CHAPTER

# 3

# Right Triangle Trigonometry

Suppose you need to calculate the distance across a river for the construction of a bridge or the height of a building or monument. Each of these distances can be calculated using the properties of right triangles, similar triangles, and trigonometry. Trigonometry is the branch of mathematics that studies the relationships between angles and the lines that form them in triangles. It was first developed for use in astronomy and geography. Today, trigonometry is used in surveying, navigation, engineering, construction, and the sciences to explore the relationships between the side lengths and angles of triangles.

## Big Ideas

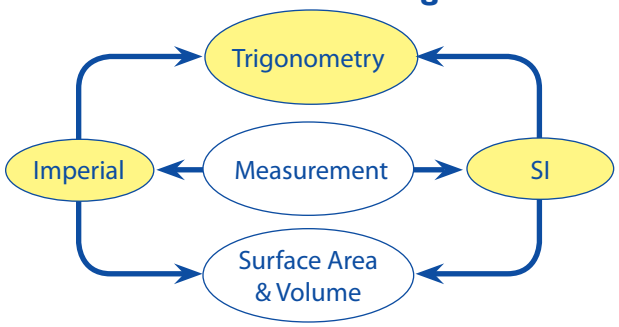
When you have completed this chapter, you will be able to ...

- apply the Pythagorean theorem and primary trigonometric ratios to solve problems involving right triangles
- solve problems involving indirect and direct measurement
- solve right triangles

**Key Terms**

hypotenuse
opposite side
adjacent side
tangent ratio
sine ratio
cosine ratio
primary trigonometric ratios

### Your Measurement Organizer



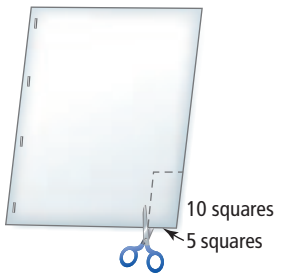
**Astronomer**

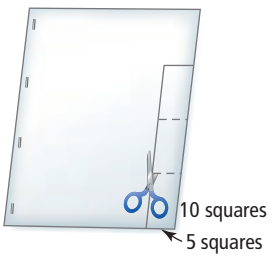
Astronomers study matter in outer space and the celestial bodies. They study their compositions, motions, and origins. Astronomers usually focus their work on planetary science, solar astronomy, the origin and evolution of stars, and the formation of galaxies. Extragalactic astronomy is the study of distant galaxies. It includes studying how distant galaxies move, when they collide, and how they transform as a result of this interaction.

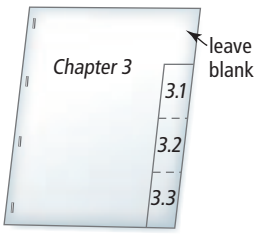
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 3.

1. Staple four sheets of single-sided grid paper together, along the left edge. Make sure the grid sides face down.
2. Make a mark ten squares up from the bottom right edge of the top sheet. Cut through the top three sheets about five squares in from this mark as shown.
3. Cut through the top two sheets up ten more squares. As you do this, you will form tabs along the right side. Continue, until you have four tabs.
4. Label the Foldable™ as shown. On the back of the Foldable™, write the title What I Need to Work On.



# 3.1 The Tangent Ratio

Vancouver, British Columbia

**Focus on ...**

- explaining the relationships between similar triangles and the definition of the tangent ratio
- identifying the hypotenuse, opposite side, and adjacent side for a given acute angle in a right triangle
- developing strategies for solving right triangles
- solving problems using the tangent ratio

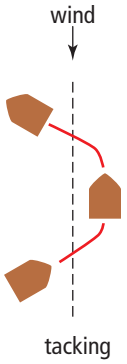
In addition to the Pacific Ocean, there are many lakes in Western Canada that are ideal for sailing. One important aspect of boating is making sure you get where you want to go. Navigation is an area in which trigonometry has played a crucial role; and it was one of the early reasons for developing this branch of mathematics.

People have used applications of trigonometry throughout history. The Egyptians used features of similar triangles in land surveying and when building the pyramids. The Greeks used trigonometry to tell the time of day or period of the year by the position of the various stars. Trigonometry allowed early engineers and builders to measure angles and distances with greater precision. Today, trigonometry has applications in navigating, surveying, designing buildings, studying space, etc.

## Investigate the Tangent Ratio

**Materials**
- grid paper
- protractor
- ruler

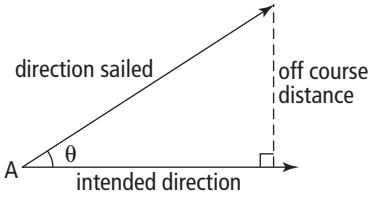
Sailing is a very popular activity. One of the limitations of sailing is that a boat cannot sail directly into the wind. Using a technique called *tacking*, it is possible to sail in almost any direction, regardless of the wind direction. When sailing *on a tack*, you are forced to sail slightly off course and then compensate for the distance sailed when you change direction. You can use trigonometry to determine the distance a boat is off course before changing direction.




**1. a)** On a sheet of grid paper draw a horizontal line 10 cm in length to represent the intended direction.

**b)** Draw a tacking angle, θ, of 30°.

**c)** Every two centimetres, along your horizontal line, draw a vertical line to indicate the off course distance. Label the five triangles you created, △ABC, △ADE, △AFG, △AHI, and △AJK.




**2.** Measure the base and the height for each triangle. Complete the following table to compare the off course distance to the intended direction. In the last column, express the ratio, $\frac{\text{off course distance}}{\text{intended direction}}$, to four decimal places.

| Triangle | Intended Direction | Off Course Distance | $\frac{\text{Off Course Distance}}{\text{Intended Direction}}$ |
|---|---|---|---|
| △ABC | | | |
| △ADE | | | |
| △AFG | | | |
| △AHI | | | |
| △AJK | | | |

**Did You Know?**

- vertices of a triangle are commonly labelled with uppercase letters, for example △ABC
- angles of a triangle are commonly labelled with Greek letter variables
- some common Greek letters used are theta, θ, alpha, α, and beta, β.

**3. a)** The diagram you drew in step 1c) forms a series of nested similar triangles. How do you know the triangles are similar?

**b)** Use your knowledge of similar triangles to help describe how changing the side lengths of the triangle affects the ratio $\frac{\text{off course distance}}{\text{intended direction}}$.

**4. a)** Use your calculator to determine the tangent ratio of 30°. To calculate the tangent ratio of 30°, make sure your calculator is in the degree mode.

Press **C** **TAN** **30** **=**.

**b)** How does the value on your calculator relate to the data in step 2?

**hypotenuse**

- the side opposite the right angle in a right triangle

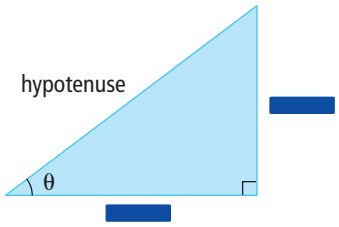
**opposite side**

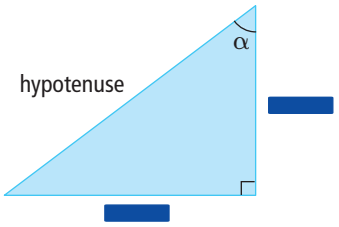
- the side across from the acute angle being considered in a right triangle
- the side that does not form one of the arms of the angle being considered

**adjacent side**

- the side that forms one of the arms of the acute angle being considered in a right triangle, but is not the hypotenuse

**5.** In the two right triangles shown, the **hypotenuse** is labelled and an angle is labelled with a variable. Copy each triangle. Use the words **opposite** and **adjacent** to label the side opposite the angle and the side adjacent to the angle.

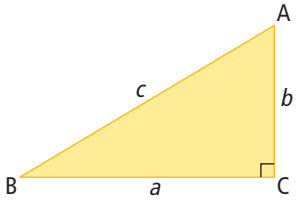



**6. Reflect and Respond**

**a)** Use your results from steps 1 to 4 and the terminology from step 5 to describe a formula you could use to calculate the tangent ratio of any angle.

**b)** Use your formula to state the tangent ratios for ∠A and ∠B in the following diagram.



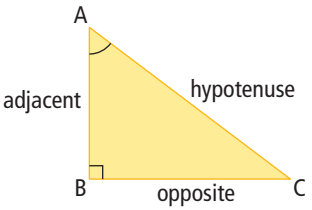
## Link the Ideas

A trigonometric ratio is a ratio of the measures of two sides of a right triangle.

One trigonometric ratio is the **tangent ratio**.

The short form for the tangent ratio of angle A is tan A.

$$\text{tangent A} = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$




**tangent ratio**

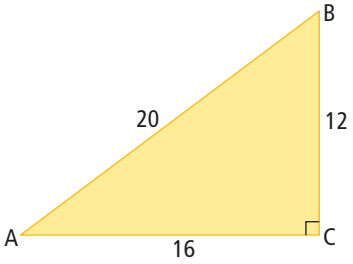
- for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side adjacent
- $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

### Example 1 Write a Tangent Ratio

Write each trigonometric ratio.

**a)** tan A

**b)** tan B




#### Solution

**a)** $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

$\tan A = \frac{BC}{AC}$

$\tan A = \frac{12}{16}$

$\tan A = \frac{3}{4}$

**b)** $\tan B = \frac{\text{opposite}}{\text{adjacent}}$

$\tan B = \frac{AC}{BC}$

$\tan B = \frac{16}{12}$

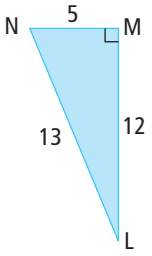
$\tan B = \frac{4}{3}$

### Your Turn

Calculate each trigonometric ratio.

**a)** tan L

**b)** tan N



## Example 2 Calculate a Tangent and an Angle

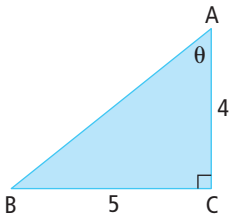
**a)** Calculate tan 25° to four decimal places.

**b)** Draw a triangle to represent $\tan \theta = \frac{5}{4}$. Calculate the angle θ to the nearest tenth of a degree.

### Solution

**a)** $\tan 25° \approx 0.4663$

**b)** Since $\tan \theta = \frac{5}{4}$, the side opposite the angle θ is labelled 5 and the side adjacent to the angle θ is labelled 4.




The inverse function on a calculator allows you to apply the tangent ratio in reverse. If you know the ratio, you can calculate the angle whose tangent this ratio represents.

$\tan \theta = \frac{5}{4}$

$\theta = \tan^{-1}\left(\frac{5}{4}\right)$

$\theta = 51.340\ldots°$

The angle θ is 51.3°, to the nearest tenth of a degree.

### Your Turn

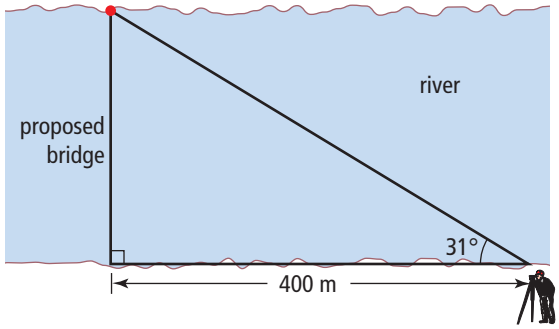
Explore your particular calculator to determine the sequence of keys required. Then, calculate each tangent ratio and angle.

| θ | Tan θ |
|---|---|
| 27° | |
| 45° | |
| 57° | |

| θ | Tan θ |
|---|---|
| | 0.5095 |
| | 0.5543 |
| | 1.4653 |

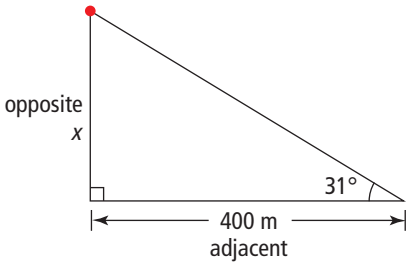
## Example 3 Determine a Distance Using the Tangent Ratio

A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor uses a theodolite to measure angles. The surveyor measures a 31° angle to the bridge site across the river. What is the width of the river, to the nearest metre?




### Solution

Let $x$ represent the distance across the river.
Identify the sides of the triangle in reference to the given angle of 31°.

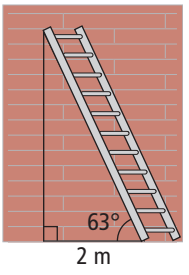



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 31° = \frac{x}{400}$$
$$400(\tan 31°) = x$$
$$240.344\ldots = x$$

To the nearest metre, the width of the river is 240 m.

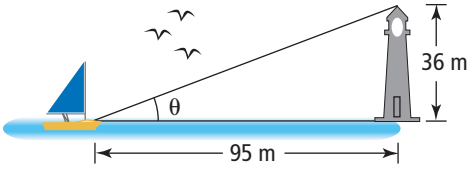
### Your Turn

A ladder leaning against a wall forms an angle of 63° with the ground. How far up the wall will the ladder reach if the foot of the ladder is 2 m from the wall?



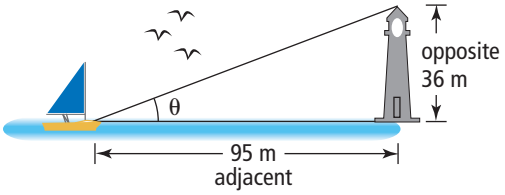
## Example 4 Determine an Angle Using the Tangent Ratio

A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Calculate the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.




### Solution

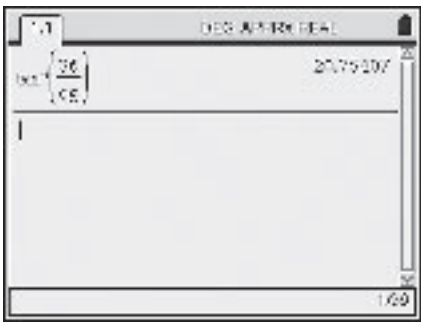
Identify the sides of the triangle in reference to the angle of θ.




$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{36}{95}$$

$$\theta = 20.754\ldots$$



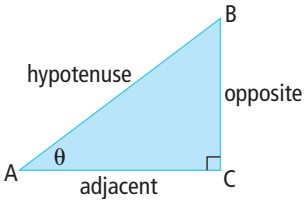
The angle from the boat to the top of the lighthouse is approximately 21°.

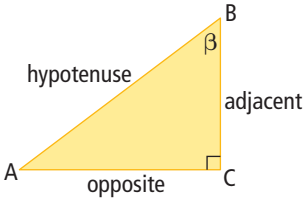
## Your Turn

A radio transmission tower is to be supported by a guy wire. The wire reaches 30 m up the tower and is attached to the ground a horizontal distance of 14 m from the base of the tower. What angle does the guy wire form with the ground, to the nearest degree?

## Key Ideas

- In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal.
- The sides of a right triangle are labelled according to a reference angle.




- The tangent ratio compares the length of the side opposite the reference angle to the length of the side adjacent to the angle in a right triangle.

$$\tan\theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta}$$

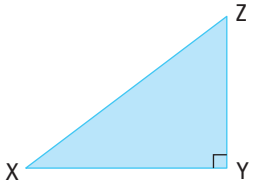
- You can use the tangent ratio to
  - determine the measure of one of the acute angles when the lengths of both legs in a right triangle are known
  - determine a side length if the measure of one acute angle and the length of one leg of a right triangle are known

## Check Your Understanding

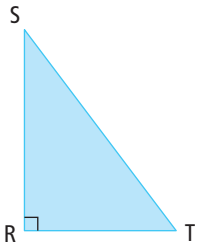
### Practise

**1.** Identify the hypotenuse, opposite, and adjacent sides associated with each specified angle.

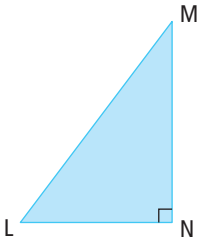
**a)** $\angle X$




**b)** $\angle T$




**c)** $\angle L$




**2.** Draw right $\triangle DEF$ in which $\angle F$ is the right angle.

**a)** Label the leg opposite $\angle D$ and the leg adjacent to $\angle D$.

**b)** State the tangent ratio of $\angle D$.

**3.** Determine each tangent ratio to four decimal places using a calculator.

**a)** tan 74°
**b)** tan 45°
**c)** tan 60°
**d)** tan 89°
**e)** tan 37°
**f)** tan 18°

**4.** Determine the measure of each angle, to the nearest degree.

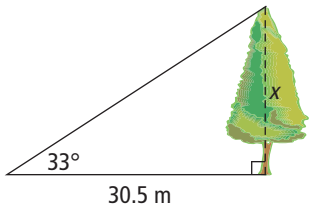
**a)** $\tan A = 0.7$
**b)** $\tan \theta = 1.75$
**c)** $\tan \beta = 0.5543$
**d)** $\tan C = 1.1504$

**5.** Draw and label a right triangle to illustrate each tangent ratio. Then, calculate the measure of each angle, to the nearest degree.

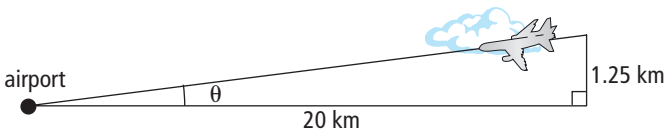
**a)** $\tan \alpha = \frac{2}{3}$
**b)** $\tan B = \frac{5}{2}$

**6.** Determine the value of each variable. Express your answer to the nearest tenth of a unit.

**a)**




**b)**




**7.** Kyle Shewfelt, from Calgary, AB, was the Olympic floor exercise champion in Athens in 2004. Gymnasts perform their routines on a 40-ft by 40-ft mat. They use the diagonal of the mat because it gives them greater distance to complete their routine.

**a)** Use the tangent ratio to determine the angle of the gymnastics run relative to the sides of the mat.
**b)** To the nearest foot, how much longer is the diagonal of the mat than one of its sides?

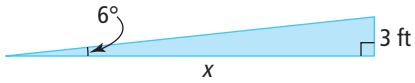
**Did You Know?**

The Franco-Albertan flag was created by Jean-Pierre Grenier. The flag was adopted by the Association canadienne-française de l'Alberta in March 1982.

## Apply

**8.** Claudette wants to calculate the angles of the triangle containing the fleur-de-lys on the Franco-Albertan flag. She measures the legs of the triangle to be 154 cm and 103 cm. What are the angle measures?

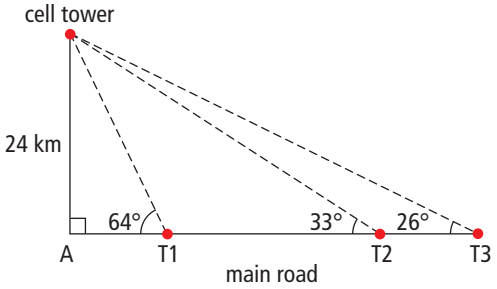
**9.** A ramp enables wheelchair users and people pushing wheeled objects to more easily access a building.



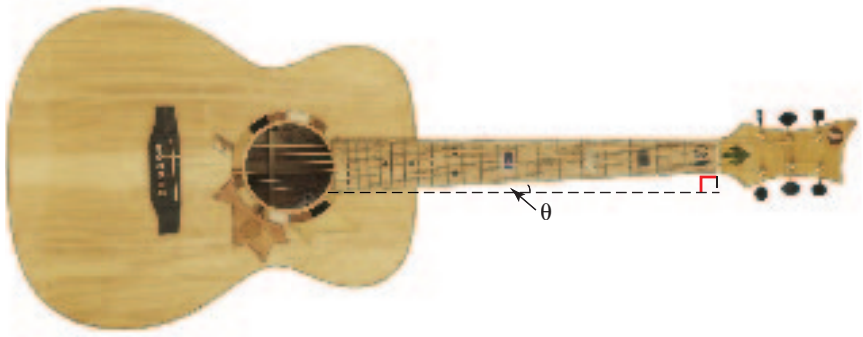

**a)** Determine the horizontal length, x, of the ramp shown. State your answer to the nearest foot.

**b)** For a safe ramp, the ratio of vertical distance : horizontal distance needs to be less than 1 : 12. Would the ramp shown be considered a safe ramp? Explain.

**10.** **Unit Project** A satellite radio cell tower provides signals to three substations, T1, T2, and T3. The three substations are each located along a stretch of the main road. The cell tower is located 24 km down a road perpendicular to the main road. A surveyor calculates the angle from T1 to the cell tower to be 64°, from T2 to the cell tower to be 33°, and from T3 to the cell tower to be 26°. Calculate the distance of each substation from the intersection of the two roads. Express your answers to the nearest tenth of a kilometre.




**11.** In the construction of a guitar, it is important to consider the tapering of the strings and neck. The tapering affects the tone that the strings make. For the *Six String Nation Guitar* shown, suppose the width of the neck is tapered from 56 mm to 44 mm over a length of 650 mm. What is the angle of the taper for one side of the guitar strings?




**Did You Know?**

The *Six String Nation Guitar*, nicknamed *Voyageur*, is made from 63 pieces of history and heritage, from every part of Canada. It represents many different cultures, communities, and characters. The guitar is made from pieces of wood, bone, steel, shell, and stone from every province and territory. It literally embodies Canadian history.

**12.** When approaching a runway, a pilot needs to maneuver the aircraft, so that it can approach the runway at a constant angle of 3°. A pilot landing at Edmonton International Airport begins the final approach 30 380 ft from the end of the runway. At what altitude should the aircraft be when beginning the final approach? State your answer to the nearest foot.

**13.** The Idaà Trail is a traditional route of the Dogrib, an Athapaskan-speaking group of Dene. It stretches from Great Bear Lake to Great Slave Lake, in the Northwest Territories. Suppose a hill on the trail climbs 148 ft vertically over a horizontal distance of 214 ft.

**a)** Calculate the angle of steepness of the hill.

**b)** How far would you have to climb to get to the top of the hill?

## Extend

**Did You Know?**

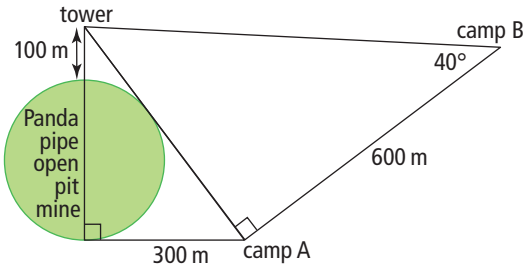
Ekati mine is Canada's first diamond mine. It is located 200 km south of the Arctic Circle in the Northwest Territories. Diamond mines contain pipes, which are cylindrical pits where diamonds are founds.

**14.** One of the Ekati mine's pipes, called the Panda pipe, has northern and southern gates. A communications tower stands 100 m outside the north gate. The tower can be seen from a point 300 m east of the south gate at camp A.

**a)** The distance between camp A and camp B is 600 m. Calculate the diameter of the Panda pipe.

**b)** Calculate the distance from camp B to the tower.

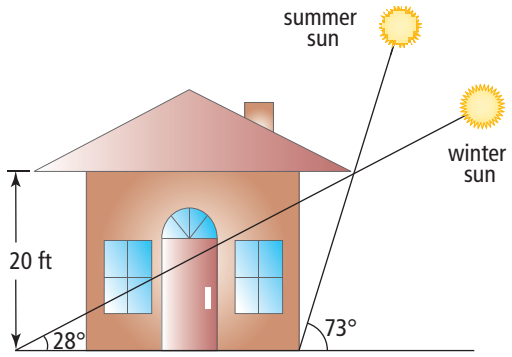
Panda Pipe



**15.** Habitat for Humanity Saskatoon has designed a home that provides passive solar features. The idea is to keep the sun off the outside south wall during the summer months and to have the wall exposed to the sun as much as possible during the winter months. The highest angle of the sun during the summer months is 73°.

**a)** Suppose the wall of the house is 20 ft tall. How much overhang on the roof trusses should be provided so that the shadow of the noonday sun reaches the bottom of the wall during the summer months?

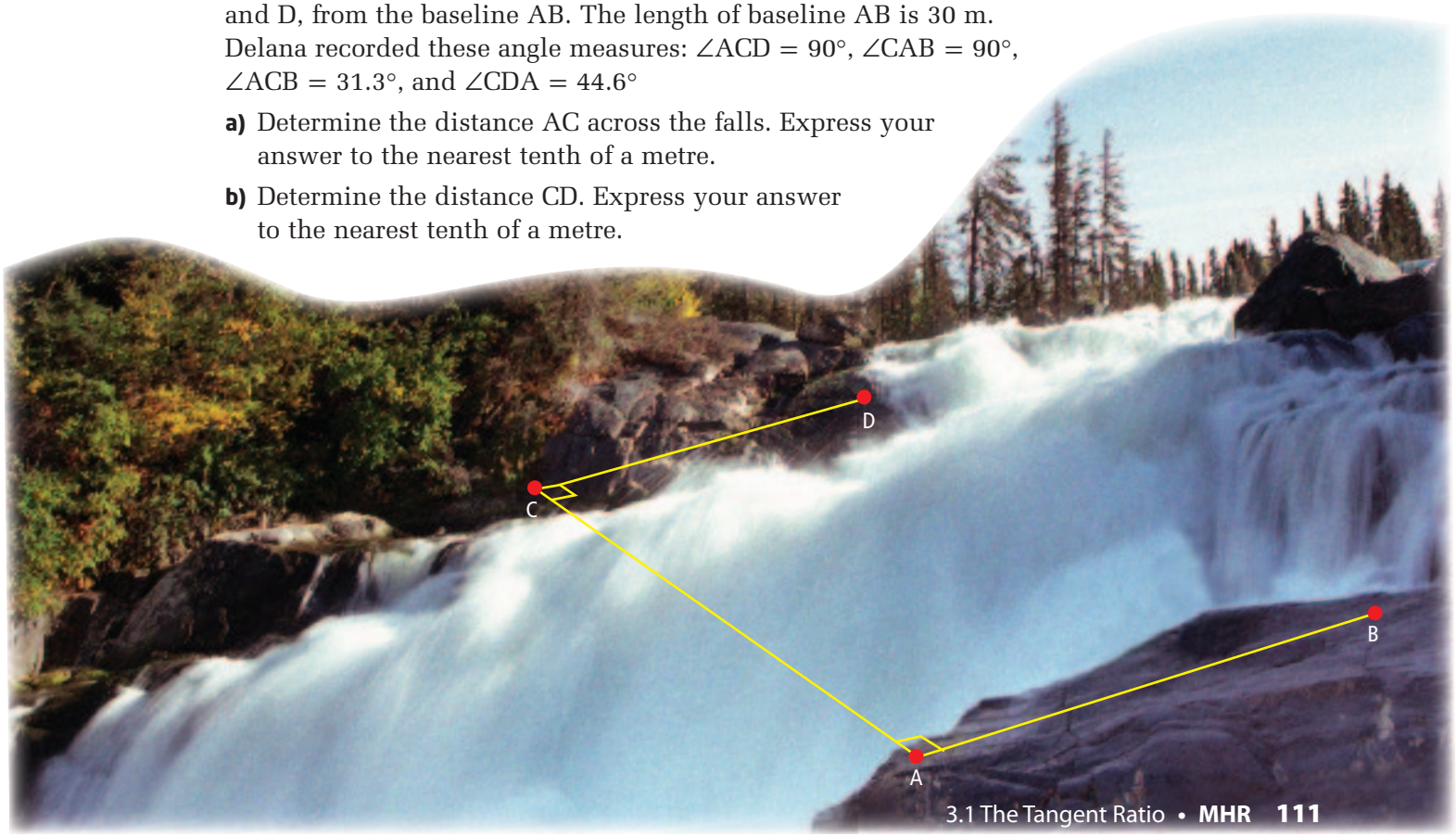
**b)** The lowest angle of the sun during the winter months is 28°. What height of the wall will be in direct sunlight during the winter months?



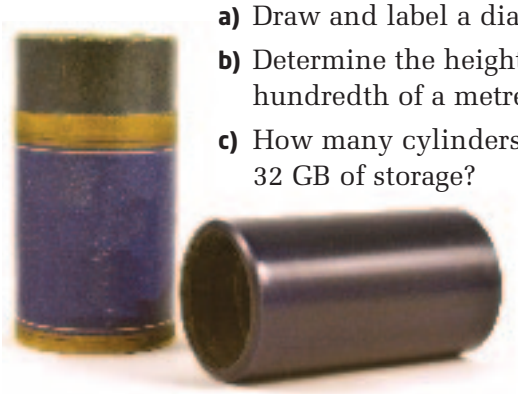

**16.** Nistowiak Falls, located in Lac LaRonge Provincial Park is one of the highest waterfalls in Saskatchewan. Delana, a surveyor, needs to measure the distance across the falls. She sighted two points, C and D, from the baseline AB. The length of baseline AB is 30 m. Delana recorded these angle measures: $\angle ACD = 90°$, $\angle CAB = 90°$, $\angle ACB = 31.3°$, and $\angle CDA = 44.6°$

**a)** Determine the distance AC across the falls. Express your answer to the nearest tenth of a metre.

**b)** Determine the distance CD. Express your answer to the nearest tenth of a metre.



**17.** **Unit Project** The first sound recordings were done on wax cylinders that were 5 cm in diameter and 10 cm long. Wax cylinders were capable of recording about 2 min of sound. Modern music storage devices can have tremendous memory and store thousands of songs. Janine calculated the number of wax cylinders needed to match a 32 GB storage capacity. Imagine that these cylinders are stacked one on top of another. From a distance of 10 m, the angle of elevation to the top of the stack would be 89.5°.



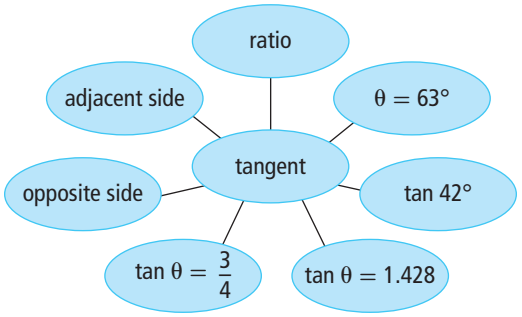
**a)** Draw and label a diagram to represent the situation.

**b)** Determine the height of the stack of cylinders, to the nearest hundredth of a metre.

**c)** How many cylinders would need to be stacked to match 32 GB of storage?

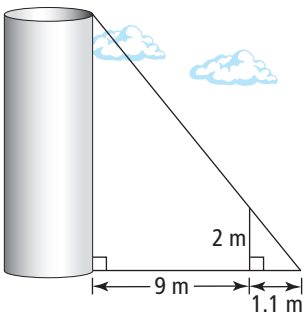
## Create Connections

**18.** Copy the following graphic organizer. For each item, describe its meaning and how it relates to the tangent ratio.




**19.** Draw a right triangle in which the tangent ratio of one of the acute angles is 1. Describe the triangle.

**20.** Devin stores grain in a cylindrical granary. Suppose Devin places a 2-m-tall board 9 m from the granary and 1.1 m away from a point on the ground. Describe how Devin could use trigonometry to calculate the angle formed with the ground and the top of the granary. Then, determine this angle.

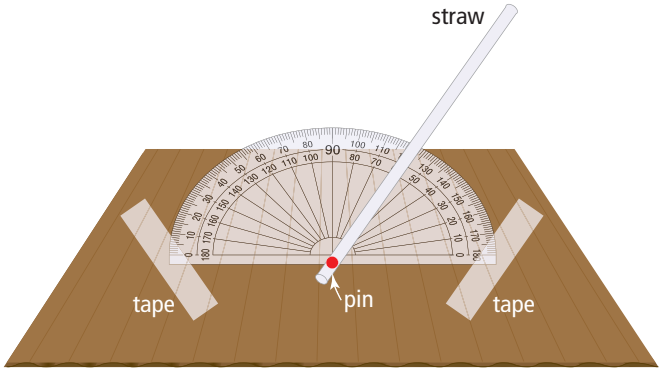


**21.** **MINI LAB** When measuring inaccessible distances, a surveyor can take direct measurements using a transit. A transit can measure both horizontal and vertical angles.

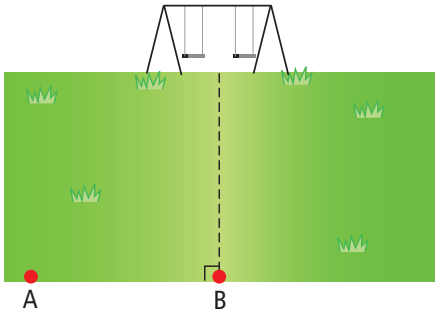
**Materials**

- piece of cardboard
- large protractor
- drinking straw
- tape
- pin
- measuring tape

**Step 1** Construct a transit as shown in the diagram. Pin the straw at the centre of the protractor.




**Step 2** Explain how a transit could be used to assess the distance to an object. Hint: You will need to draw and measure a baseline. This is the line from A to B in the diagram.




**Step 3** To calculate the distance to some objects in your schoolyard, use your transit to measure the required angles.

| Object | Length of Baseline AB | Measure of ∠A | Distance to the Object |
|---|---|---|---|
| | | | |

# 3.2 The Sine and Cosine Ratios

**Focus on ...**

- using the sine ratio and cosine ratio to solve problems involving right triangles
- solving problems that involve direct and indirect measurement

The first suspension bridge in Vancouver was built in 1889 by George Mackay. He had built a cabin along the canyon wall and needed a bridge to conveniently access his cabin. Mathematical tools, such trigonometry, can enable you to calculate distances that cannot be measured directly, such as the distance across a river canyon.

In section 3.1, you learned about the tangent ratio. This ratio compares the opposite and adjacent side lengths in reference to an acute angle in a right triangle. There are two other trigonometric ratios that compare the lengths of the sides of a right triangle. These ratios, called the sine ratio and cosine ratio, involve the hypotenuse.

**Materials**

- protractor
- ruler

## Investigate Trigonometric Ratios

**1.** Choose an angle between 10° and 80°. This will be your reference angle.

**2. a)** Draw right triangle ABC, using your reference angle.

**b)** Draw three right triangles similar to $\triangle ABC$ using the same reference angle.

3. Write the equivalency statements that show the similarity of each triangle to $\triangle ABC$.

4. Label the sides of each triangle. Use the terms hypotenuse, opposite, and adjacent according to the reference angle.

5. Measure the sides of each triangle. You may wish to record the measurements in a table similar to this one or using spreadsheet software. Express each ratio to four decimal places.

| Triangle | Length of Hypotenuse | Length of Opposite Side | Length of Adjacent Side | Ratio of Opposite to Adjacent | Ratio of Opposite to Hypotenuse | Ratio of Adjacent to Hypotenuse |
|---|---|---|---|---|---|---|
| $\triangle$ | | | | | | |

6. Complete a similar table using the other acute angle in each triangle as your reference angle.

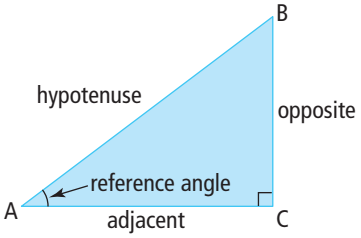
7. **Reflect and Respond** Discuss with a partner the results of the calculations of the ratios. Describe any similarities or patterns that you notice.

8. What relationships do you observe among the ratios for the angles between the two tables?

9. What conclusions can you make about how the ratios relate to your reference angle?

## Link the Ideas

The short form for the **sine ratio** of angle A is sin A. The short form for the **cosine ratio** of angle A is cos A.




$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

**sine ratio**

- for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse
- $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

**cosine ratio**

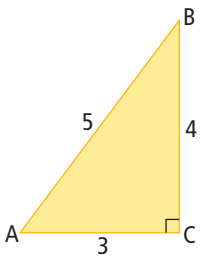
- for an acute angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse
- $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

## Example 1 Write Trigonometric Ratios

Write each trigonometric ratio.

**a)** sin A **b)** cos A

**c)** sin B **d)** cos B




### Solution

**a)** $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin A = \dfrac{BC}{AB}$

$\sin A = \dfrac{4}{5}$

**b)** $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos A = \dfrac{AC}{AB}$

$\cos A = \dfrac{3}{5}$

**c)** $\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin B = \dfrac{AC}{AB}$

$\sin B = \dfrac{3}{5}$

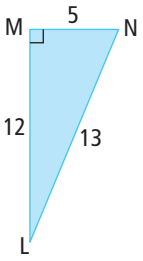
**d)** $\cos B = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos B = \dfrac{BC}{AB}$

$\cos B = \dfrac{4}{5}$

### Your Turn

Write each trigonometric ratio.




**a)** sin L **b)** cos N

**c)** cos L **d)** sin N

## Example 2 Evaluate Trigonometric Ratios

The **primary trigonometric ratios** and their inverses can be evaluated using technology.

**primary trigonometric ratios**

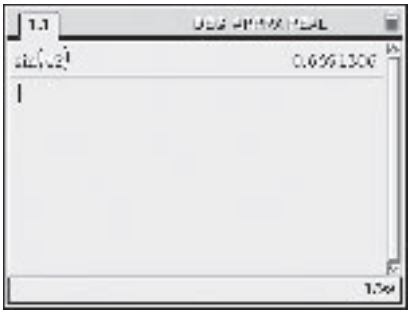
- the three ratios, sine, cosine, and tangent, defined in a right triangle

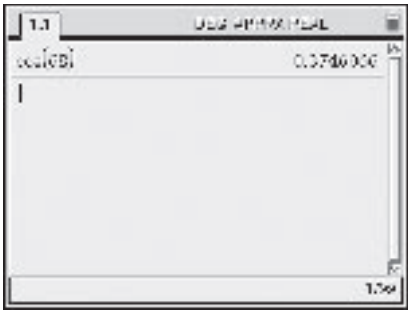
**a)** Evaluate each ratio, to four decimal places.
$\sin 42°$ $\cos 68°$

**b)** Determine each angle measure, to the nearest degree.
$\sin \theta = 0.4771$ $\cos \beta = 0.7225$

### Solution

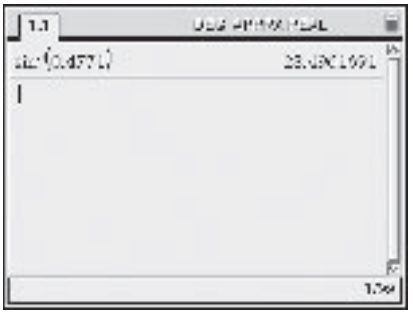
**a)** $\sin 42° \approx 0.6691$ $\cos 68° \approx 0.3746$

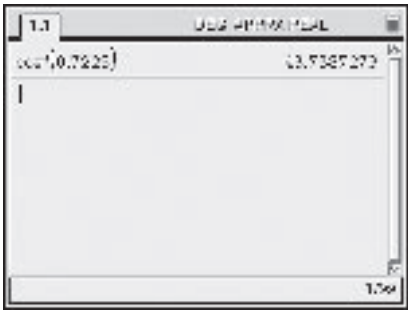




**b)** $\sin \theta = 0.4771$
$\theta = \sin^{-1}(0.4771)$
$\theta \approx 28°$

$\cos \beta = 0.7225$
$\beta = \cos^{-1}(0.7225)$
$\beta \approx 44°$





### Your Turn

**a)** Evaluate each trigonometric ratio, to four decimal places.
$\sin 60°$ $\sin 30°$ $\cos 45°$

**b)** What is the measure of each angle, to the nearest degree?
$\sin \beta = 0.4384$ $\cos \theta = 0.2079$

## Example 3 Determine an Angle Using a Trigonometric Ratio

In the World Cup Downhill held at Panorama Mountain Village in British Columbia, the skiers raced 3514 m down the mountain. If the vertical height of the course was 984 m, determine the average angle of the ski course with the ground. Express your answer to the nearest tenth of a degree.

### Solution

Visualize the problem by sketching a diagram to organize the information.


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

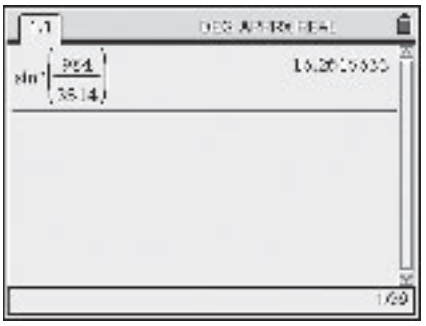
$$\sin\theta = \frac{984}{3514}$$

$$\theta = \sin^{-1}\left(\frac{984}{3514}\right)$$

$$\theta = 16.2615\ldots^\circ$$

The average angle of the ski course is 16.3°, to the nearest tenth of a degree.

For the unknown angle, the lengths of the opposite side and hypotenuse are known. So, use the sine ratio.



### Your Turn

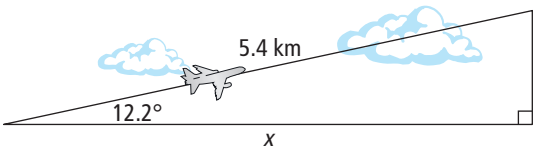
A guy wire supporting a cell tower is 24 m long. If the wire is attached at a height of 17 m up the tower, determine the angle that the guy wire forms with the ground.

## Example 4 Determine a Distance Using a Trigonometric Ratio

A pilot starts his takeoff and climbs steadily at an angle of 12.2°. Determine the horizontal distance the plane has travelled when it has climbed 5.4 km along its flight path. Express your answer to the nearest tenth of a kilometre.

### Solution

Organize the information by sketching a diagram to illustrate the problem.



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 12.2° = \frac{x}{5.4}$$

$$5.4(\cos 12.2°) = x$$

$$5.278\ldots = x$$

**How do you decide which trigonometric ratio to use?**

The horizontal distance travelled by the airplane is approximately 5.3 km.

### Your Turn

Determine the height of a kite above the ground if the kite string extends 480 m from the ground and makes an angle of 62° with the ground. Express your answer to the nearest tenth of a metre.

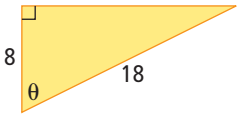
## Key Ideas

- The sine ratio and cosine ratio compare the lengths of the legs of a right triangle to the hypotenuse.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- The sine and cosine ratios can be used to calculate side lengths and angle measures of right triangles.
- Visualizing the information that you are given and that you need to find is important. It helps you determine which trigonometric ratio to use and whether to use the inverse trigonometric ratio.

Determine the value of θ, to the nearest degree.




$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{8}{18}$$

$$\theta = \cos^{-1}\left(\frac{8}{18}\right)$$

$$\theta = 63.6122\ldots°$$

Angle θ is approximately 64°.

# Check Your Understanding

## Practise

**1.** Evaluate each trigonometric ratio to four decimal places.

**a)** cos 34°

**b)** cos 56.4°

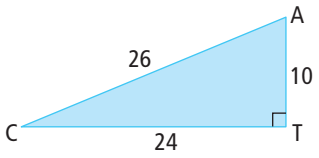
**c)** sin 62.9°

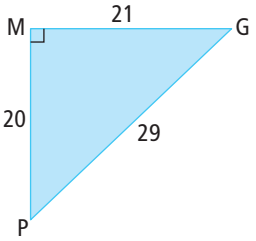
**d)** sin 19.6°

**e)** sin 90°

**f)** cos 80°

**2.** Write each trigonometric ratio in lowest terms.




**a)** sin A

**b)** sin C

**c)** cos C

**d)** cos G

**e)** sin P

**f)** cos P

**3.** Calculate the measure of each angle, to the nearest degree.

**a)** $\cos A = 0.4621$

**b)** $\cos \theta = 0.6779$

**c)** $\sin \beta = 0.5543$

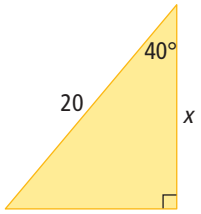
**d)** $\sin C = 1.232$

**e)** $\sin \alpha = \frac{1}{2}$

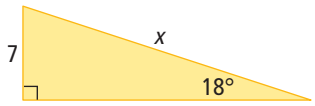
**f)** $\cos B = \frac{3}{4}$

**4.** Determine each length of $x$. Express your answer to the nearest tenth of a unit.

**a)**

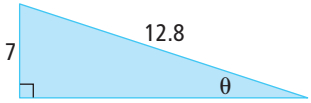



**b)**

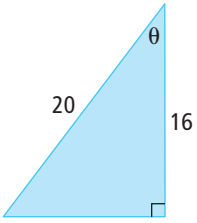



**5.** Determine the measure of each angle θ. Express your answer to the nearest tenth of a degree.

**a)**

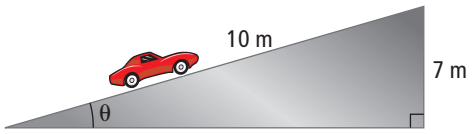



**b)**

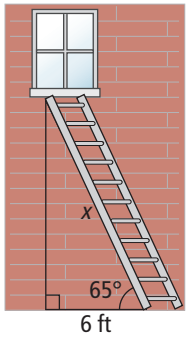


**6.** Determine the value of each variable. Express each answer to the nearest tenth of a unit.

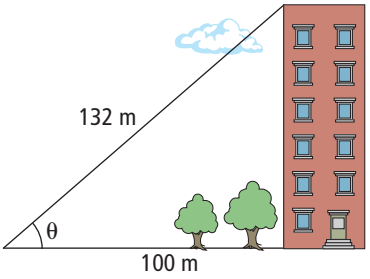
**a)**



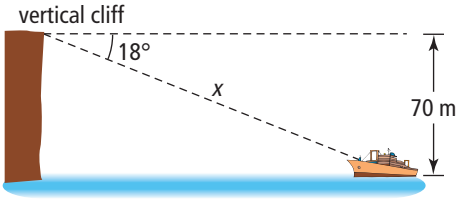

**b)**




**c)**

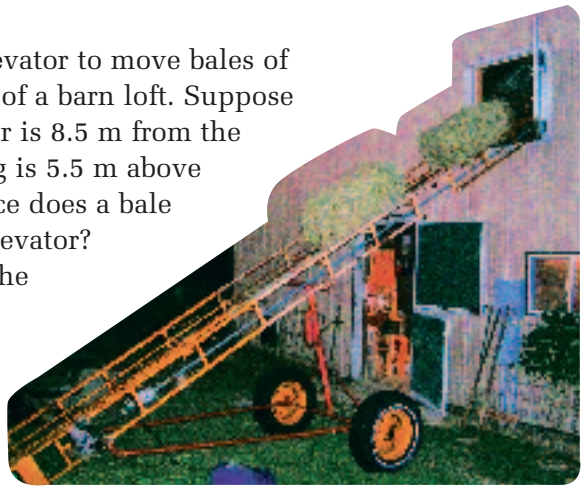



**d)**




## Apply

**7.** Some farms use a hay elevator to move bales of hay to the second storey of a barn loft. Suppose the bottom of the elevator is 8.5 m from the barn and the loft opening is 5.5 m above the ground. What distance does a bale of hay travel along the elevator? Express your answer to the nearest tenth of a metre.



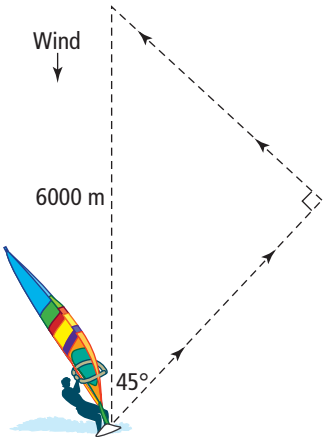
**8.** A 30-m-long line is used to hold a helium weather balloon. Due to a breeze, the line makes a 75° angle with the ground.

**a)** Draw a right triangle to model the problem. Label the measurements you know. Use variables to represent the unknown measurements.

**b)** Use trigonometry to determine the height of the balloon. Express your answer to the nearest tenth of a metre.

**9.** Oil rigs are found throughout Alberta. They play a crucial role in the search for crude oil and natural gas products. Determine the height of a rig if a 52-m-long guy wire is attached to the top of the rig and forms an angle of 50° with the ground. Express your answer to the nearest tenth of a metre.

**10.** Gerry is windsurfing at Squamish Pit, just north of Vancouver, BC. In order to get upwind 6000 m, Gerry sails at a 45° angle to the wind and then turns 90° and heads toward his original destination. How far would he have to sail to get directly upwind the 6000 m? Express your answer to the nearest tenth of a metre.



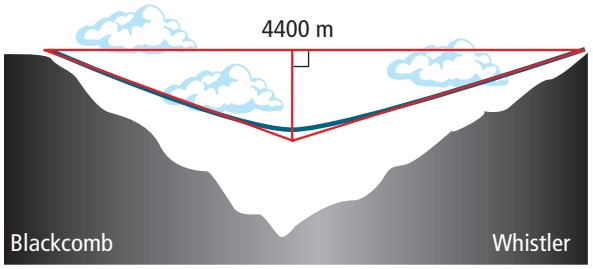

**11.** Toonik Tyme is Nunavut's biggest spring festival, celebrating the return of spring. To set up one of the holes for ice golf, the organizers cleared a track in the form of a right angle. The distance from the teeing area to the vertex of the right angle is 180 yd.

**a)** The angle from the teeing area to the flag at the other end of the track is 34°. Draw a diagram of the ice golf hole.

**b)** Determine the direct distance from the teeing area to the flag, to the nearest yard.

**c)** How much shorter would the direct distance be than following the track?

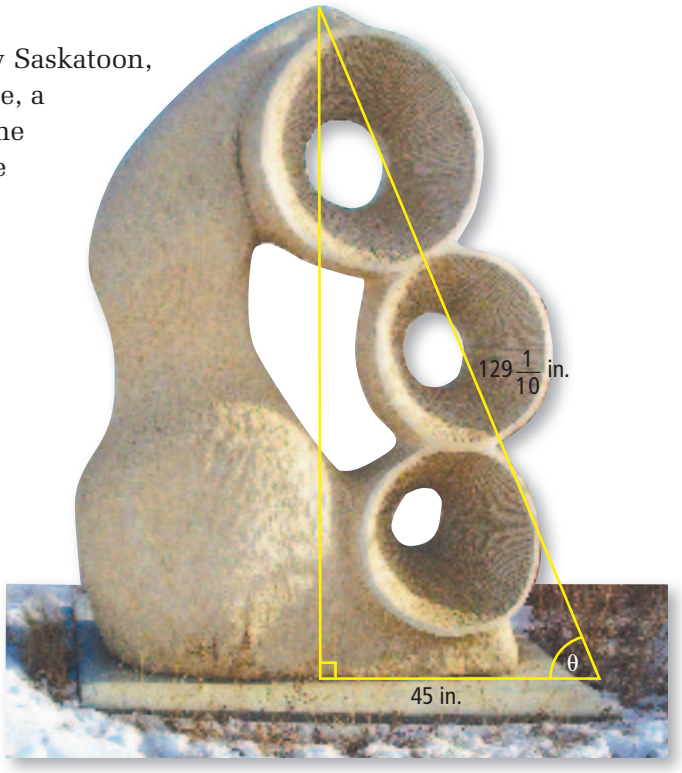
**12.** The PEAK 2 PEAK Gondola connects two mountain ski resorts, Whistler Mountain and Blackcomb Mountain, near Vancouver, BC. The straight-line distance between the two peaks is 4400 m. The gondola travels 4600 m along a cable that sags in the centre. Determine the approximate angle that the cable makes with the horizontal, to the nearest degree.



**13.** *Dream Maker* is a dolomite sculpture by Saskatoon, SK, artist Floyd Wanner. In the sculpture, a line can be drawn that passes through the centre of the two upper circles. Suppose this line is $129\frac{1}{10}$ in. long and the base line is 45 in. Describe how you might calculate

**a)** the height of *Dream Maker*

**b)** the angle between the baseline and the line through the two upper circles




**Did You Know?**

Dolomite is a rock consisting mainly of calcium and magnesium carbonate. It is mined around the world, including in western Canada. Dolomite is used to improve garden soil. It is also used as an ornamental stone, and in construction materials.

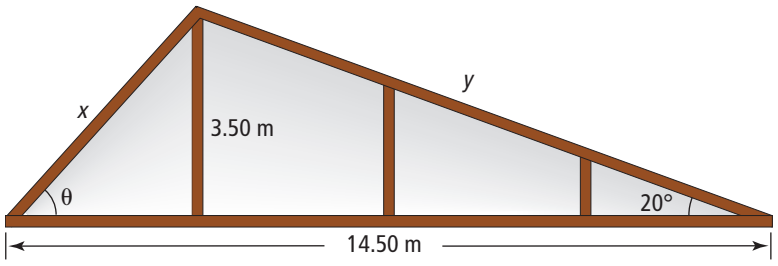
**14.** At Wapiti Valley Ski Area in Saskatchewan, the beginner slope is inclined at an angle of 11.6° from the horizontal and the advanced slope at an angle of 26.9° from the horizontal.

**a)** Suppose Francis skis 1200 m down the advanced slope while Barbara skis the same distance down the beginner slope. Predict who will cover a greater horizontal distance. Justify your prediction.

**b)** Calculate the difference between the horizontal distances for the two skiers, to the nearest tenth of a metre.

## Extend

**15.** Michael is building a cabin at Cold Lake, AB. He has drawn a diagram to design his roof truss. Determine the values of $x$, $y$, and $\theta$.




**16.** An equilateral triangle is inscribed in a circle. Determine the side length of the triangle if the diameter of the circle is 200 cm.

## Create Connections

**Materials**

- 1 m of foam pipe insulation, cut lengthwise
- marble or small steel ball
- eight to ten thick books or bricks or a chair
- masking tape
- measuring tape
- table

**17.** **MINI LAB** Work with a partner or in small groups to explore how varying the angle of a ramp in ski jumping changes the launch angle and duration of flight.

**Step 1** Build a ramp similar to the one shown. Place the edge of the ramp at the end of the table. Make a sketch of the right triangle formed by the pipe insulation, books, and table. Include measurements of the length of each leg of the triangle. Determine the angle formed between the pipe insulation and the table.

**Step 2** Place a marble at the top of the ramp. Without pushing, let it roll. Observe the flight path. Mark the place where the marble first lands on the floor, using masking tape. Repeat this step two more times and record the horizontal distance the marble lands from the edge of the table. You may wish to complete a chart similar to this one.

| Sketch of the Triangle | Measure of the Angle (°) | Distance Measured | | |
|---|---|---|---|---|
| | | Trial 1 | Trial 2 | Trial 3 |
| | | | | |

**Step 3** Adjust the ramp so that it curves downward to the table and runs flat along the table for about 20 cm before it reaches the end. Roll the marble down the track and record the distances.

**Step 4** Add a book to the end of the ramp, so that the ramp curves upward as it nears the end. Roll the marble and record your measurements.

**a)** Describe how changing the launch angle of the ramp affects the distance travelled by the marble. Explain why.

**b)** Would changing the angle of the ramp with the table affect the distance the marble travels? Explain.

# 3.3 Solving Right Triangles

Aurora borealis above Churchill, Manitoba

**Focus on ...**

- explaining the relationships between similar right triangles and the definitions of the trigonometric ratios
- solving right triangles, with or without technology
- solving problems involving one or more right triangles

The polar aurora is one of the most beautiful and impressive displays of nature. There have been various attempts to explain the phenomenon of these northern lights. Carl Stormer, a Norwegian scientist, used a network of cameras that simultaneously photographed the aurora. He used the photos to measure the parallax angle shifts and then calculate the height of the aurora.

**Materials**

- metre stick or measuring tape

## Investigate Estimation of Distance

In this investigation you will use the method of parallax to help you estimate the distance to an object.

1. Have a partner stand a distance away from you. Then, mark the floor where each of you is standing using a small piece of paper or other identifying item, such as masking tape. Stretch out your arm with your thumb pointed upward and close your right eye. Line your thumb up with your partner.

**Did You Know?**

If you stretch your arm out in front of your face with your thumb pointing upward, and then close one eye, your thumb appears to shift slightly. This shift is known as *parallax*. Your brain uses this information to figure out how far away from you objects are.

2. Open your right eye and close your left eye. Do not move your outstretched arm. Have your partner move to his or her right until he or she is in line with your thumb again. Then, mark the new location where your partner is standing.

3. Use a metre stick to measure the distance from you to your partner and the distance between your partner's locations.

4. **Reflect and Respond**

   **a)** What is the relationship between the distance to your partner and the distance between your partner's locations? Hint: You may wish to repeat your measurements to help you examine the pattern.

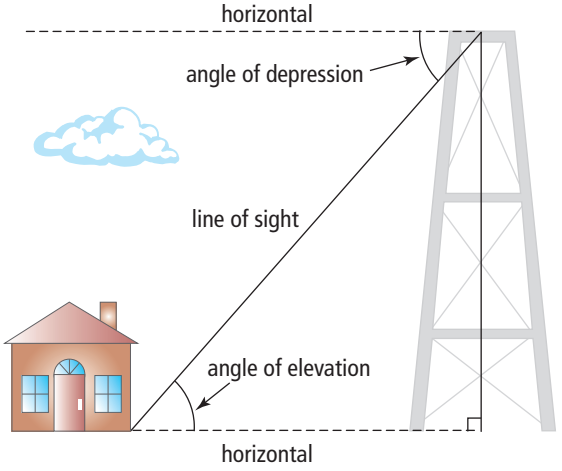
   **b)** Explain how this relationship can help you estimate your distance to an object.

## Link the Ideas

The line of sight is the invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation and an angle of depression.

- An angle of elevation is the angle formed by the horizontal and a line of sight *above* the horizontal.
- An angle of depression refers to the angle formed by the horizontal and a line of sight *below* the horizontal.

Measure the angle of elevation and the angle of depression in the diagram. How are the measures of two angles related?



## Example 1 Use Angle of Elevation to Calculate a Height

Sean wants to calculate the height of the First Nations Native Totem Pole. He positions his transit 19.0 m to the side of the totem pole and records an angle of elevation of 63° to the top of the totem pole. If the height of Sean's transit is 1.7 m, what is the height of the totem pole, to the nearest tenth of a metre?

### Solution

Let $x$ represent the height from the transit to the top of the totem pole.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 63° = \frac{x}{19.0}$$

$$x = 19.0(\tan 63°)$$

$$x = 37.289\ldots$$

Height of totem pole
= height of transit + height from transit to top of pole
= 1.7 + 37.289…
= 38.989…

The height of the First Nations Native Totem Pole is 39.0 m, to the nearest tenth of a metre.

### Your Turn

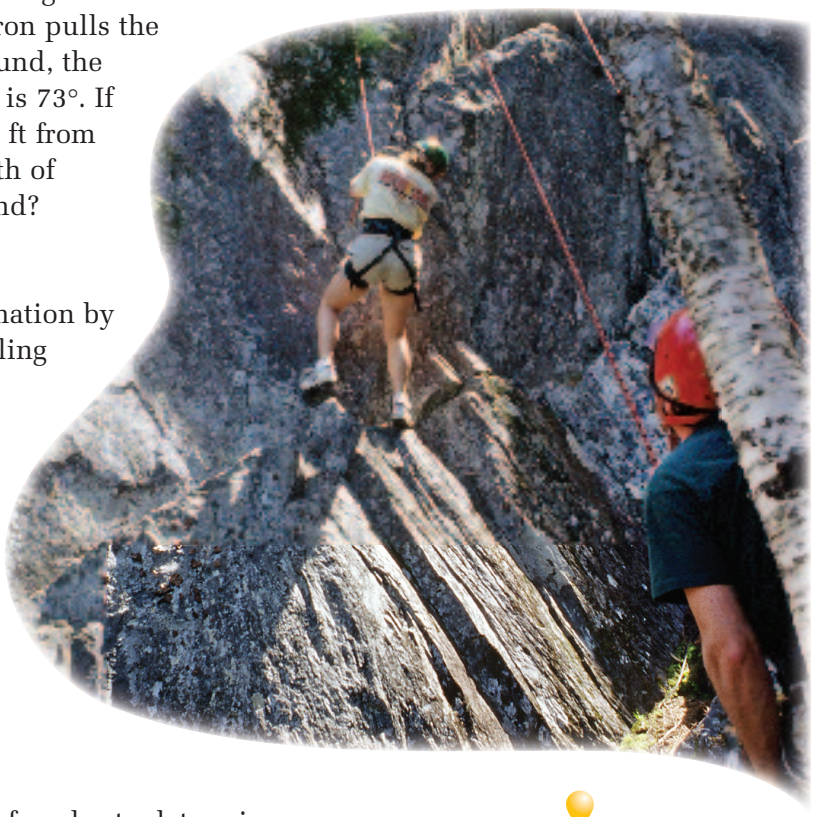
A surveyor needs to determine the height of a large grain silo. He positions his transit 65 m from the silo and records an angle of elevation of 52°. If the height of the transit is 1.7 m, determine the height of the silo, to the nearest metre.

**Did You Know?**

The First Nations Native Totem Pole is in Beacon Hill Park, in Victoria, BC. The totem pole was erected in 1956 and is one of the world's tallest totem poles.

## Example 2 Calculate a Distance Using Angle of Depression

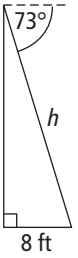
Natalie is rock climbing and Aaron is belaying. When Aaron pulls the rope taut to the ground, the angle of depression is 73°. If Aaron is standing 8 ft from the wall, what length of rope is off the ground?



### Solution

Visualize the information by sketching and labelling a diagram.

Let $h$ represent the length of rope that is off the ground.




Use the properties of angles to determine the angle measure of one of the acute angles inside the right triangle.

$\theta = 90° - 73°$
$\theta = 17°$

The angle that the rope makes at the top with the vertical is 17°.

$$\sin 17° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 17° = \frac{8}{h}$$

$$h = \frac{8}{\sin 17°}$$

$$h = 27.362\ldots$$

The rope off the ground is approximately 27 ft long.

### Your Turn

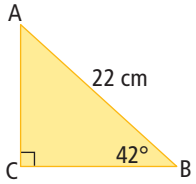
A balloonist decides to use an empty football field for his landing area. When the balloon is directly over the goal post, he measures the angle of depression to the base of the other goal post to be 53.8°. Given that the distance between goal posts in a Canadian football field is 110 yd, determine the height of the balloon.

### Did You Know?

A *belayer* is the person on the ground who secures a climber who is rock climbing. The belayer and climber each wear a harness that attaches to a rope. The belayer controls how much slack is in the rope. It takes skill and concentration to be a successful belayer.

## Example 3 Solve a Right Triangle

Solve the triangle shown. Express each measurement to the nearest whole unit.




### Solution

To *solve* a triangle means to determine the lengths of all unknown sides and the measures of all unknown angles. To solve this triangle, you need to determine the lengths of sides AC and CB and the measure of ∠A.

$\angle A = 180° - (90° + 42°)$ What is the sum of the angles in a triangle?

$\angle A = 48°$

Using ∠B as the reference angle and knowing the length of the hypotenuse, apply the cosine ratio to calculate the length of side CB.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 42° = \frac{CB}{22}$$

$$CB = 22(\cos 42°)$$

$$CB = 16.349\ldots$$

Calculate the length of side AC.

**Method 1: Apply a Trigonometric Ratio**

Since all angles are known, any of the primary trigonometric ratios could be applied.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$ How will you decide which ratio to use?

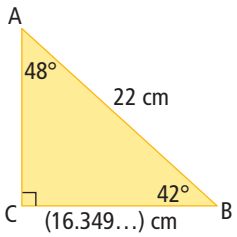
$$\sin 42° = \frac{AC}{22}$$

$$AC = 22(\sin 42°)$$

$$AC = 14.720\ldots$$

**Method 2: Apply the Pythagorean Theorem**

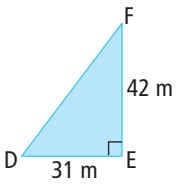
$$AB^2 = AC^2 + CB^2$$
$$22^2 = AC^2 + (16.349\ldots)^2$$
$$484 = AC^2 + 267.295\ldots$$
$$216.704\ldots = AC^2$$
$$\sqrt{216.704\ldots} = AC$$
$$14.720\ldots = AC$$




Angle A measures 48°. Side CB is about 16 cm long and side AC is about 15 cm long.

## Your Turn

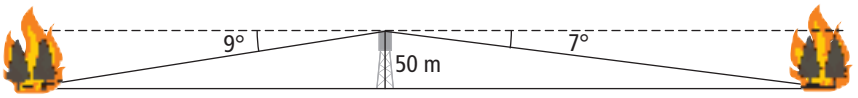
Solve the triangle shown. Express each measurement to the nearest whole unit.




What information are you given? Use the given information as much as possible in your calculations.

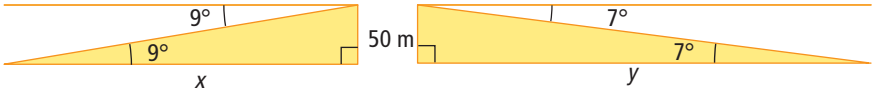
## Example 4 Solve a Problem Using Trigonometry

From a height of 50 m in his fire tower near Francois Lake, BC, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9°. The other fire is due east at an angle of depression of 7°. What is the distance between the two fires, to the nearest metre?




### Solution

Model the problem using right triangles.
Let $x$ and $y$ represent the lengths of the bases of the triangles.




$$\tan 9° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 9° = \frac{50}{x}$$
$$x = \frac{50}{\tan 9°}$$
$$x = 315.687\ldots$$

Use the given angles to find the measure of one acute angle in each right triangle.

$$\tan 7° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 7° = \frac{50}{y}$$
$$y = \frac{50}{\tan 7°}$$
$$y = 407.217\ldots$$

Add to determine the distance between the fires.
$315.687\ldots + 407.217\ldots = 722.904\ldots$

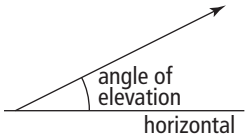
The distance between the fires, to the nearest metre, is 723 m.

### Your Turn

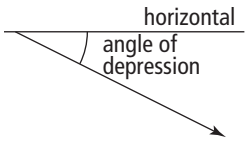
From his hotel window overlooking Saskatchewan Drive in Regina, Ken observes a bus moving away from the hotel. The angle of depression of the bus changes from 46° to 22°. Determine the distance the bus travels, if Ken's window is 100 m above street level. Express your answer to the nearest metre.

## Key Ideas

- An angle of elevation is the angle between the line of sight and the horizontal when an observer looks upward.




- An angle of depression is the angle between the line of sight and the horizontal when the observer looks downward.



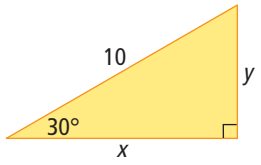

- To solve a triangle means to calculate all unknown angle measures and side lengths.

## Check Your Understanding

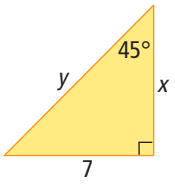
### Practise

**1.** Solve each triangle, to the nearest tenth of a unit.

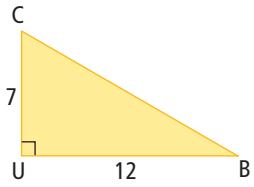
**a)**



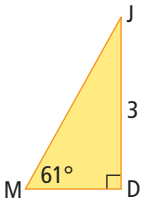

**b)**




**c)**

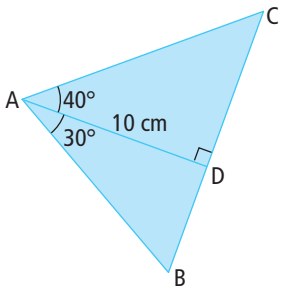



**d)**

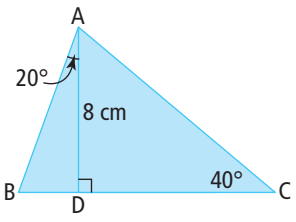



**2.** Calculate the length of BC, to the nearest tenth of a centimetre.

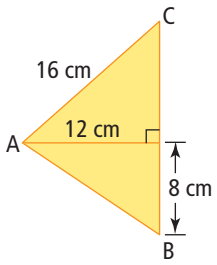
**a)**




**b)**



**3.** Determine the measure of ∠CAB, to the nearest degree.



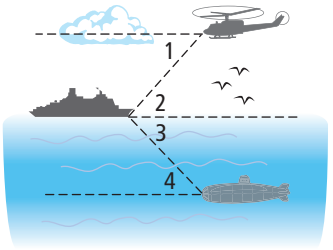

**4.** Describe each angle as it relates to the diagram.

**a)** ∠1

**b)** ∠2

**c)** ∠3

**d)** ∠4

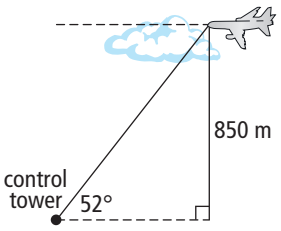



**5.** The heights of several tourist attractions are given in the table. Determine the angle of elevation from a point 100 ft from the base of each attraction to its top.

| | Attraction | Location | Height |
|---|---|---|---|
| **a)** | World's largest fire hydrant | Elm Creek, MB | 29 ft |
| **b)** | World's largest dinosaur | Drumheller, AB | 80 ft |
| **c)** | Saamis Tipi | Medicine Hat, AB | 215 ft |
| **d)** | World's largest tomahawk | Cut Knife, SK | 40 ft |
| **e)** | Igloo church | Inuvik, NT | 78 ft |

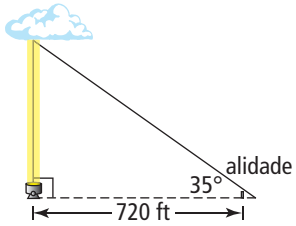
## Apply

**6.** An airplane is observed by an air traffic controller at an angle of elevation of 52°. The airplane is 850 m above the observation deck of the tower. What is the distance from the airplane to the tower? Express your answer to the nearest metre.




**7.** Cape Beale Lighthouse, BC, is on a cliff that is 51 m above sea level. The lighthouse is there to warn boats of the danger of shallow waters and the possibility of rocks close to the shore. The safe distance for boats from this cliff is 75 m. If the lighthouse keeper is 10 m above ground and observes a boat at an angle of depression of 50°, is the boat a safe distance from the cliff? Justify your conclusion.

**8.** At night, it is possible to make precise measurements of cloud height using a search light. An alidade is set 720 ft away from the search light. It measures the angle of elevation to the place where the light strikes the cloud to be 35°. What is the altitude of the cloud? Express your answer to the nearest foot.





alidade

**9.** The working arm of a tower crane is 192 m high and has a length of 71.6 m. Suppose the hook reaches the ground and moves along the arm on a trolley.

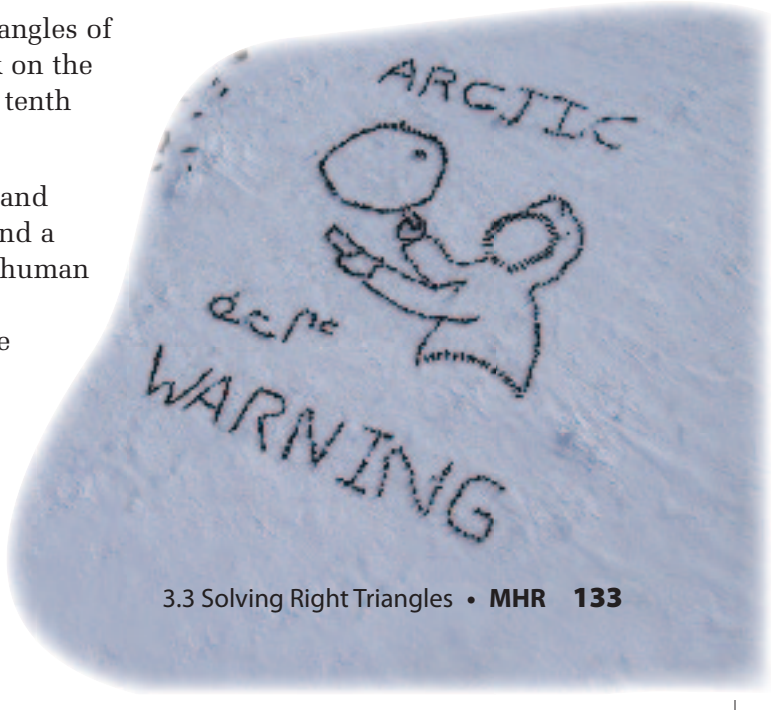

**a)** Determine the maximum distance from the hook to the operator when the trolley is fully extended at 71.6 m and the minimum distance when the trolley is closest to the operator at 8.1 m. Hint: The operator is located at the vertex of the crane.

**b)** Determine the maximum and minimum angles of depression from the operator to the hook on the ground. State your answer to the nearest tenth of a degree.

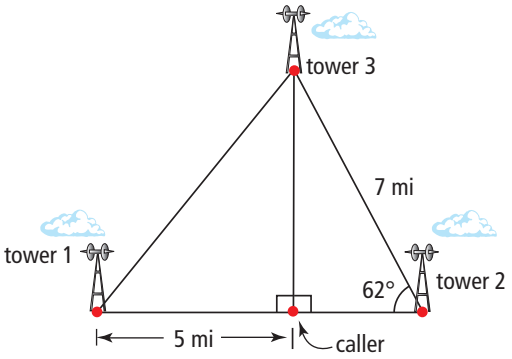
**Did You Know?**

For the 2010 Olympic Games in Vancouver, the Millennium Water Project involved building 1100 condominiums. This project made use of eight tower cranes that lifted steel, concrete, large tools, and generators. The cranes often rise hundreds of feet into the air and can reach out just as far.

**10.** Arctic Wisdom involved children, parents, and Elders gathering on Baffin Island, NU, to send a message. To achieve the best picture of the human image on the sea ice, an aerial photograph was taken. The angle of depression from the helicopter was 58° and the height of the helicopter was 140 m. How far away from the image was the helicopter?



**11.** **Unit Project** A cell phone can be used to send music, but as your location changes, you move in and out of range from one *cell* to the next. Three or more cellular towers may pick up a cell phone's signal. A cell phone signal has been located 5 mi from tower 1.




**a)** What is the distance from the caller to tower 3?

**b)** How far is tower 1 from tower 3?

**12.** The Disabled Sailing Association had its first sessions at the Jericho Sailing Centre in Vancouver, BC. At a recent regatta, a television news team tracked two sailboats from a helicopter 800 m above the water. The team observed the sailboats on the left and right sides of the helicopter at angles of depression of 58° and 36°, respectively.

**a)** Which boat is located closer to the helicopter? Explain.

**b)** Determine the distance between the two boats. Express your answer to the nearest metre.

**13.** Two tourists stand on either side of the Veterans Pole, honouring Canadian Aboriginal war veterans, in Victoria, BC. One tourist measures the angle of elevation of the top of the pole to be 21°. To the other tourist, the angle of elevation is 17°. If the height of the pole is 5.5 m, how far apart are the tourists? Express your answer to the nearest tenth of a metre.

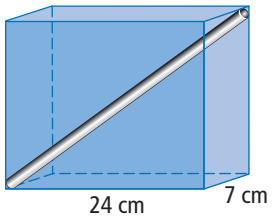
## Extend

**14.** From the top of a 35-m-tall building, an observer sees a truck heading toward the building at an angle of depression of 10°. Ten seconds later, the angle of depression to the truck is 25°.

**a)** Determine the distance that the truck has travelled. Express your answer to the nearest metre.

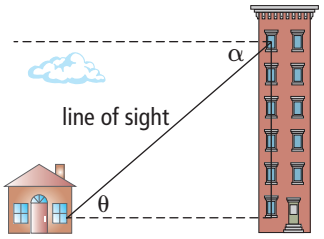
**b)** If the speed limit for the area is 40 km/h, is the truck driver following the speed limit? Explain.

**15.** A rectangular prism has base dimensions of 24 cm by 7 cm. A metal rod is run from the bottom corner diagonally to the top corner of the prism. If the rod forms an angle of 40° with the bottom of the box, calculate the volume of the box.




## Create Connections

**16.** From her apartment, Jennie measures the angle of depression to Mike's house. At the same time, Mike measures the angle of elevation to Jennie's apartment.




**a)** Mike's brother Richard observes Mike and states that Mike made an error, because the angle of elevation must be greater than the angle of depression. Is Richard correct? Explain your reasoning.

**b)** In order to calculate the measure of angle θ, you can be given any of the following measurements:

- the height of Jennie's window
- the horizontal distance between buildings
- the length of line of sight
- the measure of angle α

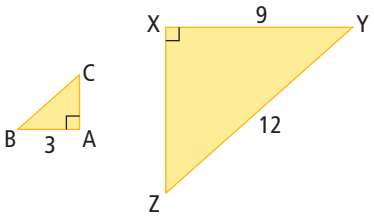
Which measurement(s) would you prefer to be given? Explain how you would use these measurements to calculate θ.

# 3 Review

## 3.1 The Tangent Ratio, pages 100–113

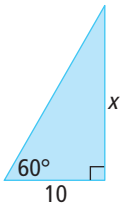
*Where necessary, express your answers to the nearest tenth of a unit.*

**1.** Triangles ABC and XYZ are similar. Calculate the lengths of the unknown sides.

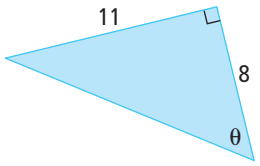



**2.** Determine the value of the variable in each triangle.

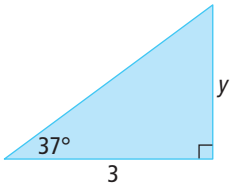
**a)**




**b)**




**c)**



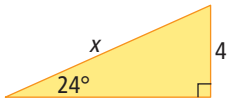

**3.** A group of conservationists needs to calculate the angle of elevation of the river bank of the North Saskatchewan River. They set up a right triangle using two measuring poles. If they measure the vertical height to be 64 cm and the horizontal distance to be 50 cm, what is the angle of elevation of the river bank?

## 3.2 The Sine and Cosine Ratios, pages 114–124

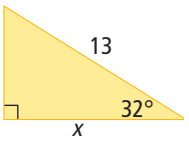
*Where necessary, express your answers to the nearest tenth of a unit.*

**4.** Determine the value of the variable in each triangle.

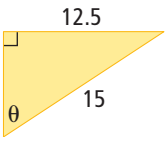
**a)**




**b)**




**c)**




**5.** Augers are used to move grain into storage bins. Suppose an auger is 67 ft long and the granary is 44 ft high. Determine the angle formed by this auger and the ground.

**6.** A 14-ft ladder leans against the bottom of a window and makes an angle of 64° with the ground. What is the height to the bottom of the window?

## 3.3 Solving Right Triangles, pages 125–135

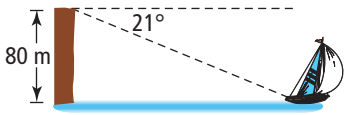
*Where necessary, express your answers to the nearest tenth of a unit.*

**7.** In $\triangle ABC$, BC = 7.4 km, $\angle B = 90°$, and $\angle A = 38°$.

**a)** Draw and label the triangle.

**b)** Solve $\triangle ABC$.

**8.** The angle of depression from the top of an 80-m-high cliff to a sailboat is 21°. Determine the distance from the base of the cliff to the sailboat.



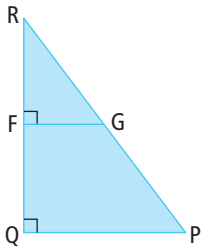

**9.** A lifeguard sitting on a platform that is 14 ft high observes someone swimming. The first sighting of the swimmer is at an angle of depression of 60°. The angle of depression becomes 30° the next time the lifeguard looks at the swimmer. Explain whether the swimmer is moving toward or away from the lifeguard. Use a diagram to support your answer. Then, determine the distance that the swimmer has travelled.

# 3 Practice Test

## Multiple Choice

*For #1 to #4, choose the best answer.*

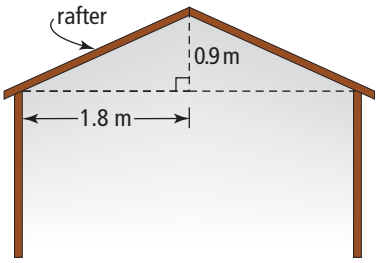
**1.** For the similar triangles shown, which expression is true?




**A** $\frac{FG}{QP} = \frac{RG}{PG}$ **B** $\frac{PQ}{GR} = \frac{RG}{QF}$

**C** $\frac{RF}{QR} = \frac{GR}{RP}$ **D** $\frac{GF}{RF} = \frac{QP}{RP}$

**2.** Madeleine's dad is designing a garage to build beside their house. He wants a 30-cm overhang on each side. How long should each rafter be?



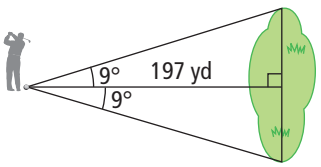

**A** 2.0 m **B** 2.3 m

**C** 3.8 m **D** 4.4 m

**3.** A gardener uses topsoil to improve garden soil for his Regina customers. He purchased a special pickup truck that acts like a dump truck. If the 80-in. truck bed is raised to a 40° angle, how high is the upper end of the truck box above the wheels?

**A** 51 in. **B** 61 in. **C** 67 in. **D** 80 in.

**4.** The 17th hole at the Rivershore Golf Course near Kamloops, BC, is 197 yd from the teeing area to the centre of the green. Suppose the largest angle at which you can drive the golf ball to the left or right and still land on the green is 9°. What is the width of the green, to the nearest yard?




**A** 15 yd **B** 31 yd **C** 47 yd **D** 62 yd

## Short Answer

**5.** During the annual Windscape Kite Festival in Swift Current, SK, Yves and Lucian's kite got caught in the top of a tree. Yves wants to use similar triangles to calculate the height of the tree. A nearby 9-m flagpole casts a shadow that is 6 m long. Yves and Lucian estimate the shadow of the tree to be 3.5 m long. What is the height of the tree, to the nearest tenth of a metre? Include a diagram of the situation.

**6.** Evaluate each trigonometric ratio, to four decimal places.

**a)** tan 17° **b)** sin 68° **c)** cos 23°

**7.** Calculate the measure of each angle, to the nearest degree.

**a)** $\sin\theta = 0.2588$ **b)** $\tan\alpha = 5.6713$ **c)** $\cos\theta = 0.7431$

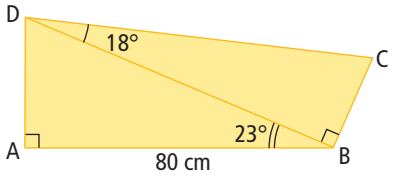
**8.** Zachary was calculating the length of side CD in the figure. His partial solution is shown.

$$\cos 23^\circ = \frac{80}{BD}$$

$$BD = \frac{80}{(\cos 23^\circ)}$$

$$\sin 18^\circ = \frac{BD}{CD}$$

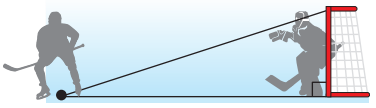
$$CD = \frac{BD}{\sin 18^\circ}$$




Before Zachary completed his work, he realized that he had made an error. Identify Zachary's error. Explain a strategy to help him avoid making this error again.

## Extended Response

**9.** The Quikcard Edmonton Minor Hockey Week is one of the largest hockey tournaments in North America. The tournament has grown to include more than 480 teams from Alberta.



**a)** Suppose the goalie's shoulder rises to 40 in., and a player takes a shot 20 ft from the net. Through what angle of elevation of the puck's flight will the goalie make the save? Give your answer to the nearest tenth of a degree.

**b)** The height of the net is 48 in. A player takes a shot over the right shoulder of the same goalie from part a) at an angle of elevation of 8.5°. If the puck travels a distance of 29 ft, will the player score a goal? Explain why.

# 1 Unit Connections

## Unit 1 Project

Use your answers to the unit project questions throughout chapters 1, 2, and 3, as well as your own research, to prepare a presentation on music distribution. Your presentation should include the following:

- research on the history of music recording
- a comparison of various storage devices
- a description of the impact technology has had on music distribution

To complete your presentation, predict what the next technological advance in music distribution might be. Include answers to the following questions:

- Describe what you think the next advance in music distribution might look like. Provide measurements for length, area, and volume of the new equipment in both SI and imperial units.
- How might this equipment work?
- What impact might the advance have on how you access music?
- How might this equipment distribute music to people around the world?

## Unit Review

### Chapter 1 Measurement Systems

**1.** Identify referents that could be used for the following linear measurements.

| | | |
|---|---|---|
| millimetre | centimetre | metre |
| inch | foot | yard |

**2.** For each total length, choose a comparable unit of measurement in the SI system. Determine the length to the nearest tenth of a unit. Justify your choice of units.

**a)** A table-tennis ball has a diameter of the width of two fingers. Each finger is half an inch wide.

**b)** Samantha often wears her hair in a ponytail. Her ponytail is 5 hand-widths long. The width of her hand is 3 in.

**c)** It takes Everrett 14 steps to leave the classroom. One of his paces measures half a yard.

**3.** Convert each measurement to the indicated unit.

**a)** 3500 mm = ■ cm

**b)** 3.5 ft = ■ in.

**c)** 8723 m = ■ km

**d)** 4.25 ft = ■ m

**e)** 67 cm = ■ in.

**f)** 14 km = ■ mi

**4.** A rectangular oak table measures 5 ft 10 in. by 3 ft 9 in. What is the perimeter of the table, in feet and inches?

**5.** An artist sculpts a $10\frac{1}{4}$-in. tall clay model of a horse. If the scale used for the sculpture is 1:6, how tall would the actual horse be, in feet and inches?

## Chapter 2 Surface Area and Volume

**6.** Calculate the area of each figure, as indicated.

**a)** a rectangle with dimensions 250 cm by 180 cm, in square metres

**b)** a square of side length 4 mi in square yards

**7.** Melody is helping prepare a cake for a banquet in Nanaimo, BC. She needs to know the amount of icing needed for the initial covering of the cake before she adds the final decorations. Assume that Melody does not ice the bottom of any layer.

**a)** What surface area does Melody need to ice for the top three layers, in square centimetres, if the top three layers are square and have the following dimensions:

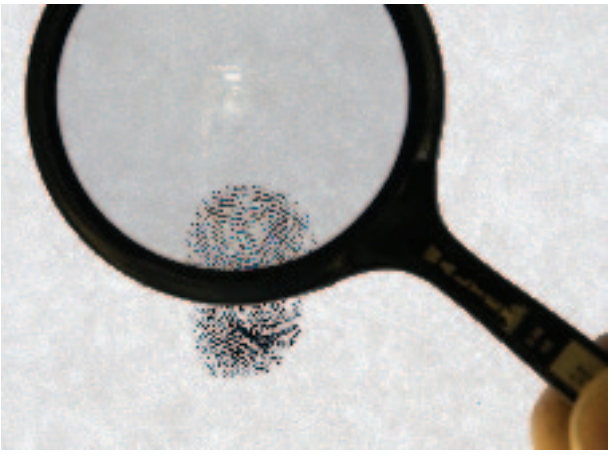
*Top layer*: side length of 10 cm and a height of 7 cm

*Second layer*: side length of 14 cm and a height of 8.5 cm

*Third layer*: side length of 18 cm and a height of 9 cm

**b)** The volume of cake used for the bottom layer is 4000 $cm^3$. The bottom square layer has a height of 10 cm. What surface area, in square centimetres, does Melody need to ice on the bottom layer?

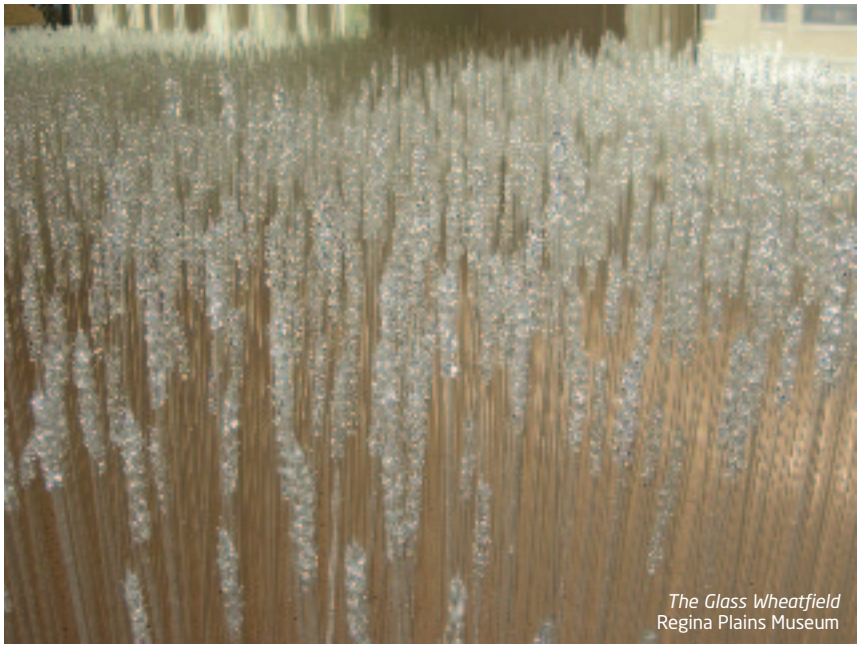
**8.** Nalze went on a field trip with his class to the RCMP detachment in Yellowknife, NT. In the Henry Larson Building, there was a display on crime investigation. Nalze used a magnifying glass to look at a fingerprint.



**a)** From the photo, estimate the diameter of the magnifying glass Nalze used.

**b)** If the glass is approximately 0.3 cm thick, what is the volume of glass used?

**9.** Saskatchewan artist Jacqueline Berting created *The Glass Wheatfield - A Salute to Canadian Farmers*. It is made up of 11 000 individually crafted waist-high stalks of glass wheat mounted in a steel base. The average cylindrical stem is 40 in. tall with a diameter of $\frac{1}{8}$ in. Each head of wheat contains the equivalent amount of glass as a cone that is 4 in. long with a base diameter of $\frac{3}{4}$ in. Approximately how much glass did Jacqueline use for the sculpture?

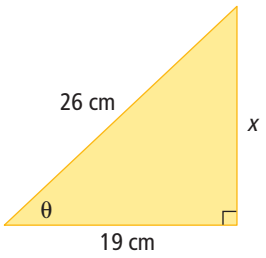


*The Glass Wheatfield*
Regina Plains Museum

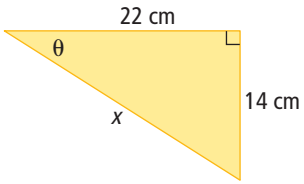
## Chapter 3 Right Triangle Trigonometry

**10.** Determine the measurements of each unknown side and unknown angle. State side lengths to the nearest tenth of a unit and angles to the nearest degree.

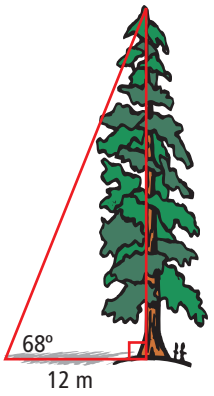
**a)**




**b)**



**11.** A tree casts a shadow 12 m long. The angle measured to the top of the tree from the end of the shadow is 68°. What is the height of the tree? Express your answer to the nearest tenth of a metre.




**12.** An oil rig is held vertical by two guy wires of unequal lengths on opposite sides of the oil rig. One of the wires makes an angle of 45° with the platform. The other wire is 90 ft long and makes an angle of 55° with the platform. Both wires are attached 8 ft down from the top of the rig.

**a)** Sketch and label a diagram of this situation.

**b)** Calculate the height of the oil rig, to the nearest foot.

**c)** Do you think the length of the unknown wire is greater than the 90-ft wire? Justify your prediction. Then, determine the measurement, to the nearest half of a foot.

**d)** Determine the distance on the platform between the two guy wires, to the nearest half of a foot.

**13.** A 15-m-long ladder is placed in a driveway between two buildings. The ladder leans against one building and reaches 12 m up the side. If the ladder is rotated to lean on the other building, it reaches 8 m up the side. How wide is the driveway between the two buildings?

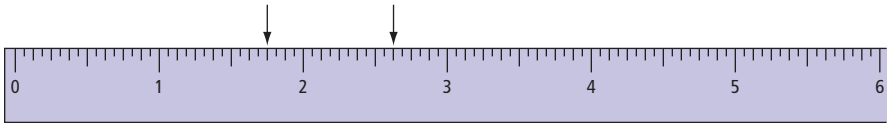
**14.** Neighbourhoods A and B are situated on opposite sides of a mountain that stands 780 m high. The angles of elevation from each neighbourhood to the top of the mountain are 67° and 54°. What would be the length of a tunnel from neighbourhood A to neighbourhood B? Express your answer to the nearest tenth of a metre.

# 1 Unit Test

## Multiple Choice

*For #1 to #4, choose the best answer.*

**1.** What is the distance measured between the two arrows on this imperial ruler?




**A** $\frac{7}{8}$ in. **B** $\frac{16}{14}$ in. **C** $\frac{7}{16}$ in. **D** $\frac{15}{16}$ in.

**2.** Elijah is helping install baseboards in a bedroom in the basement. He knows that one of his paces is approximately equal to 1 yd. If he walks 15 paces along the width of the room and 18 paces along the length, what is the approximate perimeter of the room, in feet?

**A** 99 ft **B** 198 ft **C** 270 ft **D** 792 ft

**3.** Carrie was asked to calculate the slant height of a right cone. She is given that the surface area is 251.3 cm$^2$ and the diameter is 10 cm. Her work is shown below.

Step 1 $SA = \pi r^2 + \pi r s$

Step 2 $251.3 = \pi(5^2) + \pi(5)s$

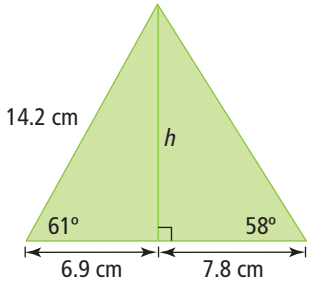
Step 3 $\frac{251.3}{(25\pi + 5\pi)} = s$

Step 4 $2.7 = s$

When Carrie examined her work, she realized that she made her first error in

**A** Step 1 **B** Step 2 **C** Step 3 **D** Step 4

**4.** The equation that could be used to calculate the value of $h$ in the diagram is




**A** $\cos 58° = \frac{h}{7.8}$

**B** $\tan 58° = \frac{h}{7.8}$

**C** $\cos 61° = \frac{h}{14.2}$

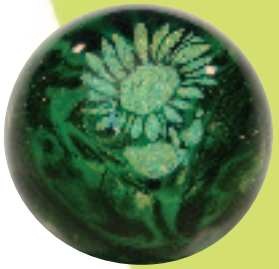
**D** $\sin 61° = \frac{h}{6.9}$

## Numerical Response

*Complete the statements in #5 to #7.*

**5.** Jett measures the diagonal of the television screen in his family room to be 117 cm. Laura measures the diagonal of her television screen to be 54 in. Laura's television is ■ in. larger than Jett's television, expressed to the nearest inch.

**6.** A glass paperweight is in the shape of a sphere and has a volume of 356 818 $mm^3$. The radius of the paperweight is ■ mm.



**7.** Your school is installing a wheelchair ramp outside the front doors. The current stairs reach a height of 0.7 m. If the ramp is 8 m long, the horizontal distance to the end of the ramp, to the nearest tenth of a metre, is ■ m.

## Written Response

**8.** Alicia found a unique gift for her friend's birthday. She bought a purse that is in the shape of a right pyramid with a square base. The dimensions of the base are 12.0 cm by 12.0 cm, and the slant height is 16.16 cm.

**a)** Determine the height of the purse.

**b)** How much space is inside the purse?

**c)** Alicia wants to place the purse in a gift box with a lid. She has gift boxes of the following volumes:
- 2100 $cm^3$
- 2200 $cm^3$

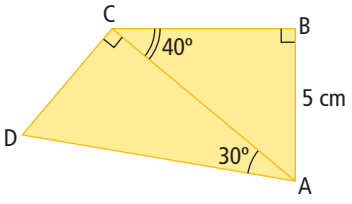
For each size of gift box, explain whether the purse will fit inside.

**9.** Given △ACD is adjacent to △ABC.

**a)** Write an equation that could be used to calculate the length of AC.

**b)** Calculate the length of AC.

**c)** Calculate the length of DC, to the nearest centimetre.




**10.** As a spectator at a hockey game, Brennan is sitting 40 m horizontally from the goal net. His seat is 10 m above ice level.

**a)** At what angle of depression is Brennan watching the goalie make a save?

**b)** A seat becomes available directly below Brennan, so he moves 3 m down. Will the angle of depression from Brennan to the goalie increase or decrease? Justify your answer.