Calculus 12 **LG 12/13 Test Version B** **/32**

Name:_______________________________ Date:___________

1. Find the extreme values for $f(x) = 2x^3 - 15x^2 + 36x$ on the interval [1, 5] and determine where those values occur. (3 marks)

$f(x) = 2x^3 - 15x^2 + 36x$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x-3)(x-2) = 0$

$x = 2, 3$

$f(1) = 23$

$f(2) = 28$

$f(3) = 27$

$f(5) = 55$

ABSOLUTE MIN OF 23 AT $x = 1$

ABSOLUTE MAX OF 55 AT $x = 5$

2. Find the extreme values for $f(x) = (x^2 - 2)^2$ on the interval $(-\infty, \infty)$ and determine where those values occur. (3 marks)

$f(x) = (x^2 - 2)^2$

$f'(x) = 2(x^2 - 2)(2x)$

$= (2x^2 - 4)(2x)$

$= 4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

$x = 0, \pm\sqrt{2}$

$\lim_{x \to -\infty} f(x) = \infty$

$f(-\sqrt{2}) = 0$

$f(0) = 4$

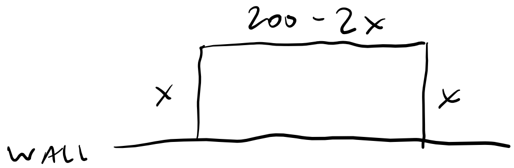
$f(\sqrt{2}) = 0$

$\lim_{x \to \infty} f(x) = \infty$

ABSOLUTE MIN OF 0 AT $x = \pm\sqrt{2}$ /6

THERE IS NO ABSOLUTE MAX.

3. A rectangular plot of land is to be bounded by a fence on 3 sides and by the wall of a building on the fourth side. Find the dimensions of the field with maximum area if you have 200m of fencing. (4 marks)



$A = l \times w$

$= x(200 - 2x)$

$A = 200x - 2x^2$

$A' = 200 - 4x = 0$

$-4x = -200$

$x = 50$

END POINTS ARE $x = 0$ & $x = 100$ BUT THEY PRODUCE AREAS OF 0.

$A(50) = 50(200 - 2(50))$

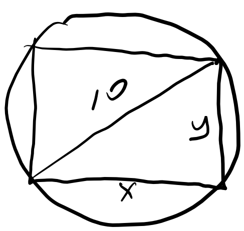
$= 50(100)$

$= 5000$

$200 - 2(50) = 100$

THE DIMENSIONS OF THE FIELD ARE 50m BY 100m.

4. Find the dimensions of a rectangle of maximum area that can be inscribed in a circle of radius 5. (4 marks)



$A = xy$

$x^2 + y^2 = 100$

$y = \sqrt{100 - x^2}$

$A = x\left(\sqrt{100 - x^2}\right)$

$A' = x\left(\frac{1}{2}(100 - x^2)^{-\frac{1}{2}}(-2x)\right) + \sqrt{100 - x^2}$

$= \frac{-x^2}{\sqrt{100 - x^2}} + \sqrt{100 - x^2}$

$= \frac{-x^2 + 100 - x^2}{\sqrt{100 - x^2}} = 0$

$-2x^2 + 100 = 0$

$x^2 = 50$

$x = \pm\sqrt{50}$

$x = \sqrt{50}$

$y = \sqrt{100 - (\sqrt{50})^2}$

$y = \sqrt{50}$

THE DIMENSIONS OF THE RECTANGLE WOULD BE $\sqrt{50}$ BY $\sqrt{50}$.

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5. Let $s = t^3 - 4t^2$. Find *s* and *v* when *a* = 0. (3 marks)

$v(t) = s'(t) = 3t^2 - 8t$

$a(t) = 6t - 8 = 0$

$6t - 8 = 0$

$t = \frac{4}{3}$

$s\left(\frac{4}{3}\right) = -\frac{128}{27}$ or -4.74

$v\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right)$

$= -\frac{16}{3}$ or -5.33

6. Let $s = t^3 - 6t^2 + 9t + 1$ be the position function of a particle. Find the maximum speed of the particle during the time interval $0 \leq t \leq 5$. (3 marks)

MAX SPEED OCCURS AT ENDPOINTS OR WHEN $v'(t) = 0$

$v(t) = 3t^2 - 12t + 9$

$a(t) = v'(t) = 6t - 12 = 0$

$t = 2$

$v(0) = 9$

$v(2) = 3(2)^2 - 12(2) + 9 = -3$

$v(5) = 3(5)^2 - 12(5) + 9 = 24$

MAXIMUM SPEED OF THE PARTICLE IS 24.

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7. The position of a particle is given by $s = t^3 - 9t^2 + 24t$ for $t \geq 0$. Describe the motion of the particle (moving right, left, stopped, speeding up, slowing down) and make a sketch. (3 marks)

$s(t) = t^3 - 9t^2 + 24t$

$v(t) = 3t^2 - 18t + 24 = 0$

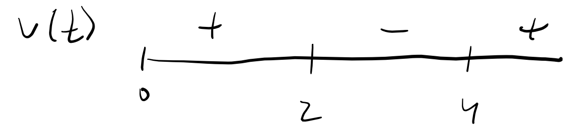
$3(t^2 - 6t + 8) = 0$

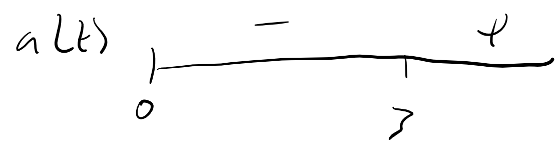
$3(t-4)(t-2) = 0$

$t = 2, 4$

$a(t) = 6t - 18 = 0$

$t = 3$





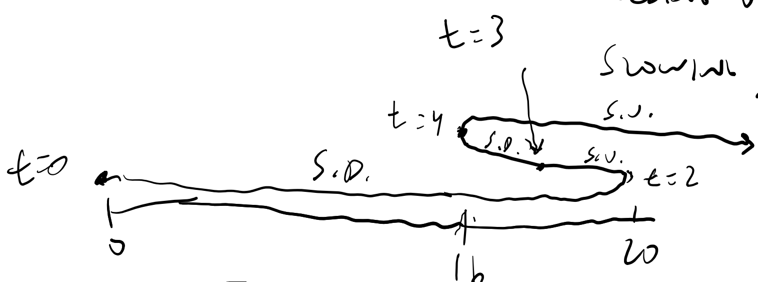
STOPPED AT $t = 2, 4$

MOVING RIGHT $(0,2)$ & $(4, \infty)$

MOVING LEFT $(2,4)$

SPEEDING UP $(2,3)$ & $(4, \infty)$

SLOWING DOWN $(0,2)$ & $(3,4)$



8. Approximate $\sqrt{5}$ to 4 decimal places by applying Newton's Method to the equation: $x^2 - 5 = 0$. (3 marks)

$f(x) = x^2 - 5 \quad f'(x) = 2x$

$x_1 = 2$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.25$

$\sqrt{5} \approx 2.2361$

$x_3 = 2.2361$

$x_4 = 2.2361$

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9. The equation $x^3 + x - 1 = 0$ has one real solution. Approximate it to 4 decimal places using Newton's Method. (3 marks)

$f(x) = x^3 + x - 1 \qquad f'(x) = 3x^2 + 1$

$x_1 = 1$ IS AN APPROXIMATE SOLUTION

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.75$

$x_3 = 0.6860$

$x_4 = 0.6823$

$x_5 = 0.6823$

THE SOLUTION IS APPROXIMATELY 0.6823

10. Verify that the Mean Value Theorm is satisfied for the function $f(x) = \sqrt{x-1}$ on [2, 5] and find all values c in that interval that satisfy the conclusion of the theorem. (3 marks)

F(x) IS DIFFERENTIABLE AND CONTINUOUS ON [2, 5] SO THE MEAN VALUE THEOREM APPLIES.

$M_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{5 - 2} = \frac{1}{3}$

$f'(x) = \frac{1}{2}(x-1)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x-1}} = \frac{1}{3}$

$2\sqrt{x-1} = 3$

$\sqrt{x-1} = \frac{3}{2}$

$x - 1 = \frac{9}{4}$

$x = 1 + \frac{9}{4}$

$x = \frac{13}{4}$

C WOULD BE $\frac{13}{4}$.

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