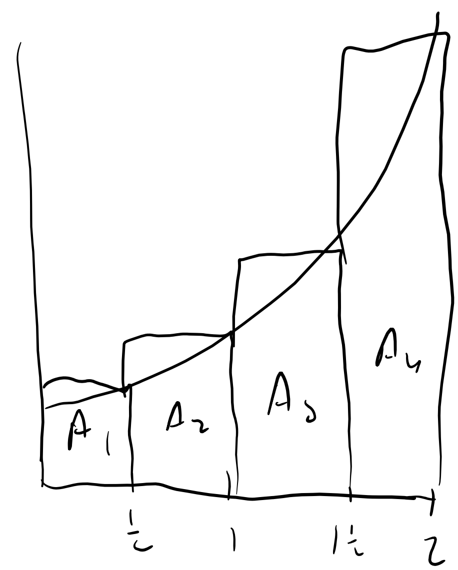
Calculus 12 **LG 15/16 Test Version A** **/30**

Name:_________________________________ Date:___________

1. Use a right Reiman sum with 4 sub-intervals to approximate the area under the curve $y = x^2 + 1$ on the interval [0, 2]. Is your approximation overestimating or underestimating the true area? (4 marks)



$A_1 = \left(\frac{1}{2}\right)\left(\left(\frac{1}{2}\right)^2 + 1\right) = \frac{5}{8}$

$A_2 = \left(\frac{1}{2}\right)\left(1^2 + 1\right) = 1$

$A_3 = \left(\frac{1}{2}\right)\left(\left(\frac{3}{2}\right)^2 + 1\right) = \frac{13}{8}$

$A_4 = \left(\frac{1}{2}\right)\left(2^2 + 1\right) = \frac{5}{2}$

TOTAL AREA = $\frac{23}{4}$ OR 5.75

THIS WOULD BE AN OVERESTIMATION.

2. Evaluate $\int x^2 + 2x + 5\, dx$ (3 marks)

$= \frac{x^3}{3} + x^2 + 5x + C$

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3. Evaluate $\int \frac{(1+x)^2}{x^{\frac{1}{2}}}\, dx$. (3 marks)

$$= \int \frac{x^2+2x+1}{x^{\frac{1}{2}}}\, dx$$

$$= \int x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}\, dx$$

$$= \frac{2x^{\frac{5}{2}}}{5} + \frac{4x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}} + C$$

4. Evaluate $\int 3x\sqrt{1-2x^2}\ dx$ (3 marks)

LET $u = 1-2x^2$

$$\frac{du}{dx} = -4x$$

$$du = -4x\,dx$$

$$\frac{du}{-4x} = dx$$

$$\int 3x\sqrt{u}\left(\frac{1}{-4x}\right)du$$

$$= \int -\frac{3}{4}u^{\frac{1}{2}}\,du$$

$$= -\frac{3}{4}\frac{u^{3/2}}{\frac{3}{2}} + C$$

$$= -\frac{3}{4}\left(\frac{2}{3}\right)u^{3/2} + C$$

$$= -\frac{1}{2}u^{\frac{3}{2}} + C$$

$$= -\frac{1}{2}\left(1-2x^2\right)^{\frac{3}{2}} + C$$

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5. Evaluate $\int_0^{\frac{\pi}{4}} x + 2sec^2x \ dx$ (4 marks)

$$= \frac{x^2}{2} + 2\tan x \Big]_0^{\pi/4}$$

$$= \left[\frac{\left(\frac{\pi}{4}\right)^2}{2} + 2\tan\frac{\pi}{4}\right] - \left[\frac{0^2}{2} + 2\tan 0\right]$$

$$= \frac{\pi^2}{32} + 2(1) - 0 \quad = \frac{\pi^2}{32} + 2 \quad \text{OR} \quad \frac{\pi^2 + 64}{32}$$

$$= 2.308$$

6. Evaluate $\int_{-3}^{5} |x + 1| \, dx$ (4 marks)

$$|x+1| = \begin{cases} x+1, & x \geq -1 \\ -x-1, & x < -1 \end{cases}$$

$$\text{So } \int_{-3}^{-1} -x-1 \, dx + \int_{-1}^{5} x+1 \, dx$$

$$= -\frac{x^2}{2} - x \Big]_{-3}^{-1} + \frac{x^2}{2} + x \Big]_{-1}^{5}$$

$$= \left(-\frac{(-1)^2}{2} + 1\right) - \left(-\frac{(-3)^2}{2} + 3\right) + \left(\frac{5^2}{2} + 5\right) - \left(\frac{(-1)^2}{2} - 1\right)$$

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$$= \left(\frac{1}{2}\right) - \left(-\frac{3}{2}\right) + \left(\frac{35}{2}\right) - \left(-\frac{1}{2}\right) = \frac{40}{2} = 20$$

7. Evaluate $\frac{d}{dx}\int_2^x \sqrt{\tan t}\ dt$ (2 marks)

$= \sqrt{\text{TAN}\ x}$

8. A particle moves along the x axis with an acceleration given by $a(t) = t - 1$. If the initial velocity was 0 at time t=0, what is the displacement and total distance travelled on the interval $0 \leq t \leq 4$. (4 marks)

$a(t) = t - 1$

$v(t) = \frac{t^2}{2} - t + C$

$v(0) = 0$ MEANS $C = 0$

$s(t) = \frac{t^3}{6} - \frac{t^2}{2} + C$

DISPLACEMENT $= \left.\frac{t^3}{6} - \frac{t^2}{2}\right]_0^4$

$= \left(\frac{64}{6} - \frac{16}{2}\right) - (0)$

$= \frac{8}{3}$

TOTAL DISTANCE TRAVELLED

$v(t) = 0$ WHEN $\frac{t^2}{2} - t = 0$

$t\left(\frac{t}{2} - 1\right) = 0 \quad t = 0 \text{ \& } t = 2$

SO $|s(4) - s(2)| + |s(2) - s(0)|$

$= \left|\frac{8}{3} - \left(-\frac{2}{3}\right)\right| + \left|\left(\frac{-2}{3}\right) - (0)\right|$

$= \frac{10}{3} + \frac{2}{3}$

$= \frac{12}{3} = 4$

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9. Find the average value of the function $y = \frac{1}{x}$ on the interval [1, 4].   (3 marks)

AVG VALUE $= \frac{1}{b-a} \int_{1}^{4} \frac{1}{x}\, dx$

$= \frac{1}{3} \left(\ln x \right) \Big]_{1}^{4}$

$= \frac{1}{3} \left(\ln 4 - \ln 1 \right)$

$= \frac{1}{3} \left(\ln 4 \right)$

$= \frac{1}{3} \ln 4 \qquad \approx 0.46$

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