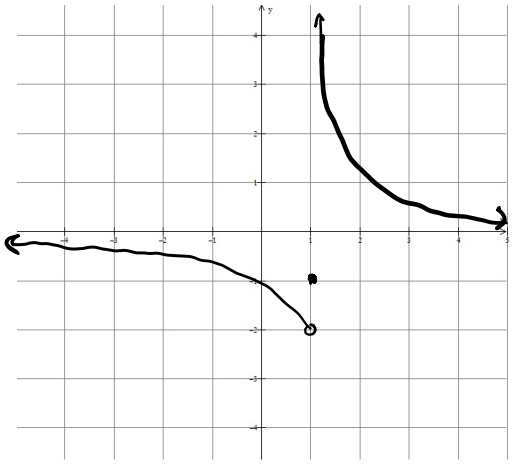
Name:_____ Date:_____

Calculus 12 LG 2-3 Quiz Ver A /26

1. Consider the following function $y = f(x)$. Find: (1/2 mark each)



a) $\lim_{x \to 1^-} f(x)$

-2

b) $\lim_{x \to 1^+} f(x)$

$+\infty$

c) $\lim_{x \to 1} f(x)$

DOES NOT EXIST

d) f(1)

-1

e) $\lim_{x \to -\infty} f(x)$

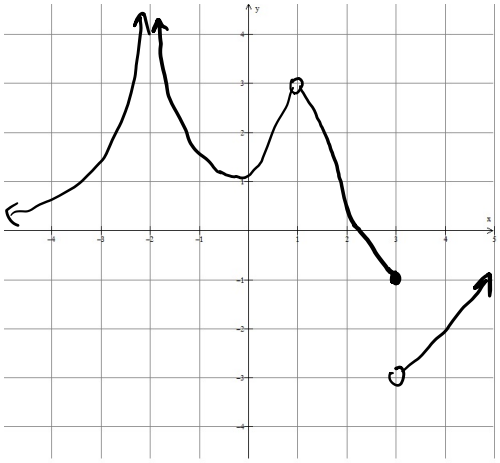
0

f) $\lim_{x \to +\infty} f(x)$

0

/3

2. For what values of x does the limit exist in the following function? (2 marks)



THE LIMIT EXISTS FOR ALL VALUES EXCEPT $x = -2$ AND $x = 3$

3. Determine the following limits. (2 marks each)

a) $\lim_{x\to 5} \frac{x-5}{x^2-25}$

$= \lim_{x\to 5} \frac{x-5}{(x+5)(x-5)}$

$= \lim_{x\to 5} \frac{1}{x+5}$

$= \frac{1}{10}$

b) $\lim_{x\to +\infty} \frac{5x-4x^3}{2x^3+4x-3}$

$= \lim_{x\to \infty} \frac{-4x^3}{2x^3}$

$= \frac{-4}{2}$

$= -2$

c) $\lim_{x\to 2^-} \frac{x}{x^2+3x-10}$

$= \lim_{x\to 2^-} \frac{x}{(x+5)(x-2)}$

$= \frac{2}{(7)(\text{SMALL}-)}$

$= \frac{2}{\text{SMALL}-}$

$= -\infty$

d) $\lim_{x\to -\infty} \frac{7x-4}{\sqrt{2+3x^2}}$

$= \lim_{x\to -\infty} \frac{7x}{\sqrt{3x^2}}$

$= \lim_{x\to -\infty} \frac{7x}{\sqrt{3}\,|x|}$

$= \frac{-7}{\sqrt{3}}$

/10

4. Determine the following limits. (2 marks each)

a) $\lim_{x\to 0} \frac{2sin3x}{x}$

$= \lim_{x\to 0} \frac{2\sin 3x}{x}\ (3)\ (1)$

$= \lim_{x\to 0} \frac{6\sin 3x}{3x}$

$= 6$

b) $\lim_{x\to 0} \frac{sin2x}{sin6x}$

$= \lim_{x\to 0} \frac{\sin 2x}{1} \cdot \frac{1}{\sin 6x}$

$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{6x}{\sin 6x} \cdot \frac{2x}{6x}$

$= (1)(1)\left(\frac{2}{6}\right)$

$= \frac{1}{3}$

5. Find the points of discontinuity. (2 marks each)

a) $f(x) = \frac{2}{x-3} + \frac{4}{x}$

$x - 3 \neq 0$

$\text{so } x \neq 3$

AND $x \neq 0$

POINTS OF DISCONTINUITY AT $x = 0$ AND 3

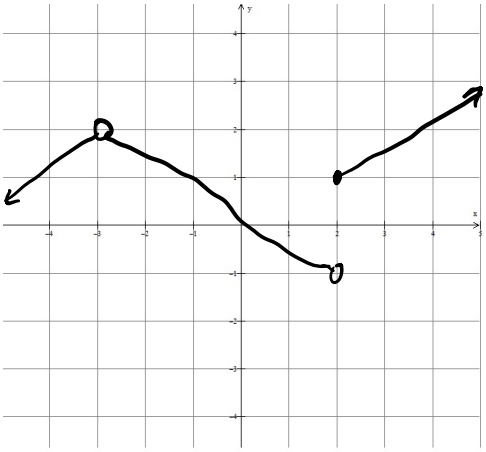
b) $f(x) = \begin{cases} 5x - 3, & x < 3 \\ 6 - x, & x \geq 3 \end{cases}$

$\lim_{x\to 3^-} f(x) = 5(3) - 3 = 12$

$\lim_{x\to 3^+} f(x) = 6 - 3 = 3$

SINCE $\lim_{x\to 3} f(x)$ DOES NOT EXIST, $f(x)$ HAS A POINT OF DISCONTINUITY AT $x = 3$

6. Sketch the graph of a function that is continuous everywhere but has a removable discontinuity at x=-3 and is also not continuous at x=2 but it continuous from the right. (2 marks)



LOTS OF OPTIONS

BUT HERE IS ONE GRAPH

/10

7. Prove that the function $f(x) = \frac{x-2}{2x^2-3x-2}$ is not continuous at x = 2. Is the discontinuity removable? (3 marks)

$$f(x) = \frac{x-2}{(x-2)(2x+1)} = \frac{1}{2x+1}, \quad x \neq 2$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{1}{2x+1} = \frac{1}{5}$$

$f(2)$ DNE.

$f(x)$ IS NOT CONTINUOUS AT $x = 2$ SINCE $f(2)$ DNE.

BUT SINCE $\lim_{x \to 2} f(x)$ EXISTS, THE DISCONTINUITY IS REMOVABLE.

/3