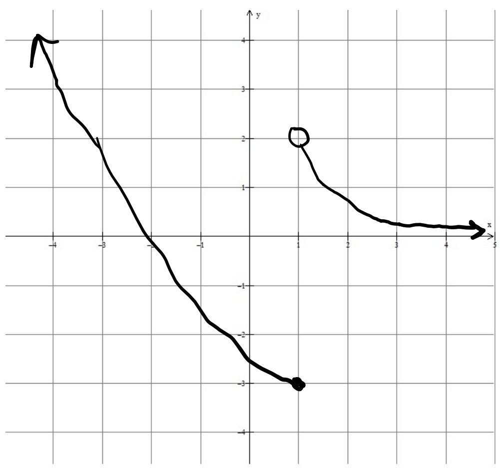
Name:______________________________ Date:__________________

Calculus 12 LG 2-3 Quiz Ver B /26

1. Consider the following function $y = f(x)$. Find: (1/2 mark each)



a) $\lim_{x \to 1^-} f(x)$

-3

b) $\lim_{x \to 1^+} f(x)$

2

c) $\lim_{x \to 1} f(x)$

D.N.E.

d) f(1)

-3

e) $\lim_{x \to -\infty} f(x)$

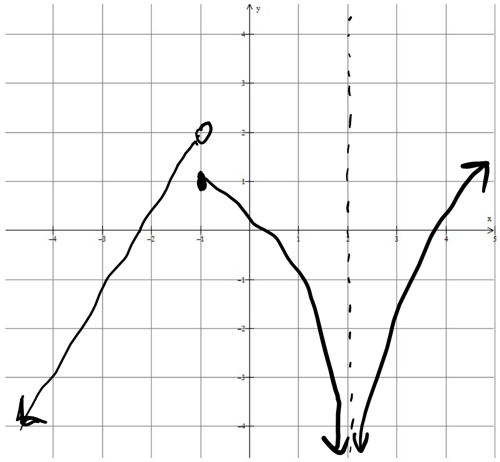
$+\infty$

f) $\lim_{x \to +\infty} f(x)$

0

/3

2. For what values of x does the limit exist in the following function? (2 marks)



THE LIMIT EXISTS FOR ALL VALUES EXCEPT $X = -1, 2$.

3. Determine the following limits. (2 marks each)

a) $\lim_{x \to 5} \frac{2x-3}{x^2-1}$

$$= \frac{2(5)-3}{5^2-1}$$

$$= \frac{7}{24}$$

b) $\lim_{x \to +\infty} \frac{4x^2-7}{6x-12x^3}$

$$= \lim_{x \to \infty} \frac{4x^2}{-12x^3}$$

$$= \lim_{x \to \infty} \frac{4}{-12x}$$

$$= 0$$

c) $\lim_{x \to 4^+} \frac{3-x}{x^2-2x-8}$

$$= \lim_{x \to 4^+} \frac{3-x}{(x-4)(x+2)}$$

$$= \frac{-1}{(\text{SMALL} +)(6)}$$

$$= -\infty$$

d) $\lim_{x \to -\infty} \frac{3-2x}{\sqrt{4x^2+1}}$

$$= \lim_{x \to -\infty} \frac{-2x}{\sqrt{4x^2}}$$

$$= \lim_{x \to -\infty} \frac{-2x}{2|x|}$$

$$= 1$$

/10

4. Determine the following limits. (2 marks each)

a) $\lim\limits_{x\to 0^+} \frac{sinx}{3x}$

$= \lim\limits_{x\to 0^+} \frac{1}{3}\left(\frac{\sin x}{x}\right)$

$= \frac{1}{3}$

b) $\lim\limits_{x\to 0} \frac{sin4x}{tan6x}$

$= \lim\limits_{x\to 0} \dfrac{\sin 4x}{\frac{\sin 6x}{\cos 6x}}$

$= \lim\limits_{x\to 0} \frac{\sin 4x}{1} \cdot \frac{1}{\sin 6x} \cdot \frac{1}{\cos 6x}$

$= \lim\limits_{x\to 0} \frac{\sin 4x}{4x} \cdot \frac{6x}{\sin 6x} \cdot \frac{1}{\cos 6x} \cdot \frac{4x}{6x} = \frac{4}{6} = \frac{2}{3}$

5. Find the points of discontinuity (if any). (2 marks each)

a) $f(x) = \frac{2x-1}{x^2-x-12}$

$x^2 - x - 12 \neq 0$

$(x-4)(x+3) \neq 0$

$x = 4, -3$

THE POINTS OF DISCONTINUITY OCCUR AT $x = -3, 4$

b) $f(x) = \begin{cases} 2x^2 - 3, & x < -2 \\ 3 - x, & x \geq -2 \end{cases}$

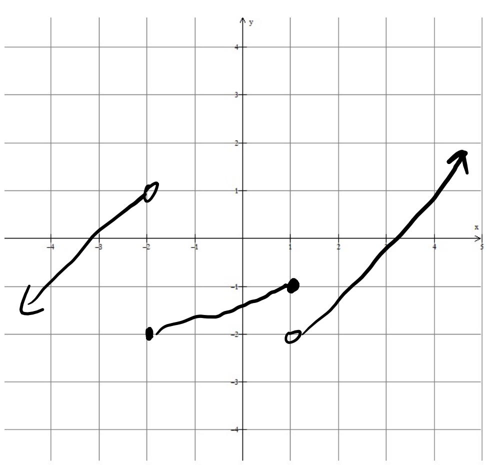
$\lim\limits_{x\to -2^-} f(x) = 5$

$\lim\limits_{x\to -2^+} f(x) = 5$

SINCE $\lim\limits_{x\to -2} f(x) = f(-2)$ AND SINCE THE 2 FUNCTIONS ARE POLYNOMIALS, $f(x)$ IS CONTINUOUS EVERYWHERE.

6. Sketch the graph of a function that is continuous everywhere but has a non-removable discontinuity at x=-2 and is also not continuous at x=1 but it continuous from the left.

(2 marks)



HERE IS ONE EXAMPLE.

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7. Prove that the function $f(x) = \frac{3x-4}{2x^2-9x-5}$ is not continuous at x = 5. Is the discontinuity removable? (3 marks)

$$f(x) = \frac{3x-4}{(2x+1)(x-5)}$$

$$\lim_{x \to 5^-} f(x) = \frac{11}{(11)(\text{small } -)} = -\infty$$

$$\lim_{x \to 5^+} f(x) = \frac{11}{(11)(\text{small } +)} = +\infty$$

$f(5)$ DNE

$f(x)$ IS NOT CONTINUOUS AT $x = 5$ SINCE $f(5)$ DNE.

SINCE $\lim_{x \to 5} f(x)$ DNE, THE DISCONTINUITY IS NON-REMOVABLE.

/3