Name:______________________________ Date:___________________

Calculus 12 LG 5-6 Quiz Ver B /30

1. If $f(x) = (x-1)^2$
   a) Find the average rate of change from *x* = -1 to *x* = 2. (2 marks)
   b) Find the instantaneous rate of change of *y* with respect to *x* at the point *x* = 2. (2 marks)
   c) Sketch the graph of $y = f(x)$ together with the secant and tangent lines whose slopes are given by the results of a) and b). (2 marks)

a) $f(-1) = (-1-1)^2 = 4$

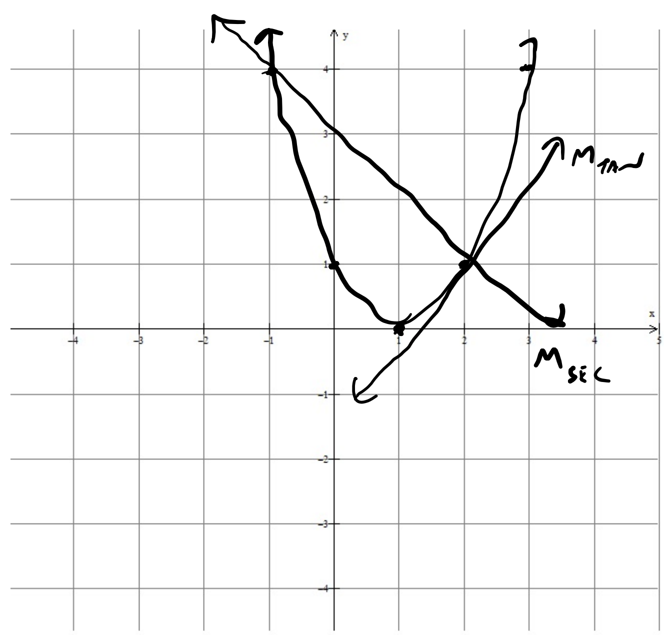
$f(2) = (2-1)^2 = 1$

AVG RATE OF CHANGE $= M_{SEC} = \dfrac{f(2) - f(-1)}{2-(-1)} = \dfrac{1-4}{2-(-1)} = \dfrac{-3}{3} = -1$

b) INSTANTANEOUS RATE OF CHANGE $= M_{TAN} = f'(x)$

$= 2(x-1) = 2x-2$

So $f'(2) = 2(2) - 2 = 2$



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2. Use the **definition of the derivative** to calculate $f'(x)$ if $f(x) = 3x^2 + 1$ and find the equation of the tangent line to the graph of *f* at *x* = -1.

(3 marks)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(3(x+h)^2+1) - (3x^2+1)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2)+1-3x^2-1}{h} = \lim_{h\to 0} \frac{3x^2+6xh+3h^2+1-3x^2-1}{h}$$

$$= \lim_{h\to 0} \frac{6xh+3h^2}{h} = \lim_{h\to 0} 6x+3h = 6x \quad \text{SO } f'(-1) = -6$$

AND $f(-1) = 3(-1)^2 + 1 = 4$

SO $y - 4 = -6(x+1)$

3. Show that $f(x) = \begin{cases} \sqrt{-2x+3}, & x \le 1 \\ 4x-3, & x > 1 \end{cases}$ is continuous but not differentiable at x = 1.

(3 marks)

$\lim_{x\to 1^-} f(x) = \sqrt{-2(1)+3} = 1$

$\lim_{x\to 1^+} f(x) = 4(1) - 3 = 1$

$f(1) = 1$

SINCE $f(1) = \lim_{x\to 1} f(x)$

$f(x)$ IS CONTINUOUS AT $x = 1$

$$f'(x) = \begin{cases} \frac{1}{2}(-2x+3)^{-\frac{1}{2}}(-2), & x \le 1 \\ 4, & x > 1 \end{cases}$$

OR $$f'(x) = \begin{cases} \frac{-1}{\sqrt{-2x+3}}, & x \le 1 \\ 4, & x > 1 \end{cases}$$

$\lim_{x\to 1^-} f'(x) = \frac{-1}{\sqrt{1}} = -1$

$\lim_{x\to 1^+} f'(x) = 4$

SINCE $\lim_{x\to 1} f'(x)$ DNE,

$f(x)$ IS NOT DIFFERENTIABLE AT $x = 1$

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4. Find $\frac{dy}{dx}$ if $y = (2x-5)(4-3x^2+2x)$ (2 marks)

$\frac{dy}{dx} = (2x-5)(-6x+2) + (4-3x^2+2x)(2)$

$= -12x^2 + 4x + 30x - 10 + 8 - 6x^2 + 4x$

$= -18x^2 + 38x - 2$

5. Find $f''(1)$ if $f(x) = \frac{x+2}{x}$ (2 marks)

$f'(x) = \frac{x(1) - (x+2)(1)}{x^2} = \frac{x - x - 2}{x^2} = \frac{-2}{x^2} = -2x^{-2}$

$f''(x) = 4x^{-3} = \frac{4}{x^3}$

$f''(1) = \frac{4}{(1)^3} = 4$

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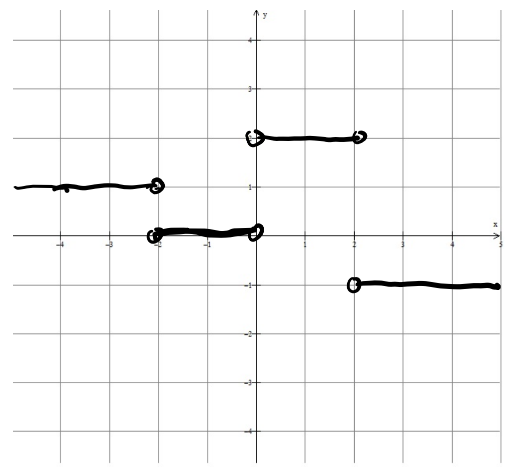
6. Sketch the graph of the derivative of the function whose graph is shown.

(2 marks)

$f(x)$

$f'(x)$



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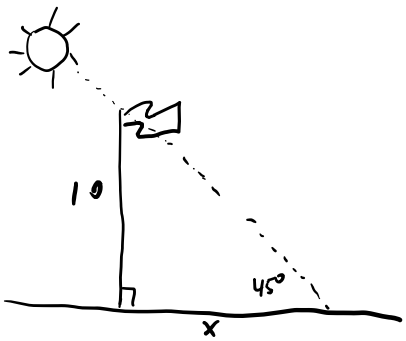
7. The sun is casting a shadow of a 10m tall 88flag pole on the ground. Find the rate at which the length x of the shadow is changing with respect to θ when θ is 45 degrees. (3 marks)



NEED TO FIND $\frac{dx}{d\theta}$

$\tan\theta = \frac{10}{x}$

$x = \frac{10}{\tan\theta}$ OR $10\cot\theta$

$\frac{dx}{d\theta} = 10(-\csc^2\theta)$. $\theta = 45°$ OR $\frac{\pi}{4}$ RADIANS

$\frac{dx}{d\theta} = 10(-\csc^2\frac{\pi}{4}) = 10(-(\sqrt{2})^2) = -20$ ft/RAD

$\frac{-20 \text{ ft}}{\text{RAD} \times \frac{180}{\pi}} = -20 \times \frac{\pi}{180} = \frac{-\pi}{9}$ ft/DEG OR -0.35 ft/DEG.

8. Find $f'(x)$ where $f(x) = (x^2 + 3)secx$ (2 marks)

$f'(x) = (x^2+3)\sec x\tan x + \sec x\,(2x)$

$= (x^2+3)\sec x\tan x + 2x\sec x$

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9. Find $\frac{dy}{dx}$ where $y = sin^2(2x^3 - x)$ (2 marks)

$$\frac{dy}{dx} = 2\sin(2x^3-x) \cdot \cos(2x^3-x) \cdot (6x^2-1)$$

$$= 2(6x^2-1)\sin(2x^3-x)\cos(2x^3-x)$$

10. Find the equation of the tangent line to the graph of $f(x) = \cos(x^2 - 9)$ at x = 3. (3 marks)

$$f'(x) = -\sin(x^2-9) \cdot (2x)$$

$$f'(3) = -\sin(0) \cdot (2(3))$$

$$= 0$$

$$f(3) = \cos(3^2-9) = \cos 0 = 1$$

So $y - y_1 = m(x-x_1)$

$$y - 1 = 0(x-3)$$

$$y = 1$$

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11. Use a local linear approximation to estimate the value of 2.02^3.
(2 marks)

$y = x^3$

SO FIND EQN OF TAN LINE AT $x = 2$

$\frac{dy}{dx} = 3x^2$ SO $\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 = 12$

IF $x = 2$, $y = 2^3 = 8$

SO EQN IS $y - y_1 = m(x - x_1)$

$y - 8 = 12(x - 2)$

$y - 8 = 12(2.02 - 2)$

$y - 8 = 12(.02)$

$y - 8 = .24$

$y = 8.24$

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