

UNIT

1

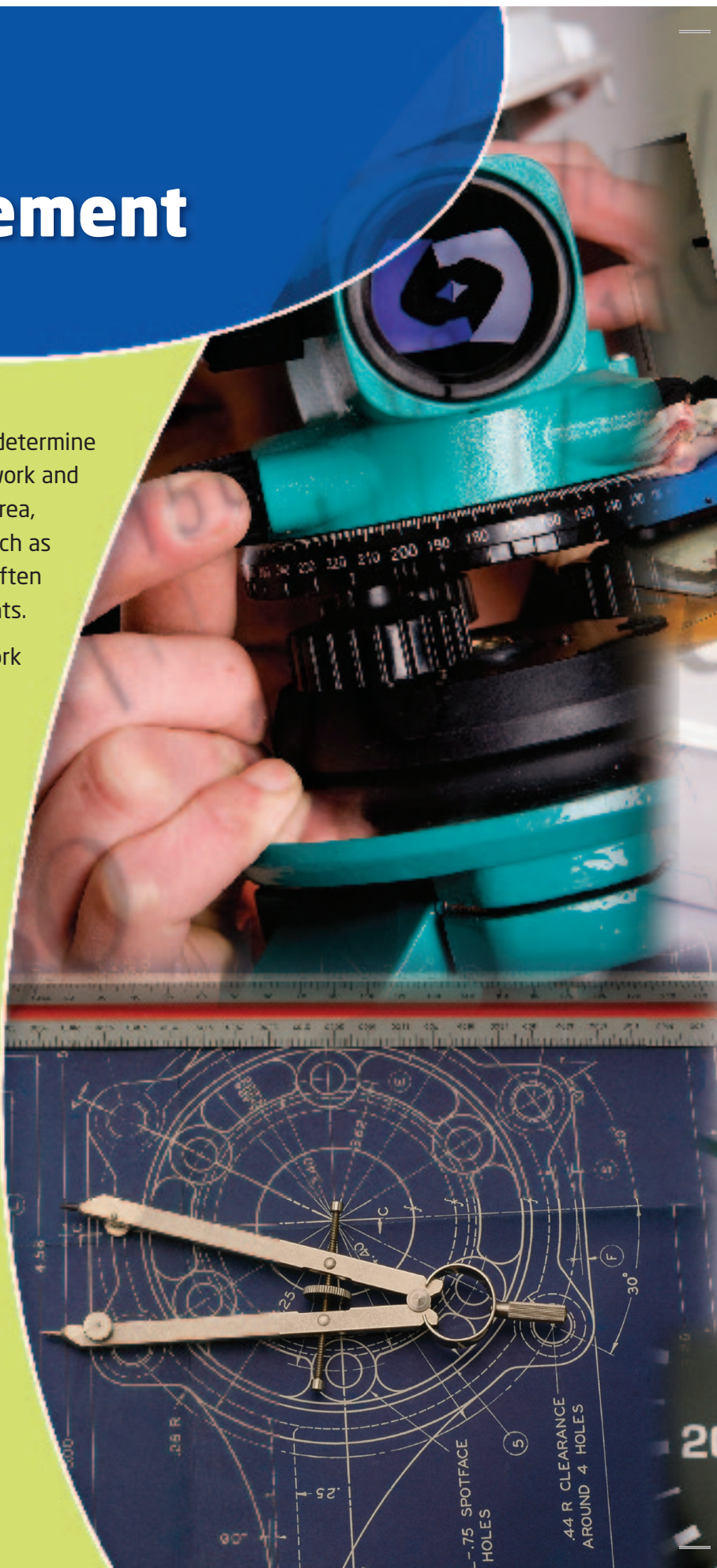
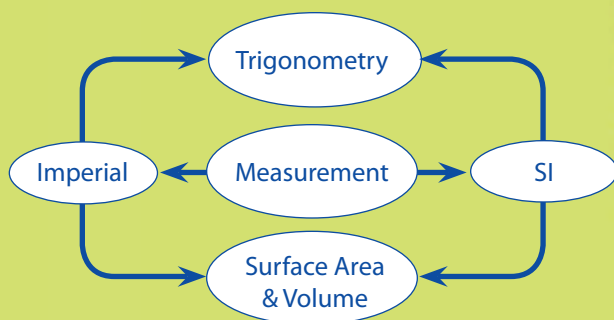
Measurement

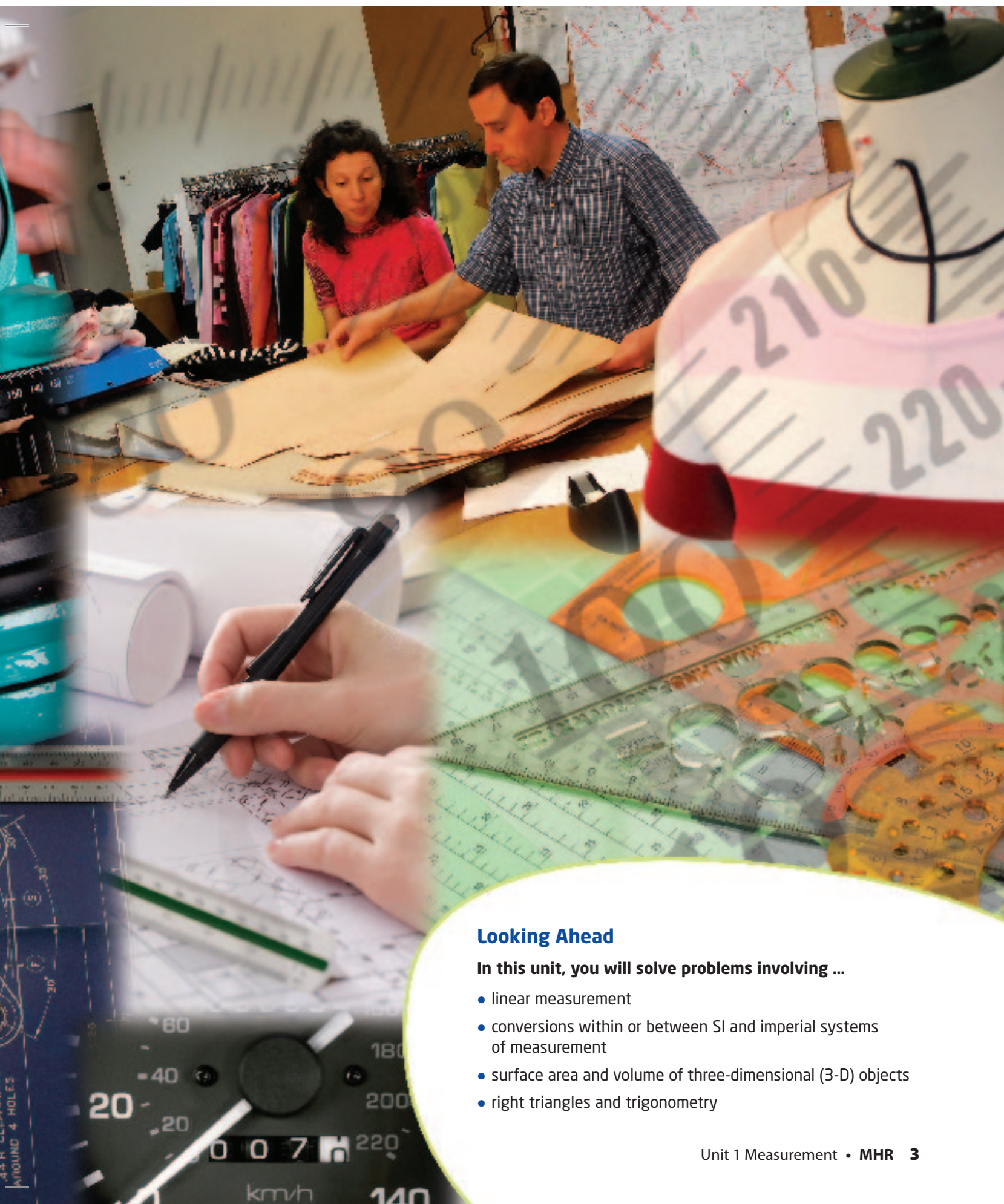
Measurement explores different ways you can determine lengths, areas, and volumes. In many fields of work and in your daily life, you may need to use length, area, and volume measurements. Workers in fields such as architecture, road construction, and surveying often use trigonometry to help calculate measurements.

Depending upon your task, you may need to work in the imperial or SI system, or even convert between systems. In this unit, you will take some measurements directly and learn how to calculate surface area and volume of three-dimensional objects. You will learn how to use trigonometry to calculate distances that are difficult to measure.

Your Measurement Organizer

You can use this measurement organizer to see how the concepts in this unit are connected. You will see this organizer on the first page of each chapter. The concepts covered in that chapter are highlighted.





Looking Ahead

In this unit, you will solve problems involving ...

- linear measurement
- conversions within or between SI and imperial systems of measurement
- surface area and volume of three-dimensional (3-D) objects
- right triangles and trigonometry

Unit 1 Project

Changes in Music Distribution

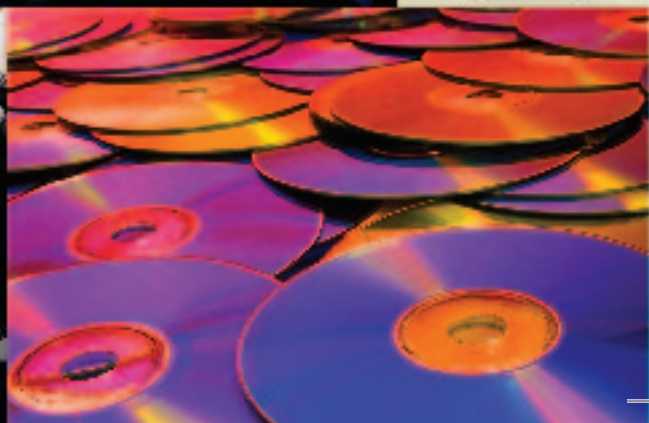
The music industry has been big business for decades. Technological advances have affected how music is recorded, distributed, and listened to. Over the years, different devices have been used to store and transfer music. Many of these devices could hold only a few songs. Today, the increasing demand for greater storage and distribution capabilities has created a need for improved communication networks.

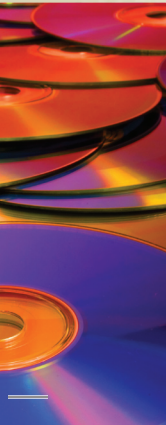
In the Unit 1 project, you will prepare a presentation that includes research on the history of music recording, a comparison of storage devices, a description of the impact of technology on music distribution, and a prediction of the next technological advance.

Throughout Chapters 1, 2, and 3, **Unit Project** questions will help you gather some of the information for your project. You may need to conduct additional research to complete your project.

While completing your project, you will ...

- research the history and storage of music over time (throughout Unit 1)
- estimate and calculate linear measurements of music storage devices (Chapter 1)
- apply your understanding of unit conversions (SI and imperial) to compare music storage devices (Chapters 1 and 2)
- use surface area and volume to compare music storage devices (Chapter 2)
- use the Pythagorean theorem and trigonometry to explore how wireless systems have affected music distribution (Chapter 3)
- predict the next technological advance in music distribution (end of Unit 1)





CHAPTER

1

Measurement Systems

Measurement is the process of finding the size of an object. It relates the unknowns in our world to accepted or standard units. Linear measurement is critical to many jobs. How is measurement being used in each picture? Challenge yourself to think of a career that does not have a connection to measurement.

Big Ideas

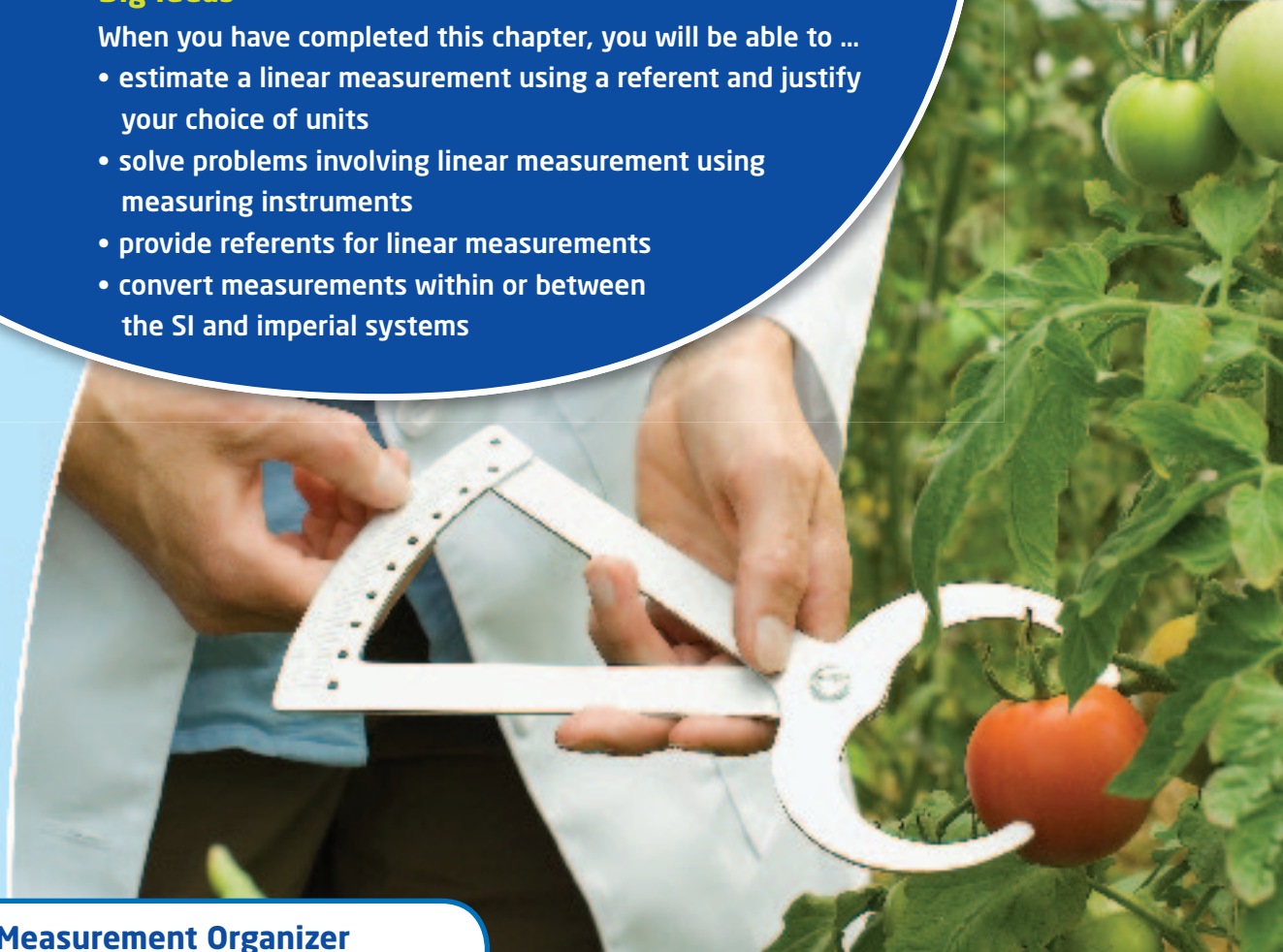
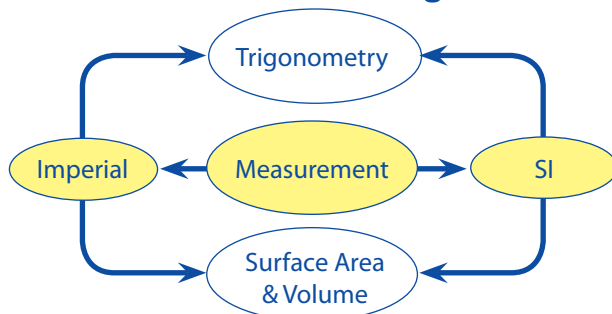
When you have completed this chapter, you will be able to ...

- estimate a linear measurement using a referent and justify your choice of units
- solve problems involving linear measurement using measuring instruments
- provide referents for linear measurements
- convert measurements within or between the SI and imperial systems

Key Terms

SI
referent
imperial system

Your Measurement Organizer





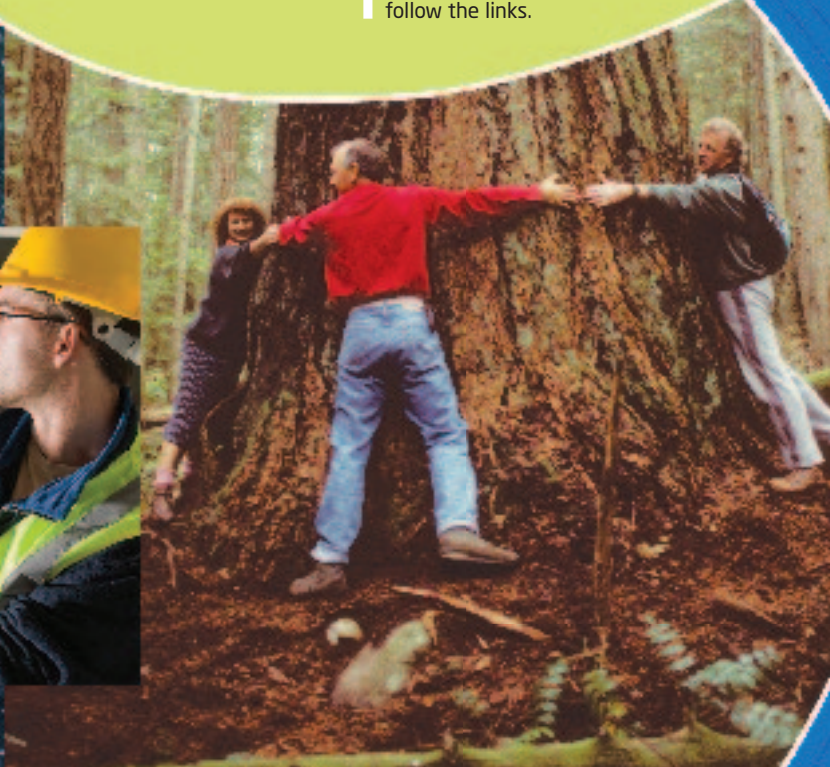
Photogrammetrist

Photogrammetrists analyse, measure, and interpret aerial photographs. Using these skills, they get information about 3-D objects. The information is used in making maps, in ecological studies, and in forestry. A photogrammetrist may be hired to track an oil spill or engineer a roadway.



WWW Web Link

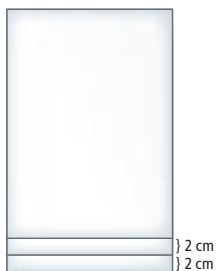
To learn more about photogrammetrists, go to www.mhrmath10.ca and follow the links.



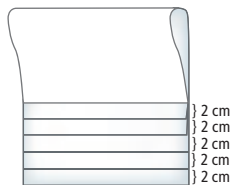
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 1.

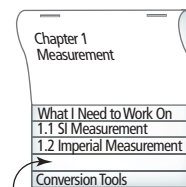
- 1 Create a booklet by staggering three sheets of $8\frac{1}{2}$ in. by 11 in. paper to create tabs that are approximately 2 cm wide.



- 2 Fold the top of the booklet toward you and align these tabs with the existing ones. All of the tabs should appear the same height.



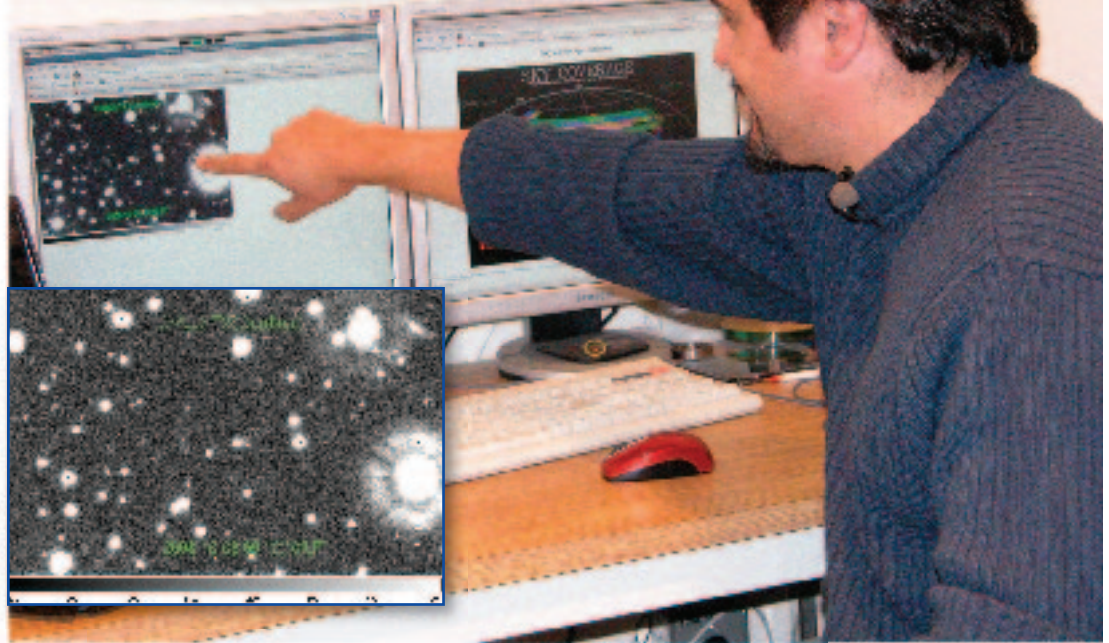
- 3 Staple the top to hold the pages in place. Write the title of each section on the tabs.



1.3 Converting Between SI and Imperial Systems

1.1

SI Measurement



Focus on ...

- justifying the units used for a measurement
- solving problems that involve linear measurement
- explaining the process used to estimate a linear measurement
- selecting appropriate referents

Rob Cardinal, who is of the Siksika First Nation, is a researcher at the University of Calgary's Rothney Astrophysical Observatory. On October 1, 2008, he took some images that he thought were of an asteroid. It turned out to be an undiscovered comet and was named Comet Cardinal. Whether you are exploring the universe, hiking in the Rockies, or travelling in the Prairies you will be using linear measurement with SI units. SI stands for *Système International d'Unités*. What SI units can you name? What type of measurement is associated with each unit?

Materials

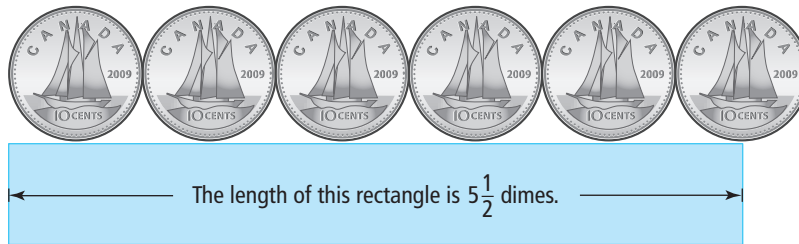
- three items that are non-standard measuring units (e.g., coin, paperclip)
- SI measuring tape
- grid paper

Investigate Dimensions of a Rectangle

What effect does the unit of measurement have on the length-to-width ratio of a rectangle?

1. Work with a partner. Draw a rectangle on half of a sheet of paper.
2. Choose three non-standard units to measure the dimensions of your rectangle. Estimate how many of each non-standard unit it takes to measure the length and width of your rectangle.

3. Measure the dimensions using each non-standard unit. Record all data in a chart or table. You may need to record measurements using fractions or decimals.



4. Estimate the length and width of your rectangle in millimetres and centimetres. Then, confirm your estimate by measuring using each of these standard units. Record your estimates and measurements.
5. For each unit of measurement, plot your measurements as (length, width) coordinate pairs on a grid.
6. **Reflect and Respond**
- Describe any patterns you see in the graph.
 - Does the ratio of length to width for your rectangle vary when you change the units of measurement? Explain.
7. Discuss the advantages and disadvantages of using standard units for measuring distance.

Link the Ideas

Canada's official measurement system is **SI (Système International d'Unités)**. Some SI units for linear measurement are listed in the table.

Unit	Abbreviation	Multiplying Factor
kilometre	km	1000
hectometre	hm	100
decametre	dam	10
metre	m	1
decimetre	dm	0.1
centimetre	cm	0.01
millimetre	mm	0.001

Various measuring instruments allow accurate measurement of distances in standard units. You can also develop personal **referents** to use when estimating measurements.

SI (Système International d'Unités)

- a system of measurement in which all units are based on multiples of ten
- the metre is the basic unit of length

referent

- an item that an individual uses as a measurement unit for estimating
- for example, the height of a doorknob above the floor is about 1 m, or the thickness of a dime is about 1 mm

Measuring Instruments

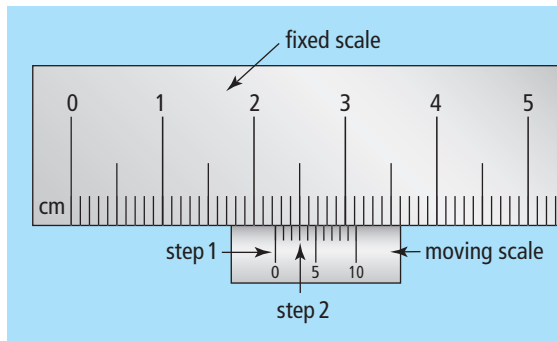
SI rulers, metre sticks, and measuring tapes give measurements to the nearest millimetre, or 0.1 cm. A caliper can accurately measure to the nearest tenth of a millimetre, or 0.01 cm, depending on the scales.

Follow these steps to read a caliper.

1. Read the value on the fixed scale that is located exactly at or just to the left of the zero on the moving scale. **For the caliper shown, the reading is 2.2.**
2. Identify the next line on the moving scale that aligns with a line on the fixed scale. Read the value on the fixed scale. **For this caliper, it is the line on the moving scale that represents 0.03.**
3. The final reading is 2.23 cm. ($2.2 + 0.03 = 2.23$)

WWW Web Link

To watch a video showing how to read an SI caliper, go to www.mhrmath10.ca and follow the links.



Referents

A non-standard measuring unit can be used as a personal referent. Referents help individuals estimate in standard units, such as SI units. For example, suppose you use the width of your fingernail to approximate 1 cm. Then, when you measure something that appears to be as wide as 4 of your fingernails, you can estimate that it is 4 cm wide.

Several referents are possible for each of the main linear SI units, the millimetre, centimetre, metre, and kilometre. Finding a referent for a kilometre is more difficult, because it is a large unit. You might choose a referent for a kilometre to be 12 or 13 city blocks.

Example 1 Estimate and Measure Using SI Units

Estimate each distance using an appropriate referent. Then, measure each distance.

- a) the thickness of a CD case
- b) the height of the seat of a chair
- c) the width of this page

Solution

- a) Use the width of one fingernail as a referent for 1 cm.



The thickness of a CD case is approximately as wide as half of one fingernail. Estimate the thickness of a CD case as 0.5 cm. Measure, using an SI ruler or caliper. The thickness of a CD case is 4.5 mm or 0.45 cm.

- b) The height of the seat of a chair is approximately half of waist height. Use waist height as a referent for 1 m. Estimate the seat of the chair as 0.5 m high, or 50 cm. Measure, using a measuring tape. The height of the seat of a chair is 46 cm.
- c) Use the width of one fingernail as a referent for 1 cm. Count the number of fingernails that fit across half of this page and double the number. An estimate of the width of this page is 22 cm. Measure the width using an SI ruler. The width of this page is 21.5 cm.

Your Turn

Estimate the height of the chalk or marker tray on a blackboard or whiteboard using an appropriate referent. Then, measure this height.

Did You Know?

In a fraction, common factors in the numerator and denominator divide to make 1. This concept is used to simplify fractions to lowest terms.

Example:

$$\frac{18}{45} = \frac{(9 \times 2)}{(9 \times 5)} \\ = \frac{2}{5}$$

In *unit analysis*, the same concept can be applied to units of measurement.

Example:

To convert from metres to millimetres,

$$(25 \text{ m}) \left(\frac{1000 \text{ mm}}{1 \text{ m}} \right) \\ = \left(\frac{25 \cancel{\text{m}}}{1} \right) \left(\frac{1000 \text{ mm}}{1 \cancel{\text{m}}} \right) \\ = 25\,000 \text{ mm}$$

Example 2 Convert Between SI Units for Length

A newspaper reported the following measurements in different stories.

The distance from Earth to the moon is 38 440 300 000 cm.

A worm measures 0.0019 m.

- For each measurement, state a more appropriate SI unit. Justify your choice.
- Convert the given measurement to the more appropriate unit.

Solution

- The distance from Earth to the moon is very large. It could be measured in kilometres rather than centimetres. The length of a worm is very small. It could be measured in millimetres rather than metres.

- Convert 38 440 300 000 cm to kilometres.

Use *unit analysis* to calculate the number of centimetres in 1 km.

$$1 \text{ km} = 1000 \text{ m} \text{ and } 1 \text{ m} = 100 \text{ cm}$$

$$\text{So, } 1 \text{ km} = (1000)(100) \text{ cm}$$

$$1 \text{ km} = 100\,000 \text{ cm}$$

$$38\,440\,300\,000 \text{ cm} \left(\frac{1 \text{ km}}{100\,000 \text{ cm}} \right) = 384\,403 \text{ km}$$

The measurement 38 440 300 000 cm can be converted to 384 403 km.

Convert 0.0019 m to millimetres.

Let x represent the number of millimetres.

Use proportional reasoning.

$$1000 \text{ mm} = 1 \text{ m}$$

$$\frac{1000 \text{ mm}}{1 \text{ m}} = \frac{x \text{ mm}}{0.0019 \text{ m}}$$

$$1000(0.0019) = x$$

$$1.9 = x$$

The measurement 0.0019 m can be converted to 1.9 mm.

How do you decide which values to use for the numerator and denominator?

Your Turn

Convert each measurement to a more appropriate SI unit. Justify your choice of unit.

- A tube of toothpaste is 205 mm long.
- The circumference of a highlighter measures 0.06 m.
- You travel 590 000 m from Regina to Winnipeg.
- The top of a door is 2110 mm high.

Example 3 Solve a Problem Involving Linear Measurement

Kyla buys an oversized wooden barrel. She cuts it in half to make a planter. She wants to place a metal band around the planter, 4 cm from the top, to hold the planter together.

- If the radius 4 cm from the top of the planter is 0.6 m, what length of band will she need? Express your answer to the nearest centimetre.
- If the bottom band of her planter is 1 m shorter than the top band, what is the radius of the planter at the bottom band? Express your answer to the nearest centimetre.
- What is the difference between the radius of the planter at the top band and the radius at the bottom band?
- Show how much the radius of any barrel increases if 1 m is added to the length of a band. State your answer as an exact value. Then, express your answer to the nearest centimetre.



Solution

- a) The length of the band is equal to the circumference of the planter at 4 cm from the top.

$$C = 2\pi r$$

$$C = 2\pi(0.6)$$

$$C = 3.7699\dots$$

How do you convert 3.77 m to centimetres?

The length of the band 4 cm from the top is 3.77 m or 377 cm.

- b) The circumference of the planter at the bottom band is equal to the length of the bottom band. The bottom band is 1.0 m shorter than the top band.

$$3.77 - 1.0 = 2.77$$

The length of the bottom band is 2.77 m.

Calculate the radius of the planter at the bottom band.

$$C = 2\pi r$$

$$2.77 = 2\pi r$$

$$\frac{2.77}{2\pi} = r$$

$$0.4408\dots = r$$

The radius of the planter at the bottom band is 0.44 m or 44 cm.

- c) Calculate the difference between the two radii.

$$0.6 - 0.44 = 0.16$$

The difference between the radius of the planter at the top band and the radius at the bottom band is 0.16 m or 16 cm.

Did You Know?

Wooden barrels bulge in the middle. The bulge or bilge, as it is known, is designed to make it easier to roll and to change direction while rolling a barrel.

- d) The formula $C = 2\pi r$ represents the circumference, C , for any barrel with radius r . A barrel whose circumference increases by 1 m and radius increases by x metres can be represented by the following formula.

$$C + 1 = 2\pi(r + x)$$

$$C + 1 = 2\pi r + 2\pi x$$

$$C + 1 = C + 2\pi x$$

Substitute C for $2\pi r$.

$$1 = 2\pi x$$

$$\frac{1}{2\pi} = x$$

The radius of the barrel increases by the exact value $\frac{1}{2\pi}$.

$$\frac{1}{2\pi} = 0.1591549431$$

The radius of the barrel increases by approximately 0.16 m or 16 cm for every 1 m increase in circumference.

How does this answer relate to your answer to part c)?

Why do you think this increase will hold true for any size barrel?

Did You Know?

Red River carts were used by Métis to transport goods and belongings. The carts were pulled by horses or oxen. Sometimes several carts were attached in a line to haul a greater amount of freight. Red River carts were built entirely of wood, sinew, and rope. This allowed the carts to float easily with the wheels removed. Prairie dust prevented the axles from being effectively greased. As a result, the carts made a loud squeaking noise.

Your Turn

Suppose the inner rim of a Red River cart wheel has a circumference of 7.1 m. Each spoke is 1 m long. What is the diameter of the centre circular hub of the wheel?



Key Ideas

- Each unit in the SI measurement system is a multiple of 10. All linear measurements are derived from the metre. The most common units are the kilometre (km), metre (m), centimetre (cm), and millimetre (mm).
- The kilometre is a large unit ($1 \text{ km} = 1000 \text{ m}$) and is suitable for measuring large distances.

- The millimetre is a small unit ($1 \text{ mm} = \frac{1}{1000} \text{ m}$) and is suitable for measuring small distances.
- A referent is a personal measurement unit that you can use to estimate measurements in standard units, such as SI units.

Estimate the length of an eyeglass case.

Use a personal referent, such as “the width of your palm is 7.5 cm.”

The length of an eyeglass case measures 2 times as wide as your palm.

$$7.5(2) = 15$$

Estimate the length of the eyeglass case to be 15 cm.

Check Your Understanding

Practise

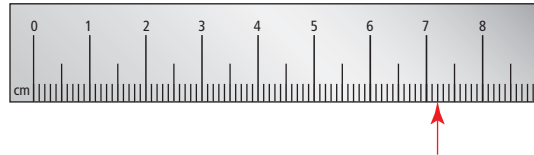
1. a) Estimate the perimeter of each figure in an appropriate SI unit.



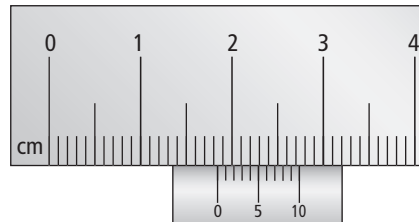
- b) Measure the perimeter of each figure. If all the angles in each figure are right angles, is it necessary to measure all sides of the figures? Explain.

2. a) On a plain piece of paper, draw a letter S whose curve length you estimate to be each distance.
- i) 25 mm ii) 20 cm
- b) Explain how you could measure the distance of each curved letter you drew.
- c) Measure each S and compare your measurements with the required distances. If you are out by more than 5 mm for part i) or 2 cm for part ii), try drawing the letter again.
3. What reading is shown on each measuring instrument?
Give each reading in both millimetres and centimetres.

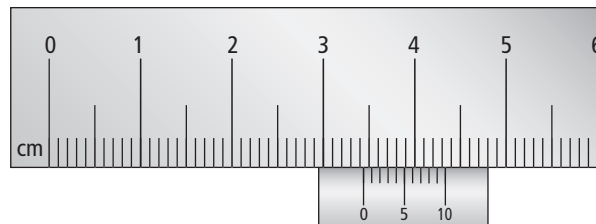
a) SI ruler



b) SI caliper



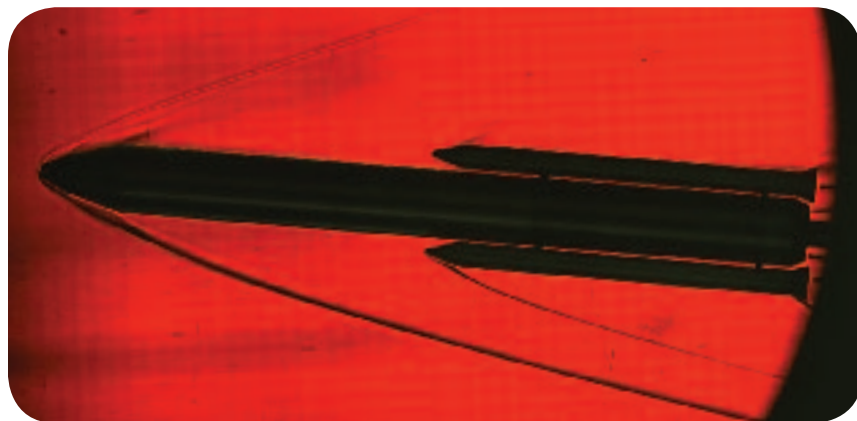
c) SI caliper



Did You Know?

The Ares V is the cargo launch component for Project Constellation. NASA plans manned space travel in this vehicle, in 2019. The Ares V can carry 188 000 kg into low Earth orbit and 71 000 kg to the moon.

4. The photograph shows a wind-tunnel test of the airflow over a model of NASA's heavy launch vehicle, called Ares V.

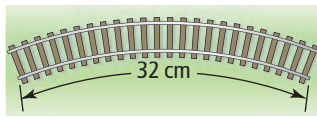


- a) Assume the photo and model show a reduction of approximately 1 : 1000. What is the actual length of Ares V?
- b) What is the diameter of one of the actual solid-rocket boosters?

5. Consider each measurement. State whether it is reported in the most appropriate unit. If it is not, explain why and convert to a more appropriate unit.
- The highest mountain in Canada, Mount Logan, has a height of 595 900 cm.
 - The diameter of a water bottle is 0.064 m.
 - The world's tallest bear was 4200 mm.
 - A whooping crane's wingspan is 0.001 95 km.

Apply

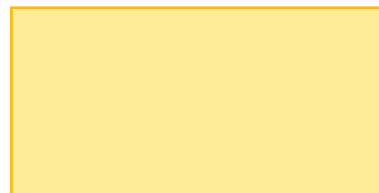
6. A circular model railway track is made by connecting 12 pieces like the one shown.



- When the 12 track pieces are assembled, what distance does a toy train travel along the inside of the track?
 - Calculate the radius of the inside edge of the track, to the nearest millimetre.
 - Estimate the radius of the outside edge of the track.
7. Give some examples of measuring instruments that are used to measure distance in the home, community, or workplace. Show or explain how one of these instruments works.
8. Jacques wants to build a trundle wheel. He wants the wheel to go around once for every metre the trundle wheel is pushed. What will be the radius of his trundle wheel?



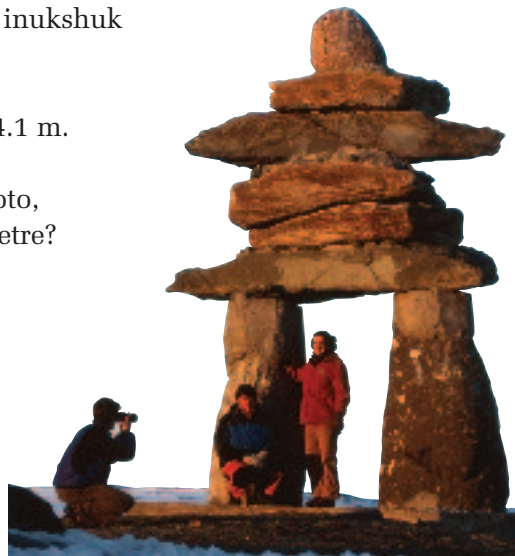
9. A magazine editor needs to assess whether the photograph can be reduced proportionately to fill the rectangle below. Can it be? Explain, using measurements and ratios.



Did You Know?

The inukshuk is a symbol of Inuit culture. It was traditionally used to mark a place of respect, to help hunt caribou, or as a landmark. Inukshuks are made by piling rocks on top of each other. They may be created in many forms; however, few have the form of a person.

10. The photograph shows an inukshuk overlooking Rankin Inlet in Nunavut. Suppose the height of the inukshuk is 4.1 m. What is the height of the person standing in the photo, to the nearest tenth of a metre?



11. a) Measure the diameter of a Canadian dime and quarter.
b) Calculate the ratio of the diameter of a dime to the diameter of a quarter.
c) If this ratio applies for a quarter and a loonie, what would the diameter of the loonie be? Does the ratio apply? Justify your reasoning.
12. Use the map of part of the Northwest Territories to help answer the following questions.

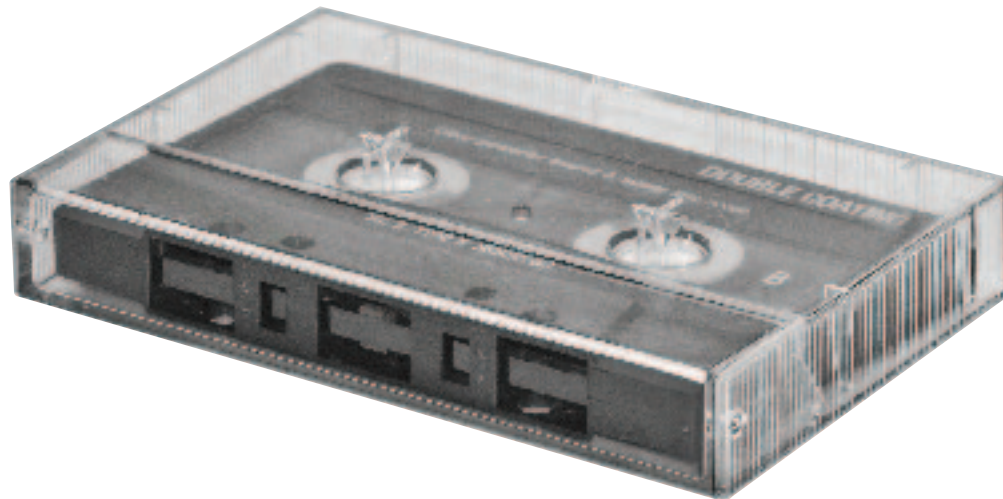


- a) Express the scale of the map as a ratio in lowest terms.
b) Estimate the distance from Fort Simpson to Moose Ponds. Measure and compare the distance with your estimate.
c) Compare the distances from Virginia Falls and Rabbitkettle Lake to Fort Simpson. How much greater is the distance from Rabbitkettle Lake?

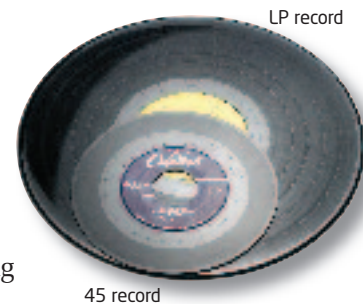
13. A geostationary satellite is in orbit 35 800 km above Earth's equator. The average radius of Earth at the equator is 6380 km.
- Draw and label a diagram of Earth and the path of the satellite.
 - What distance does an observer on the equator travel in one day due to Earth's rotation? Express your answer to the nearest tenth of a kilometre
 - How far must the geostationary satellite travel in one day to appear stationary above Earth? Express your answer to the nearest tenth of a kilometre.
 - How much faster is the satellite travelling than the observer on Earth? Hint: Use the following formula.

$$\text{Velocity (in kilometres per hour)} = \frac{\text{distance (in kilometres)}}{\text{time (in hours)}}$$

14. **Unit Project** The music industry involves the production, distribution, and sale of music in a variety of forms. Since the 1960s, music distribution has evolved from vinyl records to cassette tapes, to CDs, and to MP3s. Each change emphasized that smaller is better.
- For the actual-size cassette shown, use a suitable referent to estimate the dimensions of the cassette case. Explain why you used that referent.



- Measure and calculate the perimeter of each different face of the cassette case, in millimetres. How many perimeters do you need to calculate?
- Vinyl records are available in three sizes—45 rpm (revolutions per minute), 78 rpm, and $33\frac{1}{3}$ rpm or LP size. A 45 record has an actual diameter of 17.5 cm. Estimate the diameter of the LP in the photograph. Then, by measuring and determining a scale, calculate the actual diameter of the LP, in millimetres.



Did You Know?

A geostationary satellite is a satellite that appears in a fixed position to an observer on Earth. The satellite revolves around Earth at the same distance above the equator. These satellites are used for communications such as direct TV distribution.

WWW Web Link

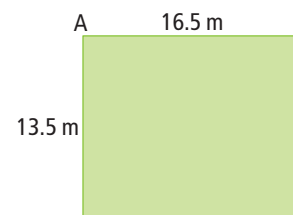
To learn more about the history of music distribution, go to www.mhrmath10.ca and follow the links.

Extend

15. The scale of an aerial photograph can be approximated as the ratio of the camera's focal length to the airplane's altitude. For this aerial photograph taken near Fort McMurray, AB, the airplane's altitude is 305 m. The camera's focal length is 45 mm. What is the greatest distance across the crater shown in the photo?



16. Your lawn has the dimensions shown. You cut the grass using a mower that cuts with a width of 52 cm.
- a) To mow the entire lawn, what is the minimum distance you must walk? You start at A and return to A when finished.
- b) Does your route, for example, along the perimeter versus in rows, affect the distance you walk? Explain.



Create Connections

17. Imagine a band or ring placed tightly around Earth at the equator. You cut the band and lengthen it by 1 m. If you could block the new band so it is equally spaced above the equator, what distance would it be away from Earth? Assume Earth is circular at the equator and the radius of Earth is about 6400 km.
18. Work with a partner. Sometimes a sprained ankle needs to be wrapped with a compression bandage.
- a) Estimate the length of elastic bandage needed to wrap your ankle then foot repeatedly for four cycles.
- b) Wrap your ankle and foot using the figure eight wrapping technique with a strip of cloth. Measure the length required. Compare your estimate with the actual measurement.
- c) Darwin estimates the length of bandage by determining the approximate circumferences of his ankle and foot and multiplying the sum by four. He notices that his estimate is not close to his actual measurement of the bandage. Explain why.

19. Sandra lives in Salmon Arm, BC. She wants blue orchard mason bees to live in her backyard. She purchases a bee nesting box and intends to place it in one of her flower beds. The front of the box measures 15 cm by 15 cm on the inside. The outer diameter of each nesting tube is 8 mm.

- Estimate the number of tubes that can fit inside the box.
- Draw and label a diagram showing the dimensions of the nesting box.
- Calculate the maximum number of tubes that can fit inside the box. Then, describe one way to check your work.



Did You Know?

Blue orchard mason bees tend to be quite gentle around people and pets. They are native to North America, and recognized as effective pollinators. Each female builds a nest by herself. She forages nectar and pollen and lays eggs.

20. A factory makes frying pans. The inside surface of each pan is coated with a non-stick coating. You are hired to verify the formula that the factory uses to calculate the area of the inside surface.

The company formula is $S = \pi d \left(\frac{d}{4} + h \right)$, where S represents the inside surface area, in square centimetres; d represents the diameter of the pan across the top, in centimetres; and h represents the height up the side of the pan, in centimetres.

- Measure a frying pan. Calculate the inside surface area using a technique of your choice. Give your answer in terms of π . Then, calculate the surface area using the company formula. Compare the values. Is the factory formula correct? Justify your response.
- Li says that most frying pans will have an inside surface area slightly greater than the one calculated using the formula. Do you agree with Li? Explain why. Use an example and a diagram to support your explanation.

21. **MINI LAB** Work in a small group to establish a personal referent for a kilometre. Mark off a square that has sides of length 25 m. For each group member, measure the time it takes to walk around the square once.

- Step 1** Estimate how long it might take to walk 1 km.
- Step 2** Measure actual times by having each group member walk at a normal pace 10 times around the square. Why might your estimate not be close to the actual time?
- Step 3** Walk along a street or road for your length of time from step 2. Measure the distance using an outdoor measuring device.
- Step 4** List some places that are about 1 km from your school or home.

Materials

- SI measuring tape
- watch
- outdoor measuring device, such as an odometer, pedometer, trundle wheel, or measuring tape

1.2

Imperial Measurement

Focus on ...

- providing referents for linear measurements
- describing a strategy for taking a linear measurement
- solving problems that involve linear measurement using instruments
- estimating linear measurements

imperial system

- a system of measurement based on British units

Most of the world uses the SI measurement system. The United States uses the **imperial system** for linear measurement. This system is based on the older English units of measurement derived from nature and everyday activities.

Canada began a transition from the imperial system to SI in 1970, but imperial measurement is still used. Why do you think this is true? Where have you seen imperial units used to measure distance? What other imperial units can you identify?



The Royal Observatory in Greenwich, United Kingdom displays metal representations for the smaller imperial distance measurements. These include the inch, foot, and yard.

Materials

- imperial linear measuring instrument

Investigate Referents for Imperial Measurement

One of the smallest imperial units for measuring distance is the inch. The next unit larger than the inch is the foot. The next unit larger than the foot is the yard. Work with a partner. Share your answers with your classmates.

1. Identify the length of one inch, one foot, and one yard on your measuring instrument.
 - a) How many inches are in one foot?
 - b) How many feet are in one yard?
 - c) How many inches are in one yard?

2. List objects at school or at home that you could use as a referent for one inch, one foot, and one yard.

3. What could you use as a referent for one mile?

4. Reflect and Respond

- a) Choose an object in your classroom. Describe how to use a referent to measure the dimensions of the object.
- b) Explain why you chose your referent.

Link the Ideas

The following units are the basic imperial units used for measuring distances. They are in order from smallest to largest. The abbreviations and symbol are in brackets.

inch (in. or ")

foot (ft or ') 1 ft = 12 in.

yard (yd) 1 yd = 3 ft or 36 in.

mile (mi) 1 mi = 1760 yd or 5280 ft

How many inches are in $3\frac{1}{2}$ yd?

$$1 \text{ yd} = 36 \text{ in.}$$

$$3\frac{1}{2} \text{ yd} = 3\frac{1}{2} \text{ yd} \left(\frac{36 \text{ in.}}{1 \text{ yd}} \right)$$

$$3\frac{1}{2} \text{ yd} = 126 \text{ in.}$$

There are 126 in. in $3\frac{1}{2}$ yd.

Approximately how many miles are in 12 640 ft?

$$1 \text{ mi} = 5280 \text{ ft}$$

$$\frac{1 \text{ mi}}{5280 \text{ ft}} = \frac{x \text{ mi}}{12\,640 \text{ ft}}$$


$$\frac{1(12\,640)}{5280} = x$$

$$2.3939 \dots = x$$

There are approximately 2 mi in 12 640 ft.

Measuring Instruments

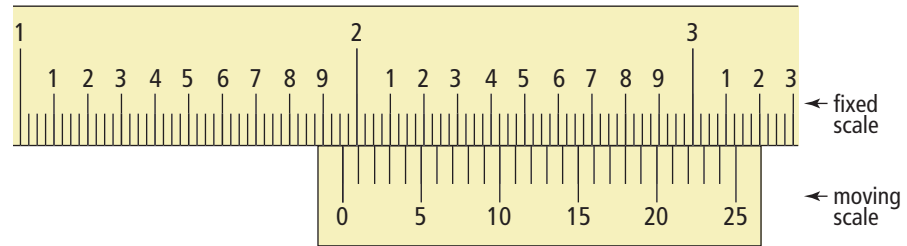
Different measuring devices are used depending on the precision required. An imperial ruler or measuring tape can measure distances to the nearest $\frac{1}{16}$ in. A caliper can measure to the nearest $\frac{1}{1000}$ in.

 **M E**
 $\left(3\frac{1}{2}\right)(36) \approx 4(30) \text{ or } 3(40)$.
So, an estimate is 120.

WWW Web Link

To watch a video showing how to read an imperial caliper, go to www.mhrmath10.ca and follow the links.

Follow these steps to read an imperial caliper.



1. Read the whole number and tenth values on the fixed scale.
This reading is 1.9 in.
2. Determine where zero on the moving scale lies relative to, in this case, the 9 on the fixed scale.
It is 2 small divisions beyond the 9.
 $\frac{2}{4}$ of $\frac{1}{10} = \frac{1}{20}$ or 0.05.
This reading is 0.05 in.
3. Identify the next line on the moving scale that aligns with a line on the fixed scale. In this example, it is 10.
This reading is 0.010 in.
4. Add the measurement readings from steps 1 to 3.
The final reading is 1.960 in. ($1.9 + 0.05 + 0.010 = 1.960$)

Example 1 Determine Imperial Distances

The photograph shows a polar bear near Churchill, MB. The scale of the photograph is 1:24.



- a) Calculate the height of the bear's back, to the nearest inch.
- b) What is the length of the bear? State your answer in feet and whole inches.

Solution

- a) Use an instrument to measure the distance from the highest point on the bear's back to the ground.

height of bear in photo = 2 in.

Let x represent the height of the actual bear.

$$\text{Scale} = \frac{\text{distance on photograph}}{\text{actual distance on ground}}$$

$$\frac{1}{24} = \frac{2}{x}$$

What measurement unit will the actual height be in? How do you know?

$$\frac{1}{24}(24x) = \frac{2}{x}(24x)$$
$$x = 48$$

The height of the bear is 48 in.

- b) Measure the distance from the bear's nose to the rear leg.

length of bear in photo = $3\frac{3}{8}$ in.

Let z represent the actual length of the bear.

$$\frac{1}{24} = \frac{3\frac{3}{8}}{z}$$

$$\frac{1}{24} = \frac{27}{z}$$

$$z = 24\left(\frac{27}{8}\right)$$

$$z = 81$$

The length of the bear is 81 in.

Convert 81 in. to feet and inches.

To do this calculate the number of whole feet and then find the number of inches remaining.

$$\frac{81 \text{ in.}}{12} = 6 \text{ ft } 9 \text{ in.}$$

The length of the bear is 6 ft 9 in.

Use 1 ft = 12 in. to find the number of whole feet.

The number of inches in 6 ft is $6(12 \text{ in.}) = 72 \text{ in.}$

$$\text{Remainder} = 81 - 72$$
$$= 9$$

Your Turn

The photograph of a muskox uses a scale of 1:30. Calculate the height of the muskox and the distance between the tips of its horns. State each answer in feet and inches.



Example 2 Apply Linear Measurement

The Carsons want to buy a 32" television. The size of a television is measured across the screen diagonally. They are choosing between a standard 4:3 television set and a widescreen 16:9 HDTV. To help them decide, calculate the screen dimensions and the viewing area for each television. Which television has the greater viewing area?

Solution

Standard 4:3 Television Screen

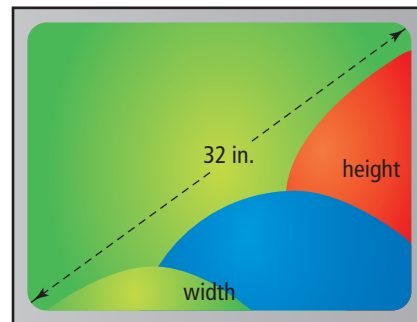
Using the ratio of width to height (4:3), draw a scale diagram to visualize the television screen.

Measure the diagonal, the width, and the height of the television screen in the diagram.

$$\text{diagonal} = 2.5 \text{ in.}$$

$$\text{width} = 2 \text{ in.}$$

$$\text{height} = 1.5 \text{ in.}$$



To calculate the width, w , and height, h , of the actual TV screen, calculate the scale factor using the measurement of the diagonal. Let s represent the scale factor.

$$32s = 2.5$$

$$s = \frac{2.5}{32}$$

$$s = 0.078125$$

Scale factor(width of actual TV) = width of screen in diagram

$$0.078125w = 2$$

$$w = \frac{2}{0.078125}$$

$$w = 25.6$$

Scale factor(height of actual TV) = height of screen in diagram

$$0.078125h = 1.5$$

$$h = \frac{1.5}{0.078125}$$

$$h = 19.2$$

Area of screen = width \times height

$$A = 25.6(19.2)$$

$$A = 491.52$$

The viewing area of the standard television is 491.52 in.².

Widescreen 16:9 HDTV

The ratio of width to height in the diagram is 16:9.

The actual TV is an enlargement of the diagram.

To determine the dimensions of the actual TV screen, you could use an enlargement factor of x .

Then, the actual width, w , is represented by $16x$ and the actual height, h , is represented by $9x$.

The actual diagonal is 32 in.

You can use the Pythagorean relationship to determine the enlargement factor.

$$\begin{aligned}(16x)^2 + (9x)^2 &= 32^2 \\ 256x^2 + 81x^2 &= 1024 \\ 337x^2 &= 1024 \\ x^2 &= \frac{1024}{337} \\ x^2 &= 3.0385\dots \\ x &= 1.7431\dots\end{aligned}$$

Calculate the actual width.

$$\begin{aligned}w &= 16x \\ w &= 16(1.7431\dots) \\ w &= 27.8904\dots\end{aligned}$$

Calculate the actual height.

$$\begin{aligned}h &= 9x \\ h &= 9(1.7431\dots) \\ h &= 15.6883\dots\end{aligned}$$

Area of screen = width \times height

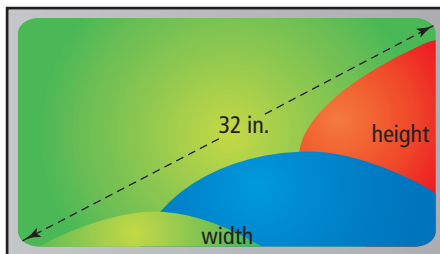
$$\begin{aligned}A &= (27.8904\dots)(15.6883\dots) \\ A &= 437.5548\dots\end{aligned}$$

The viewing area of the standard TV is about 491 in.² and the viewing area of the HDTV is about 438 in.².

The standard TV has the greater viewing area.

Your Turn

What is the difference in the viewing area for a 46" standard television (4:3) and a 46" widescreen television (16:9)?



Did You Know?

Scientists have developed an ultra-thin flexible screen that folds to fit in a pocket. These screens could be used for computers, telephones, and advertising.



Inuit drum dancers performing at the inaugural event in Iqaluit, Nunavut. Nunavut became Canada's newest territory on April 1, 1999.

Example 3 Solve a Problem Using Imperial Measurements

Alashun wants to make a drum, or qilaut, that resembles the one used by a drum dancer in Iqaluit, NU. He has a circular frame, over which to stretch caribou skin. Then, he will lash it into place along the frame with sinew. Alashun uses $3\frac{1}{2}$ in. of sinew for each inch of the frame.

- Estimate the diameter of the drum frame in imperial units. The scale of the photo is 1:15.
- Approximately what length of sinew does Alashun need to make the drum? State your answer in yards and inches.

Solution

- The diameter of the drum in the photo appears to be just over 1".
 $1''(15) \approx 15''$
 So, the diameter of the drum is approximately 15".

How would you estimate the diameter of the drum in the photo?

- Estimate the circumference of the drum frame.

$$C = \pi d$$

$$C = \pi(15)$$

$$C = 15\pi$$

The circumference of the drum frame is approximately 15π inches.

Alashun uses $3\frac{1}{2}$ in. of sinew for each inch of the frame.

Let l represent the length of sinew needed.

$$l \approx \left(3\frac{1}{2}\right)(15\pi)$$

$$l \approx 164.9336\dots$$

Round this distance to whole inches for converting.

Convert 165 in. to yards and inches.

$$\frac{165 \text{ in.}}{36} = 4 \text{ yd } 21 \text{ in.}$$

What steps do you follow to convert between imperial units?

$$\frac{160}{40} = 4$$

So, 165 in. \approx 4 yd.

Alashun needs approximately 4 yd 21 in. of sinew.

Your Turn

A round Inuit drum needs to have its skin restretched and then lashed into place with sinew. For each inch of the frame, $3\frac{1}{2}$ in. of sinew are needed. The diameter of the frame is $1\frac{1}{4}$ ft. What length of sinew is needed? Express your answer to the nearest quarter of a foot.

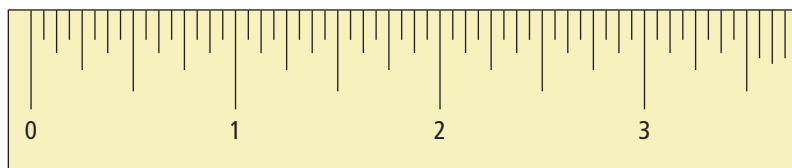
Key Ideas

- The imperial system of measurement is widely used in the United States for measuring distances.
- Even though SI is Canada's official measurement system, some Canadian industries still use imperial units.
- In the imperial system, common units for linear measurement are the inch (in.), foot (ft), yard (yd), and mile (mi). The imperial units for length are related according to the following conversions:
 $1 \text{ mi} = 1760 \text{ yd}$ $1 \text{ yd} = 3 \text{ ft}$ $1 \text{ ft} = 12 \text{ in.}$

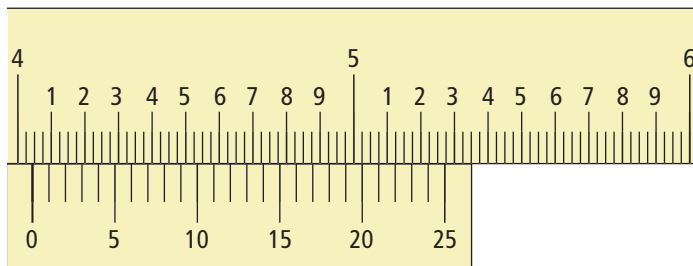
Check Your Understanding

Practise

1. a) What does the smallest subdivision on this imperial ruler represent?



- b) Look at the caliper that measures in inches. What is the value of each of the smallest subdivisions on the fixed scale? State your answer as both a fraction and a decimal.



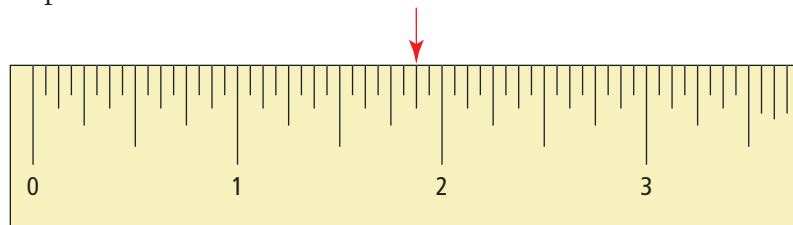
- c) What is the value of each of the smallest subdivisions on the moving scale of the caliper in part b)? State your answer in fraction and decimal form.

2. Convert each measurement to the unit indicated.

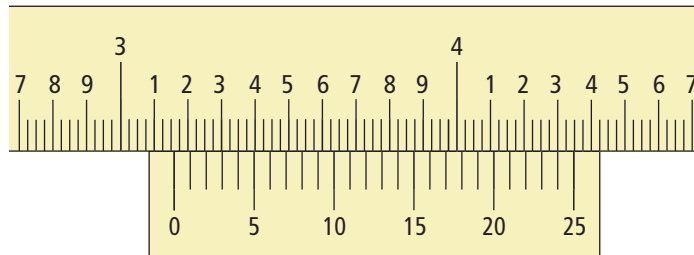
- a) The world's longest earthworm measured $1\text{ ft } 1\frac{1}{2}\text{ in.}$
(nearest half of an inch)
- b) The world's shortest man is $2' 3''.$
(nearest quarter of a yard)
- c) A rocket separates from its space capsule at $400\ 000\text{ ft.}$
(nearest mile)
- d) The altitude of a balloon is 3 mi. (nearest foot)

3. What reading is shown on each measuring scale? For each measurement, name one item that might have this dimension.

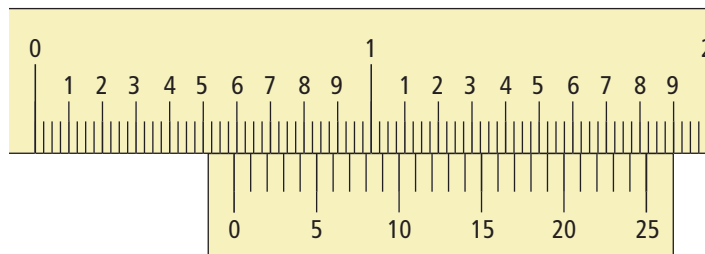
a) imperial ruler



b) imperial caliper



c) imperial caliper



4. Name a measuring device that would be appropriate to measure each distance. Explain your choices. Then, measure each distance, to the nearest sixteenth of an inch.

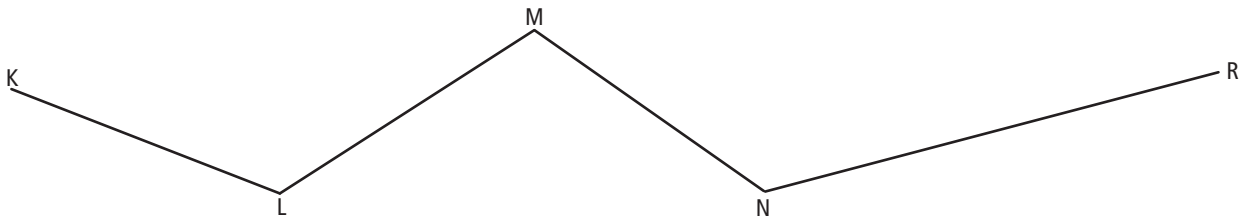
- a) the diameter of a pen
- b) the circumference of a pen
- c) the length of a pen

5. Use your referent for an inch to estimate the total length of each figure. Then, measure each distance. Express answers to the nearest quarter of an inch.

a)



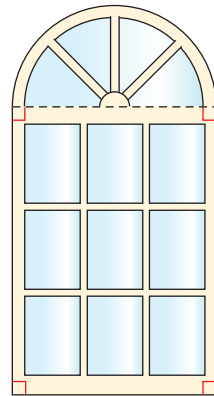
b)



6. Explain how you can use a personal referent to help you estimate. Then estimate and measure each distance. State each measurement in feet and inches.
- the width of your classroom
 - the perimeter of your desk or table top
7. Billy Loutit was a Métis mail carrier for the Hudson's Bay Company. He ran 100 mi, from Athabasca, AB, to Edmonton, in 16 h through flooded terrain.
- What was Billy's average speed in miles per hour?
 - How long did it take Billy to run a mile?

Apply

8. An interior designer wants to present a client with some options for wood trim to frame the Norman window shown in the diagram. The scale of the drawing is 1:32. What is the distance around the outside of the window? Express your answer to the nearest half inch. Assume the curve is a semicircle.



Did You Know?

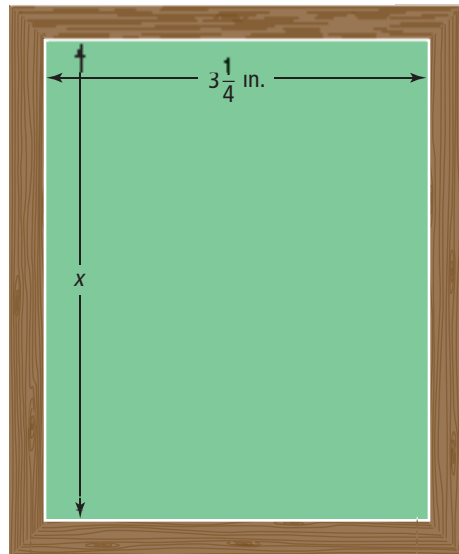
In 1904, William (Billy) Loutit was sent from Athabasca, AB, to Edmonton seeking emergency help against the flood that threatened to destroy Athabasca. By completing the 100-mi journey on foot in 16 h, he became a hero.



9. Leslie has a new manual wheelchair. It has 3 in. diameter micro-caster wheels and 24 in. diameter drive wheels.
- Leslie wants to know how many times the caster wheels rotate for each rotation of the drive wheels. Explain the calculations you would perform to obtain the answer. Then, give the answer as a ratio of drive wheel rotations to caster wheel rotations. Write the ratio in lowest terms.
 - How many rotations of the drive wheels are needed to travel 250 yd?
 - Suppose Leslie travels $1\frac{1}{2}$ mi. How many rotations will the drive wheels make?
10. Marcus works in a photography laboratory. He needs to enlarge a photograph of Virginia Falls in Nahanni National Park, NT, and make it fit into the frame shown.

Did You Know?

The water at Virginia Falls in Nahanni National Park, NT, plunges 295 ft. In the centre of the falls stands Mason's Rock. It is named after Bill Mason, a well-known Canadian adventurer and canoeist.



- What is the scale factor for the enlargement?
 - What is the length of the unknown side of the frame?
11. Gail and Bram are calculating the area of their washroom floor to order new tiles. Together, they measure the length and width of the floor to be $7\frac{1}{2}$ ft and 5 ft.
- Gail calculates the area to be $37\text{ ft}^2\ 5\text{ in.}^2$. Is Gail correct? Explain.
 - The tiles they select are 6 in. by 12 in. How many tiles are needed to cover their washroom floor?

12. **Unit Project** Today's music storage devices tend to be smaller than those of the past, but they can store many more songs. Find a cassette tape case, a CD, and an MP3 player. Use an imperial unit to measure each of the following dimensions. Justify your choice of unit.

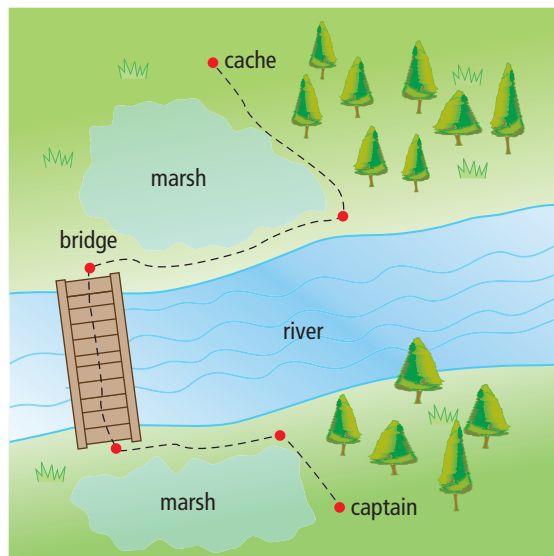


WWW Web Link

To learn more about past and present music storage devices, go to www.mhrmath10.ca and follow the links.

- the diameter of the CD
- the dimensions of the cassette case
- the perimeter of the largest face of the MP3 player

13. A geocaching team captain estimates that a cache is located 500 yd northwest from his position. However, the team must cross the river using the bridge shown.
- Estimate the distance that the team travels from the captain's position to the cache. Justify your answer.
 - Global Positioning System (GPS) readings provide straight line distances. Estimate the total of the GPS distances between the red dots on the map. Give your answer in yards and feet. How does this total distance compare with the actual distance walked? Explain why.



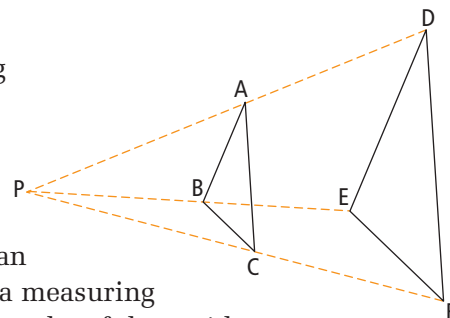
Extend

14. Sometimes it is difficult to measure the diameter of an object. It may be easier to measure its circumference. The distance around an above-ground circular pool is 60 ft.
- What is the diameter of the pool? Express your answer to the nearest inch.
 - The owner wants to build a circular wall outside the existing one to help insulate the pool. The material she wants to use is only available in 62 ft, 65 ft, and 70 ft lengths. Determine the diameter of the new insulating wall using each of the available lengths. Express your answer to the nearest inch.
 - Which length would you recommend the owner choose? Explain your reasoning.



15. The astronomical unit (AU) is a unit of length based on the average distance from Earth to the sun. The AU is currently accepted as 92 955 887.6 mi.
- Two comets appeared in rapid succession in 1996 and 1997.
- Comet Hyakutake came within 0.1018 AU of Earth.
 - Comet Hale-Bopp came within 1.315 AU of Earth.
- Within how many miles of Earth did each comet pass?
 - What is the difference in the distance from Earth between the paths of Comets Hale-Bopp and Hyakutake? Give your answer to the nearest mile.

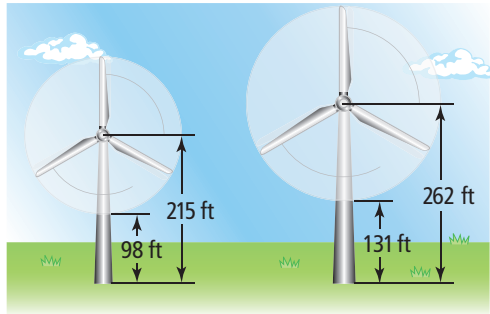
16. You can enlarge a shape by using a point, P, and measuring distances from it to the vertices of the shape and to the enlargement.



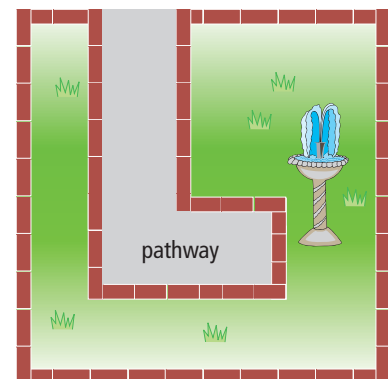
- What ratios of side lengths should be equal if $\triangle DEF$ is an enlargement of $\triangle ABC$? Use a measuring instrument to measure the lengths of these sides and compare the ratios.
- Explain the mathematics behind this method of enlarging a figure.
- Use this method to enlarge a figure by a factor of three.

Create Connections

17. Sam's room measures 11 ft by $13\frac{1}{2}$ ft. He wants to buy a new queen-size bed, if it will fit with his existing furniture. Sam has a desk, which measures 4 ft wide by 22 in. deep, and a night stand.
- Find the dimensions of a double bed and a queen-size bed.
 - Design a layout for Sam's room using a scale diagram. Hint: You will need to estimate the dimensions of a night stand.
 - Which bed do you suggest Sam buy? Why?
18. Manitoba Hydro announced plans to build a 300-MW wind farm at St. Joseph, MB. Measurements of some of the wind turbines being considered are shown below.
- What is the difference in the length of the blades for the wind turbines shown?
 - Suppose each turbine makes 30 revolutions per minute. The tip speed of the blades can be up to 6 times the wind speed. Determine the tip speed of each blade. What is the maximum wind speed for each turbine?
Hint: $\text{speed} = \frac{\text{distance}}{\text{time}}$.



19. A pathway leads to a fountain in a small park. The park and the pathway are outlined with a brick border.
- Estimate the perimeter of the border in the diagram using imperial units. Explain how you estimated your answer.
 - Measure the perimeter. How close was your estimate to the actual measure?
 - Draw a new diagram for the same park but make the pathway to the fountain half the width of the one shown in the diagram. What is the perimeter of the border in your diagram?
 - Predict how the perimeter of the border changes as the width of the pathway changes. Check your prediction. Use words, diagrams, and imperial measurements to support your answer.



1.3

Converting Between SI and Imperial Systems

Focus on ...

- comparing SI and imperial units using referents
- solving problems that involve conversion of linear measurements between SI and imperial systems
- using mental mathematics to confirm the reasonableness of a solution to a conversion problem

Materials

- compact disc (CD)



Some people use music to express their thoughts through song lyrics. Many song lyrics tell a story.

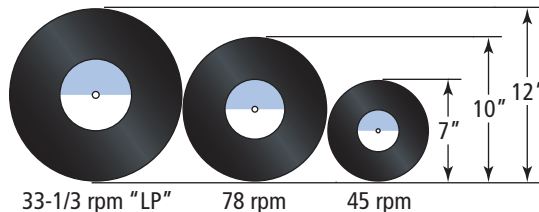
- Name some lyrics or song titles that include imperial measurements, such as miles, feet, or miles per hour.
- What SI measurements are comparable to those measurements?
- Describe how the converted measurements could change the song.

Investigate Relationships Between SI and Imperial Measurements

Unit Project

- a) Use a referent to estimate the diameter of a CD in SI units. How did you make your estimate?
 - b) Use a referent to estimate the diameter of a CD in imperial units.
2. Measure the diameter of a CD, to the nearest millimetre.

3. Calculate the diameter of each vinyl record in SI units.



4. a) Calculate the circumference of a CD and each of the three vinyl records shown, in SI units.
b) Compare the sizes of the four recording devices.

5. Reflect and Respond

- a) Discuss with a partner. Which recording device is able to store more music—a vinyl record or a CD?
b) Develop a method you could use to compare the amount of music that each device stores to the size of the device. Test your method in both imperial and SI units.
c) Describe how you think laser technology has affected the storage of music. Support your answer with information you have gathered in your research.

WWW Web Link

To learn more about laser technology, go to www.mhrmath10.ca and follow the links.

Link the Ideas

To convert from one measurement system to another, you need to understand the relationships between the units of length in each system. Conversions involve proportional reasoning and unit analysis.

Conversions between measurement systems may be approximate or exact. The imperial yard has been defined as 0.9144 m. This would be considered an exact conversion.

Since $1 \text{ yd} = 0.9144 \text{ m}$ and $1 \text{ yd} = 3 \text{ ft}$, $3 \text{ ft} = 0.9144 \text{ m}$.

$$1 \text{ ft} = 1 \text{ ft} \left(\frac{0.9144 \text{ m}}{3 \text{ ft}} \right)$$

$$1 \text{ ft} = 0.3048 \text{ m}$$

To convert from metres to yards, 1 m is often given as 1.094 yd. Verify whether this conversion is exact or approximate.

Let x represent the number of yards in 1 m.

$$0.9144 \text{ m} = 1 \text{ yd}$$

$$\frac{0.9144 \text{ m}}{1 \text{ yd}} = \frac{1 \text{ m}}{x \text{ yd}}$$

$$x = \frac{1}{0.9144}$$

$$x = 1.0936\dots$$

The conversion $1 \text{ m} = 1.094 \text{ yd}$ is approximate.

The following are some common conversions.

Exact Conversions

$$1 \text{ in.} = 2.54 \text{ cm} \qquad 1 \text{ ft} = 30.48 \text{ cm} \qquad 1 \text{ yd} = 0.9144 \text{ m}$$

Approximate Conversions

$$\begin{array}{lll} 1 \text{ mm} \approx 0.0394 \text{ in.} & 1 \text{ cm} \approx 0.3937 \text{ in.} & 1 \text{ m} \approx 1.094 \text{ yd} \\ 1 \text{ m} \approx 3.281 \text{ ft} & 1 \text{ km} \approx 0.6214 \text{ mi} & 1 \text{ mi} \approx 1.609 \text{ km} \end{array}$$

Example 1 Convert Between SI and Imperial Units for Length

Researchers at the Harvard-Smithsonian Center for Astrophysics made an announcement in January, 2001. They stated that they had “frozen light” by using super-cooled vapour to slow the speed of light waves to zero. The speed of light in a vacuum is defined as 299 792 458 m/s.

- Estimate the speed of light in miles per second.
- Predict whether the actual speed of light is greater than your estimate. Justify your prediction.
- Calculate the answer, to the nearest mile per second.

Solution

- a) Since there are 1000 m in 1 km, 299 792 458 m/s can be converted to 299 792.458 km/s or approximately 299 792 km/s.

$$299\,792 \frac{\text{km}}{\text{s}} \left(\frac{1 \text{ mi}}{1.6 \text{ km}} \right) \approx 187\,370 \frac{\text{mi}}{\text{s}} \qquad \text{Recall that } 1 \text{ mi} \approx 1.6 \text{ km.}$$

The speed of light can be estimated as 187 370 mi/s.

- b) The actual speed of light, in miles per second, is less than the estimate because the SI speed was rounded down before converting.

c) Use unit analysis. $1 \text{ mi} = 1760 \text{ yd} \left(\frac{0.9144 \text{ m}}{1 \text{ yd}} \right)$

$$1 \text{ mi} = 1609.344 \text{ m}$$

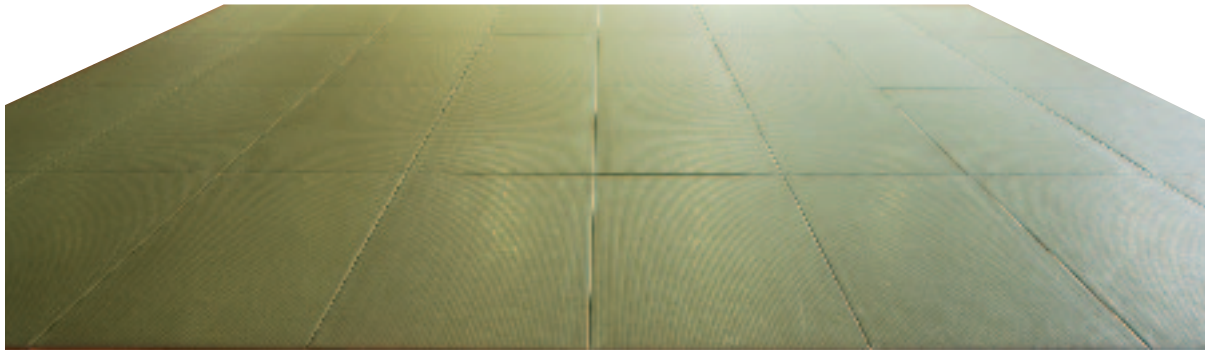
$$299\,792\,458 \frac{\text{m}}{\text{s}} \left(\frac{1 \text{ mi}}{1609.344 \text{ m}} \right) = 186\,282.397 \frac{\text{mi}}{\text{s}} \qquad \text{Why is the speed of light used here in metres per second?}$$

The speed of light is approximately 186 282 mi/s.

Your Turn

Swimmer Brian Johns of Richmond, BC, represented Canada at the 2008 Olympics in Beijing. He finished 7th in a race that one news report referred to as 400 m long and another news report referred to as $\frac{1}{4}$ mi. Are the two measurements equivalent? If not, which distance do you think is more accurate? Justify your reasoning.

Example 2 Solve a Problem Involving Linear Measurements



Your class needs to lay mats on the gymnasium floor for a gymnastics meet. The gym measures 84' by 50'. A scale drawing of one mat is shown. The scale is 1:30.5.

A classmate thinks that 131 mats are needed. Do you agree? Explain.



Solution

The mat in the diagram measures 8 cm by 4 cm.

Use the given scale.

$$\begin{aligned} \text{length of mat} &= 8(30.5) \\ l &= 244 \end{aligned}$$

$$\begin{aligned} \text{width of mat} &= 4(30.5) \\ w &= 122 \end{aligned}$$

The length of the mat is 244 cm and the width is 122 cm.

Length of mat

$$244 \text{ cm} \left(\frac{1''}{2.54 \text{ cm}} \right) \approx 96''$$

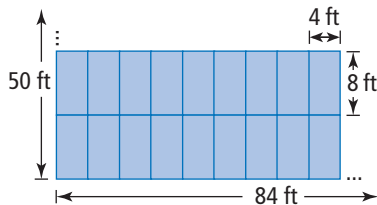
Width of mat

$$122 \text{ cm} \left(\frac{1''}{2.54 \text{ cm}} \right) \approx 48''$$

There is a 2:1 ratio of length to width in the measurements of the mat. The SI-to-imperial conversions confirm a 2:1 ratio of length to width.

The dimensions of the mat are 8 ft by 4 ft. Sketch a layout of the mats in the gym.

How do you know the dimensions are 8 ft by 4 ft?



Suppose you do not visualize the mats and you only work with areas.

$$\frac{\text{area of gym}}{\text{area of mat}} = \frac{84(50)}{8(4)}$$

Therefore, about 131 mats would be needed.

The width of each mat is 4 ft.

$$\frac{84}{4} = 21$$

So, there are 21 columns of mats.

The length of each mat is 8 ft.

$$\frac{50}{8} = 6\frac{1}{4}$$

So, there are 6 rows of mats and some uncovered space.

To cover the gym floor, there are 21 columns and 6 rows of mats.

$$21(6) = 126$$

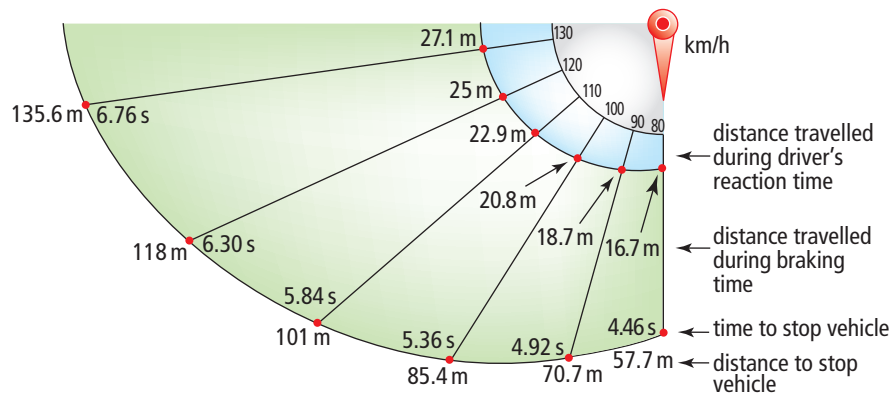
Therefore, 126 mats are needed to cover most of the gym floor.

Your Turn

- How many paving stones measuring $7\frac{1}{2}$ in. by $7\frac{1}{2}$ in. are needed to cover an area that is 1 yd by 1 yd?
- To tile a floor that is 3 m by 4 m, how many tiles measuring 30 cm by 50 cm would you buy? Add 10% extra tiles for areas that require tiles to be cut.

Example 3 Determine Stopping Distances

The distance required to stop a moving vehicle is the sum of the distances travelled during the reaction time and the braking time. The diagram shows the theoretical stopping distance at various speeds.



- What factors might affect the reaction time and braking distances?
- If a vehicle is travelling at 100 km/h, approximately what distance is travelled while the brakes are being applied?
- Convert 55 mph into kilometres per hour. What is the approximate stopping distance when a vehicle is travelling at this speed? Express your answer in feet.

Solution

- a) Factors that affect reaction time may include being tired, distraction by passengers, time in the vehicle, loud music, or reduced visibility due to weather or dirty windows.

Factors that affect braking distance may include condition of brakes, condition and/or type of tires, speed, type of road surface, outside temperature, or road conditions (dry, icy, wet).

- b) Find 100 km/h on the diagram.

The reaction-time distance at 100 km/h is 20.8 m.

The total stopping distance at 100 km/h is 85.4 m.

Total stopping distance = reaction-time distance + braking distance

$$85.4 = 20.8 + b$$

$$85.4 - 20.8 = b$$

$$64.6 = b$$

If travelling at 100 km/h, a vehicle travels approximately 65 m while the brakes are being applied.

Subtract the distances represented by the line segments on the diagram.
Stopping distance $\approx 85 - 20$ or 65



- c) Convert 55 miles to kilometres.

Use proportional reasoning.

Recall that 1 mi \approx 1.609 km.

Let x represent the number of kilometres.

$$\frac{1 \text{ mi}}{55 \text{ mi}} \approx \frac{1.609 \text{ km}}{x \text{ km}}$$

$$x \approx (55)1.609$$

$$x \approx 88.495$$

Therefore, 55 mph converts to approximately 88 km/h.

From the diagram, when a vehicle is travelling at 90 km/h, the stopping distance is 70.7 m.

The speed 88 km/h is just under 90 km/h, so convert 70 m to feet.

Use the conversion $0.9144 \text{ m} = 1 \text{ yd}$ or 3 ft.

$$70 \text{ m} \left(\frac{3 \text{ ft}}{0.9144 \text{ m}} \right) = 229.658... \text{ ft}$$

When travelling at a speed of 55 mph, a vehicle takes approximately 230 feet to stop.

Your Turn

- a) Use the diagram from Example 3 on page 40. Determine the difference between the reaction-time distances for speeds of 110 km/h and 120 km/h. Express your answer in feet.
- b) Convert 90 km/h into miles per hour. What is the approximate stopping distance for a vehicle travelling at this speed? Express your answer in yards.

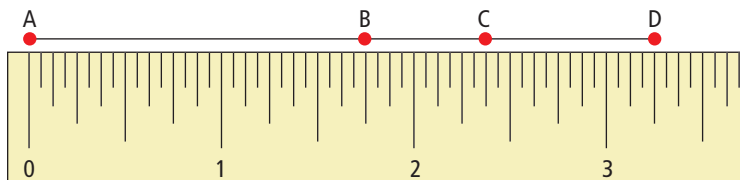
Key Ideas

- When solving problems involving measurement, it is crucial to work with the same units. You may need to convert units within one measurement system (for example, inches to feet) or between imperial and SI units.
- If an exact conversion between systems is required, use $1 \text{ yd} = 0.9144 \text{ m}$ to find a conversion between the required units.
 $1 \text{ yd} = 0.9144 \text{ m}$
 $36 \text{ in.} = 0.9144 \text{ m}$ **Divide both sides by 36.**
 $1 \text{ in.} = 0.0254 \text{ m}$
 $1 \text{ in.} = 2.54 \text{ cm}$ **This is an exact conversion**
- Sometimes you use approximate values, such as $1 \text{ in.} \approx 2.5 \text{ cm}$ or $1.6 \text{ km} \approx 1 \text{ mi}$ when estimating between measurement systems.

Check Your Understanding

Practise

1. Use the diagram of an imperial ruler to help answer the questions below.

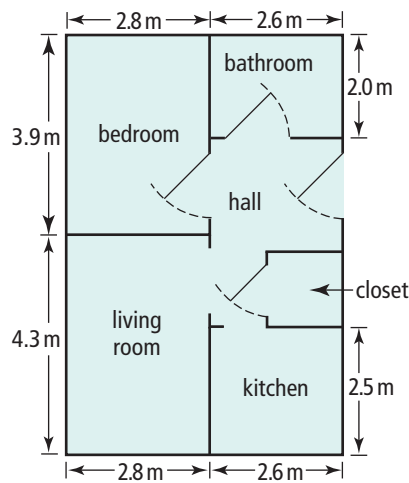


- a) What is the length of AC? Give your answer as a fraction.
 - b) Suppose you replace the imperial ruler with an SI ruler. What is the length of AD, in millimetres?
 - c) What is the difference in the lengths of segments AB and CD? Give your answer in SI and imperial units.
2. Convert each measurement to the unit specified.
 - a) The diameter of a human hair is 0.001 in. (hundredth of a millimetre)
 - b) On an NBA basketball court, the width of the key (painted area beneath the basket) is 4.9 m. (feet)
 - c) A snowmobile trail in Alberta is 26 mi 385 yd long. (hundredth of a kilometre)
 - d) An envelope has a height of $3\frac{7}{8}$ in. (hundredth of a centimetre)

3. People have always used parts of the body for measuring length. For example, you may have heard the measurements of a horse stated by the number of “hands.” Work with a partner. State your answers in both SI and imperial units.
- Estimate and then measure your hand span.
 - How many of your hand spans are needed to measure the length of your desk?
 - Estimate and then measure your pace. This is the distance between your heels when you take a step while walking. To be more accurate, you may wish to measure 10 paces and calculate the average.
 - How many of your paces would it take to walk a mile? a kilometre?
 - Which measurement system do you prefer to use when estimating? Why?



4. The floor plan for a one-bedroom apartment is shown.
- If the closet is square, estimate the length of one of its sides.
 - Determine the scale of the floor plan.
 - Calculate the length of one side of the closet. Express your answer to the nearest tenth of a metre.
 - What are the dimensions of the bedroom, in imperial units?



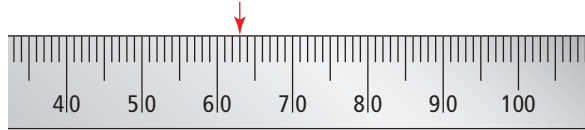
5. Read the following paragraph about the Columbia Icefield. Convert each SI measurement to an equivalent imperial measurement.

The Columbia Icefield straddles the boundary between Alberta and British Columbia. It is the largest glacier in North America, south of the Arctic Circle. It has a maximum depth of 365 m. Its highest points are Mount Columbia, at 3745 m, and Mount Athabasca, at 3491 m. The average snowfall across the icefield is 100 cm per year.

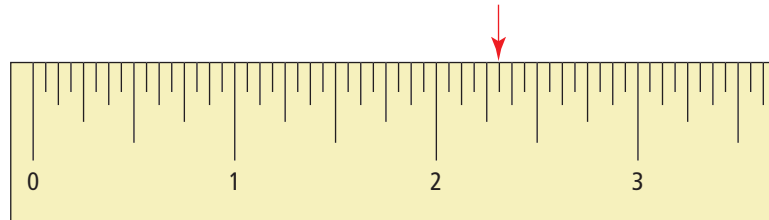


6. What is the reading represented on each measuring device? Estimate and then calculate each equivalent measurement in the other system (SI or imperial).

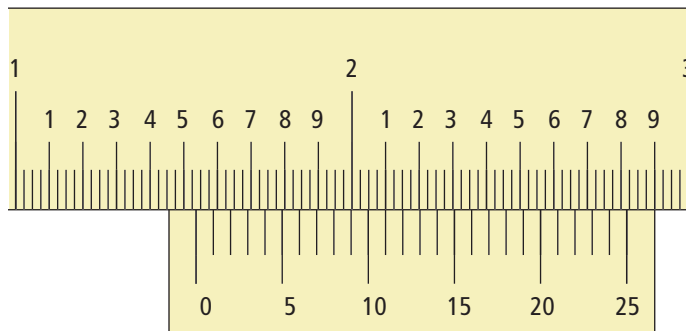
a) SI ruler



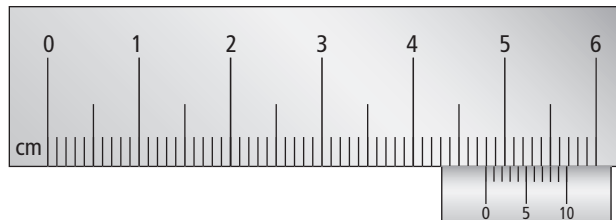
b) imperial ruler



c) imperial caliper



d) SI caliper



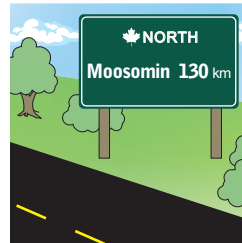
Apply

7. A traditional Inuit dog sled uses teams of Qimmiq or sled dogs on separate lines. The lines are fastened directly to the komatik or sled. Each dog has a harness with an average length of $3\frac{1}{2}$ ft. Suppose a dog sled uses a team of 13 dogs.
- Calculate the approximate total length of rope needed to harness the team.
 - Calculate the total length of the harness in SI units. Explain why you chose this particular unit.

8. Alex is on his way home to Moosomin, SK. He was visiting friends in Mohall, North Dakota. As soon as he gets on the highway, he sees the sign to the right.



As Alex passes the Canada-U.S. border, he sees this sign. What is the driving distance between Mohall and Moosomin, in SI units?



9. a) Discuss with a classmate your preferred method of converting the following units.
- metres to kilometres
 - metres to centimetres
 - yards to miles
- b) Develop a strategy for converting from a smaller unit of length to a larger unit. Does your strategy apply in both SI and imperial systems of measurement? Explain.
- c) How might your strategy change when you are converting from a larger unit of length to a smaller one?
10. Margaux and Penny each travel with their families to St. Pierre-Jolys, MB, for the annual Frog Follies. When the girls arrive, they compare how far they travelled from their homes.
- Margaux lives in Arborg, MB, and travelled 164 km.
 - Penny lives in Grand Forks, North Dakota, and travelled 113 mi.
- Penny is not familiar with SI distances. Explain how to determine the approximate conversion from kilometres to miles. Who travelled farther? Explain.
11. The deepest lake in the world is Lake Baikal, in Russia. It has a depth of 5369 ft. Canada's deepest lake is Great Slave Lake, named after the Slavey (Dene Tha) peoples who live there. It is located in the Northwest Territories and has a depth of 2015 ft. Quesnel Lake, south of Prince George, BC, is the deepest fjord lake in the world. It has a depth of 506 m.
- Compare the depth of Quesnel Lake with the depths of Lake Baikal and Great Slave Lake. Give each answer as a decimal, to the nearest metre.

Did You Know?

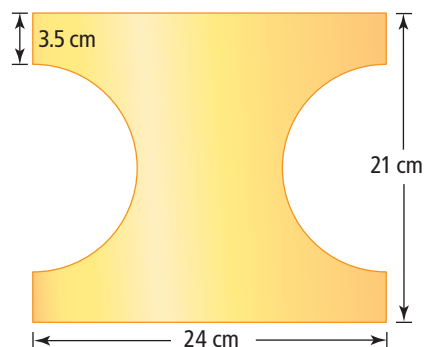
Canada has two Niagara Falls. Most people know about Niagara Falls in Ontario. The other Niagara Falls is on Quesnel Lake in British Columbia. At the mouth of Niagara Creek, water falls over 200 ft into the east arm of Quesnel Lake.

Did You Know?

The pull of gravity is greater closer to the centre of Earth. For this reason, you would weigh about 2.2 lb more at the North Pole than on the equator.

12. Earth is not a perfect sphere. It flattens slightly at the poles. A person standing at the North Pole is about 13 mi closer to the centre of Earth than a person standing on the equator. How far would you be from the centre of Earth when standing on the North Pole? Give your answer in miles. Assume the equatorial radius of Earth is 6380 km.

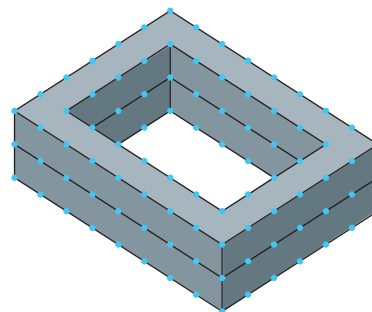
13. a) Calculate the perimeter of the figure shown. Express your answer to the nearest quarter of an inch.
- b) Discuss your steps with a partner. Then, describe what you think is the easiest way to calculate the perimeter of the figure in inches.



14. **(Unit Project)** One type of 80 GB MP3 player has dimensions of 4.14 cm (width) by 9.15 cm (height) by 0.85 cm (thickness). The storage capacity is about 20 000 songs. Each LP vinyl record holds an average of 12 songs and is approximately $\frac{1}{9}$ in. thick.
- a) Calculate the number of LPs you would need to store as many songs as the MP3 player. Use mental mathematics to show that your answer is reasonable.
- b) Suppose you stack the LPs. Calculate the height of the stack. Compare it with the height of the MP3 player. Give your answer as a ratio in lowest terms.

Extend

15. A farmer wants to build a wall using concrete blocks. She draws a diagram showing the wall and identifies the corners of the “cubes.” Each concrete block is the same size as two cubes.



- a) How many concrete blocks does the farmer need?
- b) Each concrete block is 20 cm by 20 cm by 40 cm. What are the outside dimensions of the walled enclosure, in inches?
- c) The farmer wants to build a new wall to fit tightly around the outside of the first one. How many concrete blocks are needed? Explain how to get the answer from your answer to part a).

16. Winnipeg's *River Arch* spans 23 m along the Trans-Canada Highway. The approximate area under a symmetrical arch can be calculated using the formula $A = \frac{w(H + 4h)}{6}$.

In the formula,

- A represents the area, in square metres
- w represents the width of the arch, in metres
- H represents the height of the centre of the arch, in metres
- h represents the height of the arch measured one quarter of the distance from each end of the arch, in metres



- a) Assume the arch is symmetrical. Calculate the approximate area under *River Arch*, in square metres.
- b) Would this formula work for imperial units? Explain.

Create Connections

17. a) State three distances that are often measured in SI units and three distances that are often measured in imperial units.
- b) For each distance in part a), indicate whether it is more appropriate to use an exact measurement or an approximate measurement. Justify your reasoning.
18. Using the conversion factor $1 \text{ yd} = 0.9144 \text{ m}$, show how to convert from a small imperial unit to a larger SI unit.

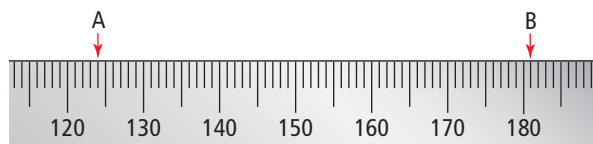
Did You Know?

River Arch symbolizes the past and present of Winnipeg. The arch contains images of bison, wheat, and ploughed fields. *River Arch* is located where two bridges cross the Red River in downtown Winnipeg.

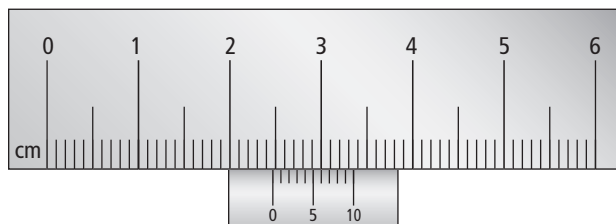
1 Review

1.1 SI Measurement, pages 8-21

1. Find an object with a curved surface, such as a can. Estimate the distance around the curved object. Explain how to measure the curved distance. Then, measure it and compare your estimate to the actual measurement.
2. Determine the distance from A to B on this SI ruler. Express your answer to the nearest tenth of a centimetre.



3. What reading is shown on this SI caliper? Name an object that could be this length.

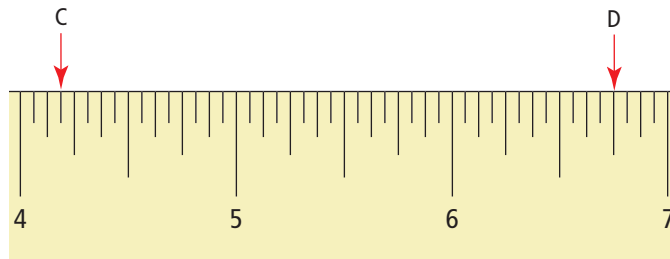


4. What is the circumference of the largest circle you could cut from a sheet of paper measuring 30 cm by 20 cm? What area of paper would you cut away?
5. The elk in the photograph stands 210 cm tall, including the antlers. Calculate the distance between the tips of the antlers, to the nearest tenth of a centimetre.

1.2 Imperial Measurement, pages 22-35

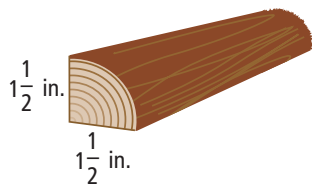
6.
 - a) On a plain piece of paper, draw a letter S. As you draw, try to make the line for the letter $4\frac{3}{4}$ in. long.
 - b) Explain how you could verify that the curve of your S is the required distance.
 - c) Measure your S. If your drawing is out by more than $\frac{3}{4}$ in., try drawing another S.

7. State the reading for point D on this imperial ruler as a mixed number in lowest terms. What is the distance from C to D? Show two ways to determine the answer.

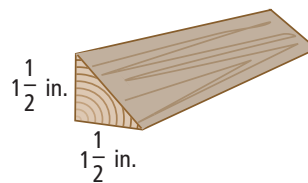


8. At a lumber yard, Jeanette buys lengths of wood with different cross-sections. For each piece of wood, explain how she could calculate the perimeter of the cross-section. Then, calculate each perimeter. Do your answers seem reasonable? Explain.

a)



b)



9. You want to enlarge the photograph of Burrard Inlet, in Vancouver, to fill a 4" by 6" frame.



- a) Estimate the dimensions of the photograph in imperial units. Then, record the actual measurements.
- b) By what scale factor do you need to enlarge the photograph? Will the enlargement need to be cropped to fit in the frame? Justify your reasoning.

1.3 Converting Between SI and Imperial Systems, pages 36-47

10. Convert each measurement to the unit specified.
- a) The distance from Calgary to Jasper is 412 km. (miles)
 - b) Twister is the highest water slide in West Edmonton Mall, at 25.3 m. (feet)



11. The world's tallest man according to *Guinness World Records* was Robert P. Wadlow at 8 ft $11\frac{1}{10}$ in. Suppose his height was stated as 2.7 m. Would this be an approximation or an exact measurement? Justify your answer.
12. Thumbelina, the world's smallest pony, is 17 in. tall. How many times as tall as Thumbelina is the horse in the photograph? Estimate what the height of the horse would be if its head were up. Could you have used different units? Explain.

13. A map of Lesser Slave Lake, AB, is shown.



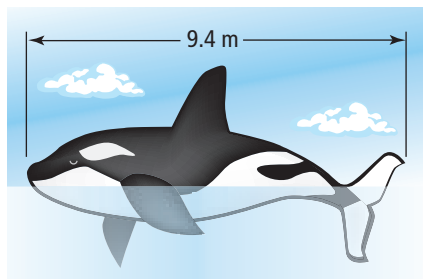
- a) How many kilometres are represented by 1 cm?
- b) How many miles are represented by one inch?
- c) Estimate the length of Provincial Road 750 from Grouard to Atikameg. Give your answer in miles.
- d) Suppose you leave Slave Lake and travel west on Highway 2 for 57 km before turning off. Where might you be going?

1 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. A student measures his pace to be $2\frac{3}{4}$ ft. How many paces will he need to take to walk 1 mi?
A 360
B 640
C 1760
D 1920
2. If 1 yd = 0.9144 m, which is the correct conversion giving the number of inches in a metre?
A 1 in. = 25.4 m
B 1 m = 39.37 in.
C 1 m = 254 cm
D 1 m = 100 in.
3. A killer whale has a length of 9.4 m. How many school door heights would it take to exceed the whale's length?



- A** 1
B 3
C 5
D 7
4. A regulation baseball bat must be $2\frac{3}{4}$ in. or less in diameter and shorter than 42 in. long. Which bat would not be allowed?
A diameter 6 cm
B length 1.0668 m
C diameter 7 cm
D length 0.9 m

5. How many of these rods can fit through a hole with a diameter of $\frac{5}{8}$ in.?



- A** 1
B 2
C 3
D 4

Short Answer

6. a) State two referents that you could use to measure the length of a van. List three appropriate units that you might measure the length of the van in.
- b) Name two units, one SI and one imperial, that are appropriate for measuring the circumference of a car steering wheel. What is an approximate conversion between these two units?
7. Sketch the approximate size of a Canadian \$5 bill. Then, estimate its dimensions in SI units.
8. Janice needs to tighten a nut on her bicycle. She finds two wrenches.
- The $\frac{1}{4}$ in. wrench is too small.
 - The $\frac{3}{8}$ in. wrench is too large.

Janice's neighbour suggests that the nut might be an SI size. Assume that the sizes of SI wrenches are only in whole millimetres. What SI wrench sizes are between the two imperial sizes Janice tried?

Extended Response

9. Two identical archways inside the Manitoba provincial legislature are shown. Suppose the space between the archways is 6 ft. Describe how to calculate the perimeter of one opening, including the bottom. Assume that the top of the archway is a semicircle. Calculate the perimeter of one archway, including the bottom. Give your answer in feet and inches.



10. A gymnasium is 40 m long. An instructor asks two students to create lines every 5 m using green masking tape across the floor. When the instructor returns, he notices that the students made the lines 5 yd apart.
- How many lines did the students make on the floor?
 - What is the distance from the last line to the end wall of the gym? Include a diagram.
 - Members of the football team train by standing at the end wall of the gym, running to the 5 yd line, and returning to the wall. Then, they run to the 10 yd line and return to the wall. They continue this pattern until they reach the opposite end. What is the minimum distance, to the nearest yard, they will run? Justify your answer.
 - Compare the total distances that the football players would run if the students had placed the lines 5 m apart versus 5 yd apart. Does your answer seem reasonable? Explain.