Name:______________________________ Date:___________________

Calculus 12 LG 7-8 Quiz Ver B /30

1. Find $f^{-1}(x)$ if $f(x) = \sqrt[3]{5x-2}$ (2 marks)

$y = \sqrt[3]{5x-2}$

$x = \sqrt[3]{5y-2}$

$x^3 = 5y-2$

$5y = x^3+2$

$y = \frac{x^3+2}{5}$

so $f^{-1}(x) = \frac{x^3+2}{5}$

2. The population of a certain city is given by:
$A(t) = 2500e^{0.03t}$ where *A* is the amount of people and *t* is in years since the year 2000.

a) How many people were present in the year 2000? (1 mark)

$A(0) = 2500\, e^{0.03(0)} = 2500$

b) What is the population in the year 2028? (1 mark)

2028 MEANS $t = 28$

so $A(8) = 2500\, e^{0.03(28)}$

$= 5790.9$ so 5790 (or 5791) PEOPLE

b) When will the population double? (1 mark)

$5000 = 2500\, e^{0.03t}$

$2 = e^{0.03t}$

$\ln 2 = 0.03t$

$t = \frac{\ln 2}{0.03} = 23.1$ YEAR

So IN 2023.

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3. Use implicit differentiation to find the equation of the tangent line to the curve $3x + y^2 = 4$ at the point (2, -1). (3 marks)

$$3 + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -3$$

$$\frac{dy}{dx} = \frac{-3}{2y}$$

$$\text{so } m = \frac{-3}{2(-1)}$$

$$= \frac{3}{2}$$

$$\text{so } y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{3}{2}(x - 2)$$

4. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 = \sin(xy) + y$ (3 marks)

$$2x = \cos(xy)\left[x\frac{dy}{dx} + y\right] + \frac{dy}{dx}$$

$$2x = x\cos(xy)\frac{dy}{dx} + y\cos(xy) + \frac{dy}{dx}$$

$$2x - y\cos(xy) = x\cos(xy)\frac{dy}{dx} + \frac{dy}{dx}$$

$$2x - y\cos(xy) = \frac{dy}{dx}\left[x\cos(xy) + 1\right]$$

$$\frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos(xy) + 1}$$

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5. Find $f'(x)$ if $f(x) = \sqrt{\ln(x^2)} + 5^x$ (3 marks)

$$f(x) = \ln(x^2)^{\frac{1}{2}} + 5^x$$

$$f'(x) = \frac{1}{2}\ln(x^2)^{-\frac{1}{2}} \cdot \frac{1}{x^2}(2x) + 5^x \ln 5$$

$$= \frac{1}{x\sqrt{\ln x^2}} + 5^x \ln 5$$

6. Find $f'(x)$ if $f(x) = \frac{(x^2)(sinx)}{\ln(x)\sqrt{x}}$ (3 marks)

TAKE ln OF BOTH SIDES FIRST.

$$\ln y = \ln x^2 + \ln \sin x - \ln(\ln x) - \ln x^{\frac{1}{2}}$$

$$\ln y = 2\ln x + \ln \sin x - \ln(\ln x) - \frac{1}{2}\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{1}{\sin x}(\cos x) - \frac{1}{\ln x} \cdot \frac{1}{x} - \frac{1}{2x}$$

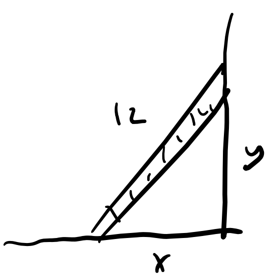
$$\frac{dy}{dx} = y\left(\frac{2}{x} + \cot x - \frac{1}{x\ln x} - \frac{1}{2x}\right)$$

7. Find $f'(x)$ if $f(x) = e^{5x} sin^{-1} x^2$ Note: $\frac{d}{dx} sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$ (3 marks)

$$f'(x) = e^{5x}\left(\frac{1}{\sqrt{1-x^4}}(2x)\right) + \sin^{-1} x^2 (e^{5x})(5)$$

$$= \frac{2xe^{5x}}{\sqrt{1-x^4}} + 5e^{5x} \sin^{-1}(x^2)$$

8. A 12 foot ladder is leaning against a wall. If the bottom of the ladder is pulled away from the wall at a rate of 2 feet per second, how fast is the top of the ladder falling down the wall when it is 9 feet above the ground? (3 marks)



$\frac{dx}{dt} = 2$ FIND $\frac{dy}{dt}$ WHEN $y = 9$

$$x^2 + y^2 = 12^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

WHEN $y = 9$, $x^2 + 9^2 = 12^2$ so $x^2 = 144 - 81$ AND $x = 7.937$

so $2(7.937)(2) + 2(9)\frac{dy}{dt} = 0$

$$\frac{dy}{dt} = \frac{-2(7.937)(2)}{2(9)} = -1.76 \text{ ft/s}$$

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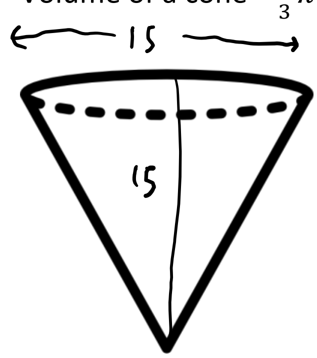
9. Coffee is draining from a conical filter into a coffee pot at the rate of 20 cubic cm per minute. The filter is 15cm tall and 15cm in diameter. How fast is the level of the coffee dropping in the filter when the depth is 10cm? (3 marks)

Volume of a cone = $\frac{1}{3}\pi r^2 h$



$\frac{r}{h} = \frac{7.5}{15}$

$r = \frac{7.5h}{15}$

so $V = \frac{1}{3}\pi\left(\frac{7.5}{15}h\right)^2 h$

$V = \frac{\pi}{3}\frac{56.25}{225}h^3$

$\frac{dV}{dt} = \pi\frac{56.25}{225}h^2\frac{dh}{dt}$

$-20 = \pi\frac{56.25}{225}(10)^2\frac{dh}{dt}$ $\qquad \frac{dh}{dt} = -0.25\text{ cm/min}$

10. Determine the following limits. (2 marks each)

a) $\lim_{x\to 0}\frac{e^x - 1}{\sin x}$

$\lim_{x\to 0}\frac{e^x}{\cos x}$

$= \frac{1}{1} = 1$

b) $\lim_{x\to\infty}\frac{x^2}{e^x}$

$= \lim_{x\to\infty}\frac{2x}{e^x}$

$= \lim_{x\to\infty}\frac{2}{e^x}$

$= 0$

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