Calculus 12 **LG 17/18 Test Version A** **/25**

Name:________________________________ Date:___________

1. Find the area of the region between the curves $y = x^2$ and $y = x + 6$ on the interval [0, 2]. (3 marks)

$$\int_0^2 (x+6) - x^2 \, dx$$

$$= \frac{x^2}{2} + 6x - \frac{x^3}{3} \Big]_0^2$$

$$= 2 + 12 - \frac{8}{3}$$

$$= 14 - \frac{8}{3} = \frac{34}{3}$$

2. Find the area enclosed by the curves $x^2 = y$ and $x = y - 2$. (3 marks)

FIND WHERE CURVES INTERSECT

$y = x^2$

$y = x + 2$

$x = x^2 - 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$(2, 4), (-1, 1)$

$$\int_{-1}^{2} (x+2) - x^2 \, dx$$

$$= \int_{-1}^{2} \frac{x^2}{2} + 2x - \frac{x^3}{3} \, dx$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = \frac{9}{2}$$

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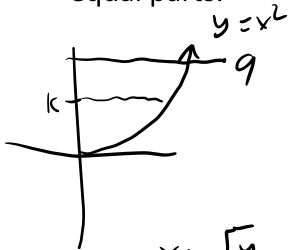
3. Find a horizontal line $y = k$ that divides the area between $y = x^2$ and $y = 9$ into two equal parts. (3 marks)



$x = \sqrt{y}$

$\int_0^9 \sqrt{y}\, dy$

$= \frac{2y^{\frac{3}{2}}}{3}\Big]_0^9 = 18$

HALF OF 18 IS 9

So $\int_0^k \sqrt{y}\, dy = 9$

$= \frac{2}{3} y^{3/2}\Big]_0^k = 9$

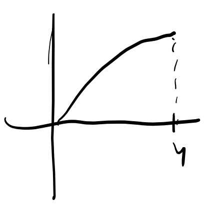
$\frac{2k^{\frac{3}{2}}}{3} = 9$

$k^{3/2} = \frac{27}{2}$

$k = \left(\sqrt[3]{\frac{27}{2}}\right)^2$

≈ 5.67

4. The base of a certain solid is the region enclosed by $y = \sqrt{x}, y = 0$, and $x = 4$. Every cross section perpendicular to the x-axis is a square. Determine the volume. (3 marks)



$\int_0^4 (\sqrt{x})^2\, dx$

$= \int_0^4 x\, dx$

$= \frac{x^2}{2}\Big]_0^4 = 8$

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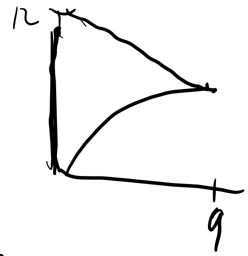
5. Find the volume of the solid that results when the region $y = \sqrt{x},\ y = 12 - x$, and $x = 0$ is revolved about the x-axis. (3 marks)

FIND POINT OF INTERSECTION



$12 - x = \sqrt{x}$

$144 - 24x + x^2 = x$

$x^2 - 25x + 144 = 0$

$(x-9)(x-16) = 0$

$x = 9 \quad x = 16$ ← REJECT

$(9, 3)$

$$\int_0^9 \pi\left[(12-x)^2 - (\sqrt{x})^2\right] dx$$

$$= \int_0^9 \pi\left(144 - 24x + x^2 - x\right) dx$$

$$= \int_0^9 \pi\left(144 - 25x + x^2\right) dx$$

$$= \pi\left(144x - \frac{25x^2}{2} + \frac{x^3}{3}\right)\Big|_0^9$$

$$= \frac{1053}{2}\pi \approx 1654$$

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6. Solve the differential equation: $\frac{dy}{dx} = \cos 2x$ (3 marks)

$$dy = \cos 2x \, dx$$

$$\int dy = \int \cos 2x \, dx$$

$$y = \frac{\sin 2x}{2} + C$$

7. Solve the initial value problem. $\frac{dy}{dx} = y(1 - x^2), \;\; y(0) = 2$ (3 marks)

$$\frac{dy}{dx} = y(1-x^2)$$

$$\int \frac{1}{y} dy = \int 1 - x^2 \, dx$$

$$\ln y = x - \frac{x^3}{3} + C$$

$$y = Ce^{x - \frac{x^3}{3}}$$

$$2 = Ce^{0 - \frac{0^3}{3}}$$

$$2 = C$$

So $y = 2e^{x - \frac{x^3}{3}}$

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8. In the year 2000, the population of a town was 12,000 people. Since then, the population of the town has been growing by 2% per year. Let $y = y(t)$ be the population of the town t years later.

a) Find a formula for $y(t)$. (2 marks)

$$\frac{dy}{dt} = .02y$$

$$\int \frac{1}{y}\,dy = \int .02\,dt$$

$$\ln y = .02t + c$$

$$y = e^{.02t+c}$$

$$y = Ce^{.02t}$$

$$y(0) = 12000$$

so $12000 = Ce^{.02(0)}$

$$12000 = C$$

$$y(t) = 12000e^{.02t}$$

b) When will the population of the town reach 20,000 people? (2 marks)

$$20000 = 12000e^{.02t}$$

$$\frac{5}{3} = e^{.02t}$$

$$\ln\frac{5}{3} = .02t$$

$$t = \frac{\ln\left(\frac{5}{3}\right)}{.02} \approx 25.54\text{ yrs}$$

So in 2026.

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