume of a rectangular prism is $x^3y + 63y^2 - 7x^2 - 9xy^3$. Determine expressions for the dimensions of the prism.



22. The area of the square shown is $16x^2 - 56x + 49$. What is the area of the rectangle in terms of x?



23. a) The difference of squares of two numbers is the same as their sum. What integers satisfy this condition? Show how you determined your answer.

b) Based on your observations in part a), identify two integers from 11 to 20 which have a difference of squares that can be expressed as the sum of the integers.

Create Connections

24. a) If $x^2 + bx + c$ is a perfect square, how are b and c related?

b) If $ax^2 + bx + c$ is a perfect square, how are a, b, and c related?

25. Use two ways to show that $a^2 - b^2 = (a - b)(a + b)$.

26. What is the difference in factoring $x^2 + 2bx + b^2$ and $x^2 - 2bx + b^2$?

27. To determine the product of two numbers that differ by 2, square their average and then subtract 1. Use this method to find the following products.

$(29)(31) = \blacksquare$
$(59)(61) = \blacksquare$

a) Explain this method using a difference of squares.

b) Develop a similar method for multiplying two numbers that differ by 6.

c) Explain your method from part b) using a difference of squares.

5 Review

5.1 Multiplying Polynomials, pages 204–213

1. Draw a diagram to model each product.

a) $(x + 5)(x - 3)$ **b)** $(y + 3)^2$

2. Determine the product and then combine like terms.

a) $(x + 7)(x + 3)$ **b)** $(b + 9)(b - 9)$

c) $(y - 11)(y + 11)$ **d)** $(3a + 8b)(5a + 6b)$

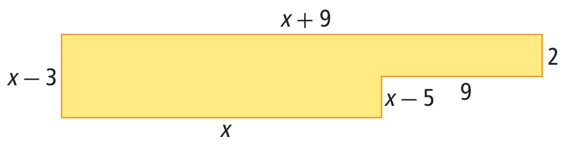
e) $-5(2x + 5b)^2$ **f)** $-(a - 6b)(a + 6b)$

3. Multiply and then combine like terms.

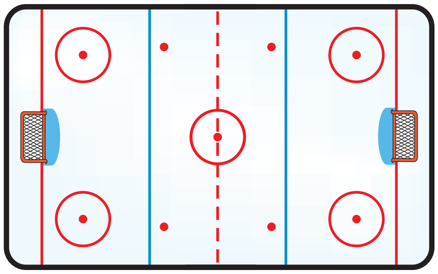
a) $(a^2 + 6a + 2)(a - 3)$

b) $2b(4b - 7)(3b + 2) - b(5b + 2)(b - 6)$

4. Write an expression to represent the area of the figure. Simplify.



5. The length of the ice surface of a hockey rink is represented by $5x + 25$. The width is represented by $2x + 10$. What expression represents the area of the ice surface?



5.2 Common Factors, pages 214–223

6. Identify the GCF of each set of terms.

a) 16 and 64 **b)** 81 and 108

c) 144, 60, and 54 **d)** $2x^2$ and $4x$

e) $10x^2$ and $20y^2$ **f)** $3xy$ and $7xz$

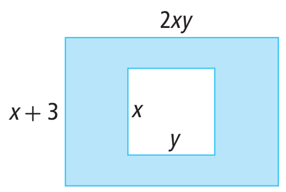
7. Identify the LCM of the following pairs of numbers.

a) 18 and 27 **b)** 125 and 15

8. Use algebra tiles or a diagram to factor each polynomial.

a) $x^2 + 5x$ **b)** $8x^2 + x$

9. Write an expression in fully factored form for the shaded area.

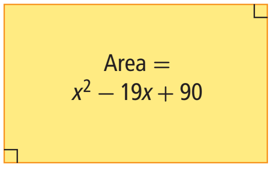


5.3 Factoring Trinomials, pages 224–237

10. Use algebra tiles or a diagram to factor each trinomial.
    a) $x^2 + 6x + 9$
    b) $x^2 + 12x + 35$
    c) $12x^2 - 5x - 3$
    d) $3x^2 - 13x + 10$

11. Factor, if possible.
    a) $x^2 - 4x - 12$
    b) $x^2 - 7x + 12$
    c) $30x^2 + 9x - 12$
    d) $-6x^2 - 34x + 12$
    e) $-2x^2 + 16x - 30$
    f) $x^3 + 3x^2 - 28x$

12. Identify binomials to represent the length and width of the rectangle. Then, calculate the dimensions of the rectangle if $x = 11$ cm.

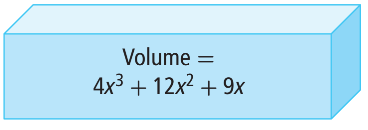


5.4 Factoring Special Trinomials, pages 238–251

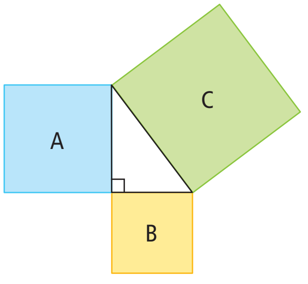
13. Factor fully.
    a) $x^2 - 100$
    b) $c^2 - 25$
    c) $9x^2 - 16$
    d) $128 - 18x^2$
    e) $1 - 225y^2$
    f) $-3x^2 + 27y^2$

14. Verify that each trinomial is a perfect square. Then, factor.
    a) $y^2 + 16y + 64$
    b) $x^2 - 20x + 100$
    c) $225 - 90y + 9y^2$
    d) $121c^2 + 308cd + 196d^2$

15. a) Write algebraic expressions for the dimensions of the rectangular prism.
    b) Describe the faces of the prism.
    c) Calculate the surface area if $x = 3$ cm.



16. The expression x^2 represents the area of square A, y^2 the area of square B, and $x^2 + y^2$ the area of square C. The side lengths of squares A and B are increased by 2 units. Write an expression for the new area of square C.

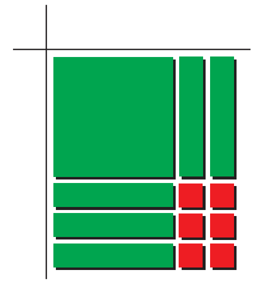


5 Practice Test

Multiple Choice

For #1 to #4, choose the best answer.

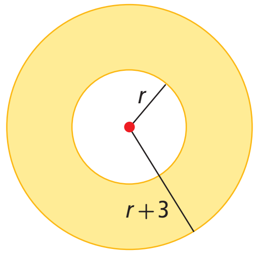
1. What binomial product does the diagram represent?



A $(x + 2)(x + 3)$ **B** $(x + 2)(x - 3)$

C $(x - 2)(x - 3)$ **D** $(x - 2)(x + 3)$

2. What is an algebraic expression for the area of the shaded region?



A $\pi(r^2 - 9)$ **B** $3\pi(2r + 3)$

C $\pi(2r^2 + 6r + 9)$ **D** 9π

3. Fully factored, $8m^2 + 16m - 10$ can be represented as

A $2(2m - 1)(2m + 5)$ **B** $2(4m + 10)(2m - 5)$

C $(4m^2 + 20)(2m - 4)$ **D** $(4m - 2)(2m + 5)$

4. Fully factored, $4y^2 - 64$ can be represented as

A $y(y^2 - 16)$ **B** $4(y - 4)^2$

C $4(y - 4)(y + 4)$ **D** $4(y - 2)(y + 2)$

Short Answer

5. What is the GCF of 56, 20, and 228? What is their LCM?

6. Simplify.

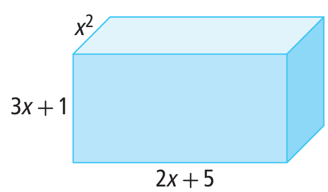
a) $(x - 3)(x - 9)$ **b)** $(2x + 3)(2x - 1)$

c) $-3(x - 4)^2 + 2(x - 3)(x + 3)$ **d)** $(3c + d)^2 + 2c(c - d)$

e) $2(x - 1)(x - 6) - 3(2x - 1)^2$ **f)** $(2c + 3d)^2 - 3(c + 1)^2$

7. a) Write an algebraic expression for the volume of the rectangular prism. Simplify the expression.

b) Calculate the volume if $x = 2$ cm.



8. Factor fully.

a) $x^2 + 10x + 25$ **b)** $25r^2 - 20rs + 4s^2$ **c)** $5x^2 - 5$

d) $1 - 49m^2$ **e)** $5m^2 + 17m + 6$ **f)** $m^2 - 9mn + 14n^2$

9. Factor, if possible.

a) $3y^3 - 7y^2 + 2y$ **b)** $4m^2 + 16$

c) $6y^2 + y - 1$ **d)** $x(m - 2) - 4(m - 2)$

e) $y^2 + 2x + 2y + xy$ **f)** $9t - 4t^3$

10. The face of a Canadian $20 bill has an area that can be represented by the expression $10x^2 + 9x - 40$.

a) Factor $10x^2 + 9x - 40$ to find expressions to represent the dimensions of the bill.

b) If x represents 32 mm, what are the dimensions of the bill? Express your answer in millimetres.

11. Trafalgar Fountain in Regina is a circular fountain with a radius of $2x$ metres. The circular pool surrounding the fountain is an additional 3 m in radius. What is an expression for the area of the base of the pool? Multiply and combine like terms.

Did You Know?

Trafalgar Fountain is one of a pair of fountains made in London, England. The fountain in Regina commemorates the founding of the Northwest Mounted Police headquarters in 1882. The other fountain is located in Ottawa.

Extended Response

12. Brendan factored the trinomial $8y^2 - 12y - 18$ in this way:

$$\begin{aligned} 8y^2 - 12y - 18 &= 2(4y^2 - 6y - 9) \\ &= 2(2y - 3)(2y - 3) \\ &= 2(2y - 3)^2 \end{aligned}$$

Is Brendan's factoring correct? Explain.

13. The volume of a rectangular prism is represented by $12x^3 - 3x$.

a) Factor the expression fully.

b) Sketch the prism and label its dimensions.

c) If x represents 6 cm, what are the dimensions of the prism?

2 Unit Connections

Unit 2 Project

In this unit, you have seen how artists use mathematics in their work. Now, it is your turn to create art from mathematics. Your artwork can be historical or contemporary. It can involve nature, stylized arrangements of models, or another idea of your choice that relates to this unit.

You can present your creation as a video, cartoon, painting, or in another form of your choice. Include a brief report, describing how the mathematics from Unit 2 specifically relates to your work of art.

Unit Review

Chapter 4 Exponents and Radicals

Write the letter of the value that represents each term.

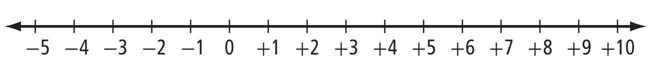
1. radicand	**A** the number 4 in $\sqrt[4]{16}$
2. rational exponent	**B** the number 3 in $2\sqrt{3}$
3. irrational number	**C** $\sqrt{20}$
4. index	**D** $3^{\frac{2}{3}}$

5. Sort the following numbers into perfect squares, perfect cubes, or neither. Identify the value of each perfect root.

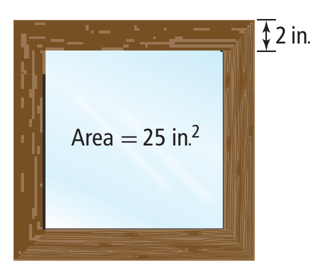
16, 15, 27, −4, 125, 1000, 169, −8, 99

6. Copy the number line shown. Order each of the following numbers on your number line.

$\sqrt{5}$, $\sqrt[3]{-8}$, $\sqrt{12}$, $\sqrt[5]{32}$, $\sqrt{65}$



7. The area of the glass within a square picture frame is 25 in.^{2}. The width of the frame around the glass is 2 in. Determine the area of the entire framed picture.



8. A child's toy cube is made up of 26 small cubes, plus one invisible small cube in the centre of the same dimension. The volume of the large toy cube is 729 cm^3. What is the area of one side of a small cube?

9. Express each radical as a mixed radical in simplest form.

a) $\sqrt{12}$ **b)** $\sqrt{162}$ **c)** $\sqrt[3]{16}$

10. Express each mixed radical as an entire radical.

a) $2\sqrt{5}$ **b)** $5\sqrt{3}$ **c)** $2\sqrt[3]{5}$

11. Express each power as an equivalent radical.

a) $7^{\frac{4}{5}}$ **b)** $\left(\frac{27}{8}\right)^{\frac{1}{3}}$ **c)** $(6x^2)^{\frac{1}{4}}$

12. Express each radical as a power.

a) $\sqrt{\frac{m^3}{n^2}}$ **b)** $\sqrt[4]{6^3}$ **c)** $\sqrt[3]{8s^4}$

13. Simplify each expression. Write your answers with positive exponents.

a) $\left(\frac{1}{3}\right)^5\left(\frac{1}{3}\right)^{\frac{1}{2}}$ **b)** $\frac{(x^3y^{-1})}{(x^6y^4)^{\frac{1}{2}}}$ **c)** $(\sqrt{81})^{\frac{3}{2}} \div 3^4$

14. Determine the exact value of each expression.

a) $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$ **b)** $5^3 + 5^2$ **c)** $\frac{4}{(2^3 - 3)^{-2}}$

Chapter 5 Polynomials

15. Model each product using a diagram or algebra tiles. Determine the simplified expression representing the product.

a) $(2x + 1)(x + 3)$

b) $(x - 2)(x + 2)$

c) $(x + 3)^2$

16. Multiply each product and then combine like terms.

a) $(a - 4)(a + 7)$

b) $(2x + 3)(5x + 2)$

c) $(-x + 5)(x + 5)$

d) $(3y + 4)^2$

e) $(a - 3b)(4a - b)$

f) $(v - 1)(2v^2 - 4v - 9)$

17. Determine a value for k that allows each trinomial to be factored over the integers.

a) $3x^2 + kx - 10$

b) $24x^2 + 47x - k$

18. Rachel's solution to the multiplication of a binomial and a trinomial is shown below.

$$(4x - 1)(2x^2 + 11x - 7) = 8x^3 + 44x - 24x - 2x^2 - 11x + 6$$
$$= 6x^2 - 9x + 6$$

a) Check Rachel's solution for $x = 2$.

b) Does Rachel have a correct solution? If not, identify her error and provide the correct product in simplified form.

19. Factor, if possible.

a) $9y^2 + 24y - 16$

b) $50x^2 - 60xy + 18y^2$

c) $x^2 + 9y^2$

20. It is possible to factor out a common factor from the expression $2x^2 + ky + 4$, where k is an integer. What can you state about the values of k? Explain.

21. Determine the greatest common factor in the terms of each polynomial.

a) $14x^2 - 21x$

b) $-10x^4 - 5x^3 + 15x^2$

c) $15a^3b(a - 1) - 12ab^2(a - 1)$

22. Express each polynomial as a product of its factors, if possible.

a) $x^2 + 8x + 9$

b) $2v^2 + 3v - 9$

c) $-2x^2 - 6x + 20$

d) $4y^2 - 25$

e) $20 - 21x + x^2$

f) $-15x^2 + x + 6$

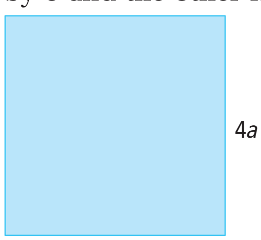
23. Julio was asked to factor the expression $2x^2 + 12x + 18$. His solution is shown.

$$\begin{aligned} 2x^2 + 12x + 18 &= 2(x^2 + 10x + 9) \\ &= 2(x + 1)(x + 9) \end{aligned}$$

a) Identify the error that Julio made.

b) Determine the correct factorization.

24. A square has a side length of $4a$. One dimension is increased by 6 and the other is decreased by 6.

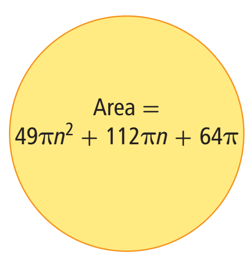


a) Write an algebraic expression to represent the area of the resulting rectangle.

b) Multiply this expression and combine like terms.

c) Write an algebraic expression for the difference between the area of the square and the area of the rectangle. Combine like terms.

25. What is the radius of a circle with an area of $49\pi n^2 + 112\pi n + 64\pi$?



2 Unit Test

Multiple Choice

For #1 to #5, choose the best answer.

1. Selena said, "A square root is the same as a perfect square."
Anoop said, "A cube root is always positive."
Danielle said, "A number is either rational or irrational, but not both."
Which of the students are correct?

A All three students are correct.

B Selena is correct, Anoop is incorrect, and Danielle is incorrect.

C Selena is incorrect, Anoop is correct, and Danielle is correct.

D Selena is incorrect, Anoop is incorrect, and Danielle is correct.

2. Which expression is equivalent to $\left(\frac{9}{4}\right)^{-2}$?

A $\frac{2}{3}$ **B** $\frac{3}{2}$ **C** $\frac{16}{81}$ **D** $\frac{81}{16}$

3. When the numbers $\sqrt{35}$, $\sqrt[3]{27}$, $\left(\frac{1}{3}\right)^{-2}$, $2\left(9^{\frac{1}{2}}\right)$ are written in ascending order, the correct sequence is

A $\sqrt[3]{27}$, $\sqrt{35}$, $2\left(9^{\frac{1}{2}}\right)$, $\left(\frac{1}{3}\right)^{-2}$

B $\left(\frac{1}{3}\right)^{-2}$, $2\left(9^{\frac{1}{2}}\right)$, $\sqrt[3]{27}$, $\sqrt{35}$,

C $\sqrt[3]{27}$, $\left(\frac{1}{3}\right)^{-2}$, $\sqrt{35}$, $2\left(9^{\frac{1}{2}}\right)$

D $\sqrt{35}$, $\left(\frac{1}{3}\right)^{-2}$, $2\left(9^{\frac{1}{2}}\right)$, $\sqrt[3]{27}$

4. Which equation is not correct?

A $(b + a)(b - a) = b^2 - a^2$

B $(a + b)^2 = b^2 + 2ab + a^2$

C $(a - b)^2 = a^2 - 2ab + b^2$

D $(a - b)(b - a) = a^2 - b^2$

5. Express the product of $3x^3 - 5$ and $3x^3 + 5$ in simplified form.

A $9x^6 - 25$ **B** $9x^3 - 25$

C $3x^3 - 5$ **D** $9x^9 - 25$

Numerical Response

Complete the statements in #6 to #9.

6. Given the trinomial $9x^2 + kx + 4$, the positive value for k that makes the expression a perfect square trinomial is ■.

7. When $\sqrt{80}$ is written in the simplified form $a\sqrt{b}$, the value of the radicand is ■.

8. You multiply the binomials $2x + 5$ and $x + 7$ and express the product in the form $ax^2 + bx + c$. The value of the coefficient of the x term is ■.

9. The value of the expression $\left(\dfrac{(2^{-3})(2^5)}{2^8}\right)^{-\frac{1}{3}}$ is ■.

Written Response

10. Simplify the expression $\left(\sqrt[3]{\sqrt{20}}\right)^{-1}$. Write it in exponential form with positive exponents only.

11. Multiply and then combine like terms.

a) $(x + 5y)(2x - y)$

b) $(2a - 3)(3a^2 + 2a - 7)$

c) $3x(x^2 - 2x + 4) - (x^2 + 5x - 1)$

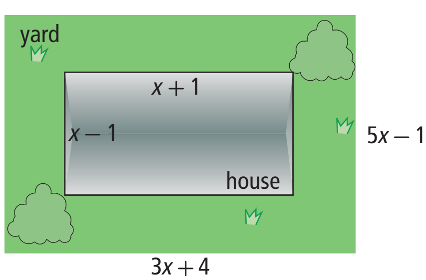
12. Factor.

a) $x^2 - 10x + 9$

b) $4a^2 - 5a - 6$

c) $16x^2 - y^2$

13. The diagram below represents a house located on a property.



a) Write an expression for the area of the house in simplified form.

b) Write an expression for the area of the yard in simplified form.