Name:______________________________ Date:___________________

Calculus 12 LG 7-8 Quiz Ver A /30

1. Find $f^{-1}(x)$ if $f(x) = 4 + x^3$ (2 marks)

$y = 4 + x^3$

$x = 4 + y^3$

$x - 4 = y^3$

$\sqrt[3]{x-4} = y$

so $f^{-1}(x) = \sqrt[3]{x-4}$

2. A radioactive isotope is transformed into another more stable isotope of a certain element by:

$A(t) = 0.0125e^{\frac{-t}{500}}$ where *A* is in mg and *t* is in years.

a) How much of the isotope was originally present? (1 mark)

$A(0) = 0.0125\, e^{\frac{0}{500}}$

$= 0.0125$ mg

b) When will half of the original amount be transformed? (1 mark)

$0.0625 = 0.0125\, e^{\frac{-t}{500}}$

$0.5 = e^{-t/500}$

$\ln 0.5 = \frac{-t}{500}$

$-t = 500 \ln 0.5$

$t = -500 \ln 0.5$

$t = 346.6$ yrs

b) When will 0.005 milligrams of the original isotope remain? (1 mark)

$0.005 = 0.0125\, e^{-t/500}$

$0.4 = e^{-t/500}$

$\ln 0.4 = \frac{-t}{500}$

$-t = 500 \ln 0.4$

$t = -500 \ln 0.4$

$t = 458.1$ yrs.

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3. Use implicit differentiation to find the equation of the tangent line to the ellipse $3x^2 + y^2 = 4$ at the point (1, 1). (3 marks)

$$6x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -6x$$

$$\frac{dy}{dx} = -\frac{6x}{2y}$$

$$\frac{dy}{dx} = -\frac{3x}{y}$$

SO AT $(1,1)$, $\frac{dy}{dx} = -3$

EQ'N OF TANGENT LINE

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -3(x-1)$$

OR $y = -3x + 4$

4. Use implicit differentiation to find $\frac{dy}{dx}$ if $xy = \ln(xtany)$ (3 marks)

$$x\frac{dy}{dx} + y = \frac{1}{x\tan y}\left(x\sec^2 y\frac{dy}{dx} + \tan y\right)$$

$$x\frac{dy}{dx} + y = \frac{\sec^2 y}{\tan y}\frac{dy}{dx} + \frac{1}{x}$$

$$x\frac{dy}{dx} - \frac{\sec^2 y}{\tan y}\frac{dy}{dx} = \frac{1}{x} - y$$

$$\frac{dy}{dx}\left(x - \frac{\sec^2 y}{\tan y}\right) = \frac{1}{x} - y$$

$$\frac{dy}{dx} = \frac{\frac{1}{x} - y}{x - \frac{\sec^2 y}{\tan y}} \quad \frac{(x\tan y)}{(x\tan y)}$$

$$\frac{dy}{dx} = \frac{\tan y - xy\tan y}{x^2\tan y - x\sec^2 y}$$

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5. Find $f'(x)$ if $f(x) = x^4 4^x$ (3 marks)

$$f'(x) = x^4 4^x \ln 4 + 4^x (4x^3)$$

$$= \ln 4 \, x^4 4^x + 4x^3 4^x$$

6. Find $f'(x)$ if $f(x) = \frac{(x^2)(sinx)(e^x)}{\sqrt{x}}$ (3 marks)

$$y = \frac{(x^2)(\sin x)(e^x)}{x^{\frac{1}{2}}}$$

$$\ln y = \ln x^2 + \ln \sin x + \ln e^x - \frac{1}{2}\ln x$$

$$\ln y = 2\ln x + \ln \sin x + x\ln e - \frac{1}{2}\ln x$$

$$\ln y = 2\ln x + \ln \sin x + x - \frac{1}{2}\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{1}{\sin x}(\cos x) + 1 - \frac{1}{2}\left(\frac{1}{x}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \cot x + 1 - \frac{1}{2x}$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} + \cot x + 1 - \frac{1}{2x}\right)$$

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7. Find $f'(x)$ if $f(x) = e^{2x}tan^{-1}3x$ (3 marks)

$$f'(x) = e^{2x}\left(\frac{1}{1+(3x)^2}\right)(3) + \tan^{-1}3x \; e^{2x}(2)$$

$$= \frac{3e^{2x}}{1+9x^2} + 2e^{2x}\tan^{-1}3x$$

8. A rock is dropped into a pond and creates a circular ripple whose radius increases at a constant rate of 0.5m/s. How rapidly is the circular area of the ripple increasing at the end of 7s? (3 marks)

$\frac{dr}{dt} = 0.5$ IF $t = 7$, THEN $r = 0.5 \times 7 = 3.5m$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(3.5)(0.5) = 11 m^2/s$

9. Sand is falling into a conical pile so that the radius of the base of the pile is always equal to one half its altitude. If the sand is falling at the rate of 10 cubic feet per minute, how fast is the altitude of the pile increasing when the pile is 5 feet deep? (3 marks)

Volume of a cone = $\frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 10 \quad h = 5 \quad$ Find $\frac{dh}{dt}$

$V = \frac{1}{3}\pi r^2 h$

$r = \frac{1}{2}h$

NEED TO ELIMINATE r.

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{3}\left(\frac{1}{4}\right)h^3$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{3\pi}{12} h^2 \frac{dh}{dt}$

$10 = \frac{\pi}{4}(5)^2 \frac{dh}{dt}$

$10 = \frac{25\pi}{4}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{40}{25\pi}$ ft/min

$\frac{dh}{dt} \approx 0.51$ ft/min

10. Determine the following limits. (2 marks each)

a) $\lim\limits_{x\to 1} \frac{lnx}{x-1}$

$= \lim\limits_{x\to 1} \frac{\frac{1}{x}}{1}$

$= \frac{1}{1}$

$= 1$

b) $\lim\limits_{x\to\infty} \frac{e^{3x}}{x^2}$

$= \lim\limits_{x\to\infty} \frac{e^{3x}(3)}{2x}$

$= \lim\limits_{x\to\infty} \frac{3e^{3x}(3)}{2}$

$= \lim\limits_{x\to\infty} \frac{9e^{3x}}{2} = +\infty$

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