Name:______________________________ Date:___________________

Calculus 12 LG 5-6 Quiz Ver A /30

1. If $f(x) = x^2 + 1$
a) Find the average rate of change from *x* = -2 to *x* = -1. (2 marks)
b) Find the instantaneous rate of change of *y* with respect to *x* at the point *x* = -2. (2 marks)
c) Sketch the graph of $y = f(x)$ together with the secant and tangent lines whose slopes are given by the results of a) and b). (2 marks)

a) $f(-2) = (-2)^2 + 1 = 5$

$f(-1) = (-1)^2 + 1 = 2$

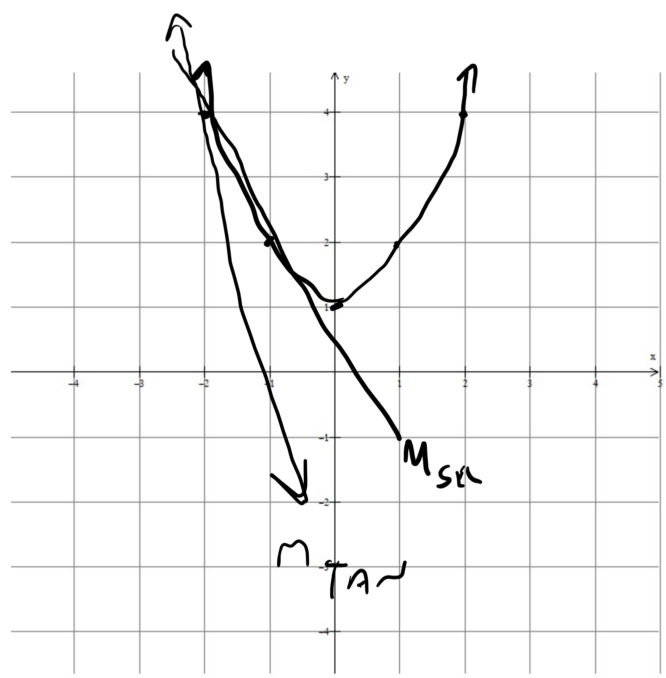
AVG SLOPE $= m_{SEC} = \dfrac{f(-2) - f(-1)}{-2 - (-1)} = \dfrac{5-2}{-2-(-1)} = \dfrac{3}{-1} = -3$

b) INSTANTANEOUS SLOPE

$= f'(-2)$

$f'(x) = 2x$

$f'(-2) = -4$



/6

2. Use the **definition of the derivative** to calculate $f'(x)$ if $f(x) = 3x^2 - x$ and find the equation of the tangent line to the graph of *f* at *x* = 1.
(3 marks)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(3(x+h)^2 - (x+h)) - (3x^2 - x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2 + 2xh + h^2) - x - h - 3x^2 + x}{h} = \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2 - h}{h} = \lim_{h\to 0} 6x + 3h - 1 = 6x - 1$$

so $f'(1) = 6(1) - 1 = 5$

$f(1) = 3(1)^2 - 1 = 2$

SO EQ'N OF TAN LINE:

$$y - 2 = 5(x - 1)$$

3. Show that $f(x) = \begin{cases} x^2 - 5, & x \le 1 \\ x - 5, & x > 1 \end{cases}$ is continuous but not differentiable at x = 1.
(2 marks)

$\lim_{x\to 1^-} f(x) = 1^2 - 5 = -4$

$\lim_{x\to 1^+} f(x) = 1 - 5 = -4$

$f(1) = -4$

SINCE $f(1) = \lim_{x\to 1} f(x)$

$f(x)$ IS CONTINUOUS AT $x = 1$

$$f'(x) = \begin{cases} 2x & x \le 1 \\ 1, & x > 1 \end{cases}$$

$\lim_{x\to 1^-} f'(x) = 2$

$\lim_{x\to 1^+} f'(x) = 1$

SINCE $\lim_{x\to 1^-} f'(x) \neq \lim_{x\to 1^+} f'(x)$

$f(x)$ IS NOT DIFFERENTIABLE AT $x = 1$

/5

4. Find $\frac{dy}{dx}$ if $y = \frac{3x^3+5x}{2x-7}$ (2 marks)

$$\frac{dy}{dx} = \frac{(2x-7)(9x^2+5) - (3x^3+5x)(2)}{(2x-7)^2}$$

$$= \frac{18x^3+10x-63x^2-35-6x^3-10x}{(2x-7)^2}$$

$$= \frac{12x^3-63x^2-35}{(2x-7)^2}$$

5. Find $f''(2)$ if $f(x) = \frac{-8}{x^2} + \frac{1}{5}x^5$ (2 marks)

$$f(x) = -8x^{-2} + \frac{1}{5}x^5$$

$$f'(x) = 16x^{-3} + x^4$$

$$f''(x) = -48x^{-4} + 4x^3$$

$$f''(2) = \frac{-48}{2^4} + 4(2)^3$$

$$= -3 + 32$$

$$= 29$$

/4

6. Show that the parabola $y = x^2$ and the line $x + 2y - 3 = 0$ intersect at right angles at one of their points of intersection. (3 marks)
HINT: Find the points of intersection and find their slopes.

FIND POINTS OF INTERSECTION:

IF $y = x^2$ THEN $x + 2(x^2) - 3 = 0$

$$2x^2 + x - 3 = 0$$

$$(2x + 3)(x - 1) = 0$$

$$x = -\frac{3}{2}, 1$$

FIND SLOPE OF LINE:

$$x + 2y - 3 = 0$$

$$2y = -x + 3$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

$$m = -\frac{1}{2}$$

FIND SLOPES OF PARABOLA:

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

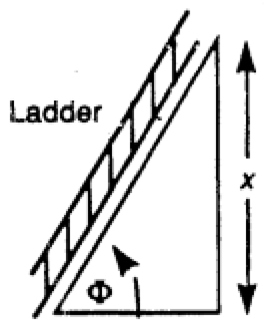
AT $x = -\frac{3}{2}$, $\frac{dy}{dx} = 2\left(-\frac{3}{2}\right) = -3$

AT $x = 1$, $\frac{dy}{dx} = 2(1) = 2$

SINCE THE SLOPES OF THE LINE AND PARABOLA ARE NEGATIVE RECIPROCALS AT $x = 1$, THEY INTERSECT AT RIGHT ANGLES.

/3

7. A 12 foot ladder leans up against a wall at an angle θ with the horizontal as shown in the figure. The top of the ladder is x feet above the ground. If the bottom of the ladder is pushed towards the wall, find the rate at which x changes with respect to θ when $\theta = 60°$. Express the answer in units of feet/degree. (3 marks)



$$\sin\theta = \frac{x}{12}$$

$$x = 12 \sin\theta$$

$$\frac{dx}{d\theta} = 12\cos\theta$$

$\theta = 60°$ OR $\frac{\pi}{3}$ RADIANS

So $\frac{dx}{d\theta} = 12\cos\frac{\pi}{3} = 12\left(\frac{1}{2}\right) = 6$ ft/RAD

$$\frac{6\text{ ft}}{\text{RAD}} = \frac{6\text{ ft}}{1 \times \frac{180}{\pi}} = \frac{6 \times \pi}{180} \text{ ft/DEG} = 0.10 \text{ ft/DEG}$$

8. Find $f'(x)$ where $f(x) = x^2(\sin 2x)^3$ (2 marks)

$$f'(x) = x^2\, 3(\sin(2x))^2(\cos(2x))(2) + (\sin 2x)^3(2x)$$

$$= 6x^2 \sin^2(2x)\cos(2x) + 2x\sin^3(2x)$$

/5

9. Find $\frac{dy}{dx}$ where $y = \left(\frac{x+1}{x-1}\right)^2$ (2 marks)

$$\frac{dy}{dx} = 2\left(\frac{x+1}{x-1}\right)\left[\frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}\right]$$

$$= \frac{2(x+1)(x-1-x-1)}{(x-1)^3}$$

$$= \frac{-4(x+1)}{(x-1)^3}$$

10. Find the equation of the tangent line to the graph of $f(x) = \sin(4 - x^2)$ at x = 2. (3 marks)

$$f'(x) = \cos(4-x^2)(-2x)$$

$$f'(2) = \cos(0)\ (-2(2))$$

$$= -4$$

$$f(2) = \sin(4-2^2)$$

$$= 0$$

so $y - y_1 = m(x - x_1)$

$$y - 0 = -4(x-2)$$

$$y = -4x + 8$$

/5

11. Use a local linear approximation to estimate the value of $\sqrt[4]{14}$.
(2 marks)

$y = \sqrt[4]{x}$ OR $x^{\frac{1}{4}}$

FIND EQ'N OF TANGENT LINE AT $x = 16$

$\frac{dy}{dx} = \frac{1}{4} x^{-3/4}$

$\left.\frac{dy}{dx}\right|_{x=16} = \frac{1}{4}(16)^{-3/4} = \frac{1}{4(8)} = \frac{1}{32}$

IF $x = 16$, THEN $y = \sqrt[4]{16} = 2$

SO EQN OF TAN LINE IS $y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{32}(x - 16)$

NOW AT $x = 14$, $y - 2 = \frac{1}{32}(14 - 16)$

$y - 2 = \frac{1}{32}(-2)$

$y - 2 = -\frac{1}{16}$

$y = 2 - \frac{1}{16}$

$y = \frac{32}{16} - \frac{1}{16}$

$y = \frac{31}{16}$ OR 1.9375

/2