Calculus 12 **LG 17/18 Test Version B** **/25**

Name:________________________________ Date:___________

1. Find the area of the region between the curves $y = x$ and $y = -2x + 8$ on the interval [0, 2]. (3 marks)

$$\int_0^2 (-2x+8) - x \, dx$$

$$= \int_0^2 -3x + 8 \, dx$$

$$= -\frac{3x^2}{2} + 8x \Big]_0^2$$

$$= \frac{-12}{2} + 16 = 10$$

2. Find the area enclosed by the curves $y^2 = 2x + 6$ and $y = x - 1$. (3 marks)

FIND INTERSECTION POINT:

$2x+6 = (x-1)^2$

$2x+6 = x^2 - 2x + 1$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = -1, 5$

$(-1, -2)$, $(5, 4)$

$\frac{y^2}{2} - 3 = x \quad x = y+1$

$$\int_{-2}^{4} (y+1) - \left(\frac{y^2}{2} - 3\right) dy$$

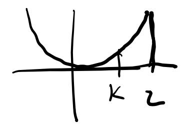
$$= \int_{-2}^{4} -\frac{y^2}{2} + y + 4 \, dy$$

$$= \frac{-y^3}{6} + \frac{y^2}{2} + 4y \Big]_{-2}^{4}$$

$$= \left(-\frac{4^3}{6} + \frac{4^2}{2} + 4(4)\right) - \left(-\frac{(-2)^3}{6} + \frac{(-2)^2}{2} + 4(-2)\right)$$ /6

$$= 18$$

3. Find a vertical line $x = k$ that divides the area between $x = \sqrt{y}$ and $x = 2$ and $y = 0$ into two equal parts. (3 marks)



FIND POINT OF INTERSECTION

$\sqrt{y} = 2$

$y = 4$

$x = 2$

$(2, 4)$

$\int_0^2 x^2 \, dx$

$= \frac{x^3}{3} = \frac{8}{3}$

HALF OF $\frac{8}{3}$ IS $\frac{8}{6}$ OR $\frac{4}{3}$

So $\int_0^k x^2 \, dx = \frac{4}{3}$

$\left.\frac{x^3}{3}\right]_0^k = \frac{4}{3}$

$\frac{k^3}{\cancel{3}} = \frac{4}{\cancel{3}}$ $\quad k^3 = 4$ $\quad k = \sqrt[3]{4}$

4. The base of a certain solid is the region enclosed by $y = x$ and $y = x^2$. Every cross section perpendicular to the x-axis is a semi-circle. Determine the volume. (3 marks)

FIND INTERSECTION POINTS:

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

DIAMETER OF SEMICIRCLE

$= x - x^2$

So RADIUS $= \frac{x - x^2}{2}$

AREA OF SEMICIRCLE $= \frac{\pi \left(\frac{x - x^2}{2}\right)^2}{2}$

So $\int_0^1 \frac{\pi \left(\frac{x - x^2}{2}\right)^2}{2} \, dx$

$\int_0^1 \frac{\pi \frac{(x - x^2)^2}{4}}{2} \, dx$

$= \int_0^1 \frac{\pi}{8} (x - x^2)(x - x^2) \, dx$

$= \int_0^1 \frac{\pi}{8} (x^2 - 2x^3 + x^4) \, dx$

$= \frac{\pi}{8} \left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \Big]_0^1$

$= \frac{\pi}{8} \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right)$

$= \frac{\pi}{8} \left(\frac{1}{30} \right)$

$= \frac{\pi}{240}$

/6

5. Find the volume of the solid that results when the region $y = x^2$ and $y = x + 2$ is revolved about the x-axis. (3 marks)

Intersection points:

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

$$\int_{-1}^{2} \pi \left[(x+2)^2 - (x^2)^2 \right] dx$$

$$\int_{-1}^{2} \pi \left(x^2 + 4x + 4 - x^4 \right)$$

$$= \pi \left(\frac{x^3}{3} + \frac{4x^2}{2} + 4x - \frac{x^5}{5} \right) \Big]_{-1}^{2}$$

$$= \pi \left[\left(\frac{8}{3} + \frac{16}{2} + 8 - \frac{32}{5} \right) - \left(\frac{-1}{3} + 2 - 4 + \frac{1}{5} \right) \right]$$

$$= \frac{72}{5}\pi$$

6. Solve the differential equation: $\frac{dy}{dx} = 4x + \sin x$ (3 marks)

$$\int dy = \int (4x + \sin x)\, dx$$

$$y = 2x^2 - \cos x + C$$

/6

7. Solve the initial value problem. $\frac{dy}{dx} = 4x^2y^2$, $y(1) = -1$ (3 marks)

$$\frac{dy}{dx} = 4x^2y^2$$

$$\int \frac{1}{y^2}\,dy = \int 4x^2\,dx$$

$$\frac{y^{-1}}{-1} = \frac{4x^3}{3} + C$$

$$\frac{1}{y} = \frac{-4x^3}{3} + C$$

$$\frac{1}{-1} = \frac{-4}{3} + C$$

$$\frac{1}{3} = C$$

$$\frac{1}{y} = -\frac{4}{3}x^3 + \frac{1}{3}$$

$$\frac{1}{y} = \frac{-4x^3 + 1}{3}$$

$$y = \frac{3}{1 - 4x^3}$$

/3

8. The cost of producing x tennis rackets is given by $\frac{dC}{dx} = \frac{300}{\sqrt{x}}$ per racket. Find the cost C of manufacturing 500 tennis rackets if C = \$5200 when x =100. (4 marks)

$$\frac{dC}{dx} = \frac{300}{x^{\frac{1}{2}}}$$

$$\int dC = \int 300x^{-\frac{1}{2}}\,dx$$

K IS A CONSTANT.

$$C = \frac{300x^{\frac{1}{2}}}{\frac{1}{2}} + K$$

$$C = 600x^{\frac{1}{2}} + K$$

$$5200 = 600(100)^{\frac{1}{2}} + K$$

$$5200 = 6000 + K$$

$$5200 - 6000 = K$$

$$-800 = K$$

$$\text{So } C = 600\sqrt{x} - 800$$

$$C = 600\sqrt{500} - 800$$

$$= \$12616.41$$

TO PRODUCE 500 TENNIS RACKETS WILL COST \$12616.41

/4