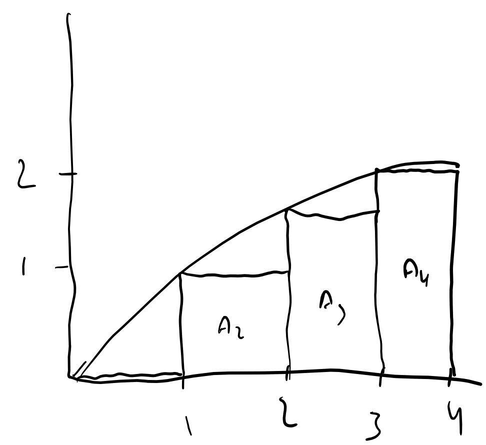
Calculus 12 LG 15/16 Test Version B /30

Name:________________________________ Date:___________

1. Use a left Reiman sum with 4 sub-intervals to approximate the area under the curve $y = \sqrt{x}$ on the interval [0, 4]. Is your approximation overestimating or underestimating the true area? (4 marks)



$A_1 = 0$

$A_2 = (1)(1) = 1$

$A_3 = (1)(\sqrt{2}) = \sqrt{2}$

$A_4 = (1)(\sqrt{3}) = \sqrt{3}$

AREA $= 1 + \sqrt{2} + \sqrt{3}$ OR ABOUT 4.15

THIS WOULD BE AN UNDERESTIMATION OF THE TRUE AREA.

2. Evaluate $\int x^2 - 3x + 1\, dx$ (3 marks)

$$= \frac{x^3}{3} - \frac{3x^2}{2} + x + C$$

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3. Evaluate $\int \frac{5}{x} - 2e^x \, dx$. (3 marks)

$$= 5\ln x - 2e^x + C$$

4. Evaluate $\int \frac{7x}{\sqrt{2x^2+1}} \, dx$ (3 marks)

Let $u = 2x^2+1$

$$\frac{du}{dx} = 4x$$

$$dx = \frac{du}{4x}$$

so $\int \frac{7x}{\sqrt{u}} \cdot \frac{du}{4x}$

$$= \int \frac{7}{4} \cdot \frac{1}{\sqrt{u}} \, du$$

$$= \frac{7}{4} \int u^{-\frac{1}{2}} \, du$$

$$= \frac{7}{4} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{14}{4} \left(2x^2+1\right)^{\frac{1}{2}} + C$$

$$= \frac{7}{2} \sqrt{2x^2+1} + C$$

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5. Evaluate $\int_0^4 x - \frac{1}{\sqrt{x}}\ dx$ (4 marks)

$$= \int_0^4 x - x^{-\frac{1}{2}}\, dx$$

$$= \frac{x^2}{2} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \Big]_0^4$$

$$= \frac{x^2}{2} - 2\sqrt{x} \Big]_0^4$$

$$= \left(\frac{4^2}{2} - 2\sqrt{4}\right) - \left(\frac{0^2}{2} - 2\sqrt{0}\right)$$

$$= 8 - 4$$

$$= 4$$

6. Evaluate $\int_{-3}^{2} 1 + |x|\, dx$ (4 marks)

$$= \int_{-3}^{0} 1 - x\, dx + \int_0^2 1 + x\, dx$$

$$= x - \frac{x^2}{2} \Big]_{-3}^{0} + x + \frac{x^2}{2} \Big]_0^2$$

$$= \left[\left(0 - \frac{0^2}{2}\right) - \left(-3 - \frac{(-3)^2}{2}\right)\right] + \left[\left(2 + \frac{2^2}{2}\right) - \left(0 + \frac{0^2}{2}\right)\right]$$

$$= -\left(-3 - \frac{9}{2}\right) + (2 + 2)$$

$$= \frac{15}{2} + 4$$

$$= \frac{15}{2} + \frac{8}{2}$$

$$= \frac{23}{2} \text{ or } 11.5$$

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7. Evaluate $\frac{d}{dx}\int_{-2}^{x} \sin t^3 \ dt$ (2 marks)

$= \sin x^3$

8. A particle moves along the x axis with an acceleration given by $a(t) = 2t + 1$. If the initial velocity was 0 at time t=0, what is the displacement and total distance travelled on the interval $0 \leq t \leq 4$. (4 marks)

$a(t) = 2t+1$

$v(t) = t^2 + t + C$

SINCE $v(0) = 0$, $C = 0$

SO $v(t) = t^2 + t$

$s(t) = \frac{t^3}{3} + \frac{t^2}{2} + C$

DISPLACEMENT $= s(4) - s(0)$

$= \left(\frac{4^3}{3} + \frac{4^2}{2}\right) - \left(\frac{0^3}{3} + \frac{0^2}{2}\right)$

$= \frac{64}{3} + 8$

$= \frac{88}{3}$

TOTAL DISTANCE

$v(t) = 0$ WHEN $t(t+1) = 0$

$t = 0$ AND -1

SINCE THERE ARE NO TIMES WHEN IT IS STOPPED ON $[0,4]$ THE TOTAL DISTANCE TRAVELLED WILL BE EQUAL TO DISPLACEMENT.

TOTAL DISTANCE $= \frac{88}{3}$

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9. Find the average value of the function $y = \sin x$ on the interval $[0, \frac{\pi}{4}]$. (3 marks)

$$\text{AVG VALUE} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\frac{\pi}{4}-0}\int_0^{\pi/4} \sin x\,dx$$

$$= \frac{4}{\pi}\left(-\cos x\right]_0^{\pi/4}$$

$$= \frac{4}{\pi}\left(-\cos\frac{\pi}{4} + \cos 0\right)$$

$$= \frac{4}{\pi}\left(-\frac{1}{\sqrt{2}} + 1\right)$$

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