Calculus 12 **LG 12/13 Test Version A** **/32**

Name:________________________________ Date:___________

1. Find the extreme values for $f(x) = 2x^3 - 3x^2 - 12x$ on the interval [-2,3] and determine where those values occur. (3 marks)

$f'(x) = 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$

$x = -1, 2$

$f(-2) = -4$

$f(-1) = 7$

$f(2) = -20$

$f(3) = -9$

ABSOLUTE MIN OF -20 WHEN $x = 2$

ABSOLUTE MAX OF 7 WHEN $x = -1$

2. Find the extreme values for $f(x) = \begin{cases} 1 - x^2, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$ on the interval [-2,1] and determine where those values occur. (3 marks)

$f'(x) = \begin{cases} -2x, & x < 0 \\ 2x, & x \geq 0 \end{cases}$

SO CRITICAL POINT AT $x = 0$

$f(-2) = 1 - (-2)^2 = -3$

$f(0) = 0^2 + 1 = 1$

$f(1) = 1^2 + 1 = 2$

ABSOLUTE MIN OF -3 AT $x = -2$

ABSOLUTE MAX OF 2 AT $x = 1$

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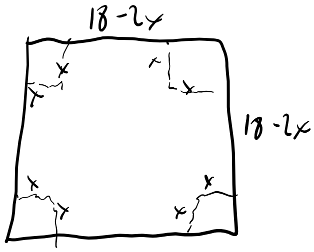
3. A sheet of cardboard 18 inch square (18 inches by 18 inches) is used to make an open box by cutting squares of equal size from corners and folding up the sides. What size squares should be cut to obtain a box with largest possible volume? (4 marks)



$V = L \times W \times H$

$= (18-2x)(18-2x)(x)$

$= (324 - 36x - 36x + 4x^2)(x)$

$= 4x^3 - 72x^2 + 324x$

$V' = 12x^2 - 144x + 324 = 0$

$x^2 - 12x + 27 = 0$

$(x-9)(x-3) = 0$

$x = 9$ or $x = 3$

$V(9) = 0$

$V(3) = 432$

3 INCH SQUARES SHOULD BE CUT.

4. Find values for x and y such that their product is a minimum if $y = 2x - 10$. (4 marks)

MINIMIZE xy LET P = PRODUCT

$P = x(2x-10)$

$P = 2x^2 - 10x$

$P' = 4x - 10 = 0$

$4x = 10$

$x = \frac{5}{2}$

X SHOULD BE $\frac{5}{2}$

AND y SHOULD BE -5.

$y = 2\left(\frac{5}{2}\right) - 10$

$= 5 - 10$

$= -5$

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5. Let $s = 2t^3 - 12t^2 + 4t + 9$. Find s and v when $a = 0$. (3 marks)

$v = s'(t) = 6t^2 - 24t + 4$

$a = v'(t) = 12t - 24 = 0$

$12t = 24$

$t = 2$

$v(2) = 6(2)^2 - 24(2) + 4$

$= -20$

$s(2) = 2(2)^3 - 12(2)^2 + 4(2) + 9$

$= -15$

6. Let $s = t^2 - 5t - 6$ be the position function of a particle. Find the maximum speed of the particle during the time interval $0 \le t \le 6$. (3 marks)

MAXIMUM SPEED OCCURS AT END POINTS AND WHEN $v'(t) = 0$

$v(t) = s'(t) = 2t - 5$

$a(t) = v'(t) = 2$

$a(t) \neq 0$

So $v(0) = -5$

$v(6) = 7$

MAXIMUM SPEED IS 7 WHEN $x = 6$

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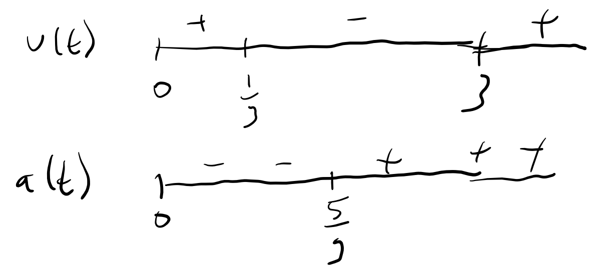
7. The position of a particle is given by $s = t^3 - 5t^2 + 3t$ for $t \geq 0$. Describe the motion of the particle (moving right, left, stopped, speeding up, slowing down) and make a sketch. (3 marks)

$v(t) = s'(t) = 3t^2 - 10t + 3 = 0$

$(3t - 1)(t - 3) = 0$

$t = \frac{1}{3} \quad t = 3$



$a(t) = v'(t) = 6t - 10 = 0$

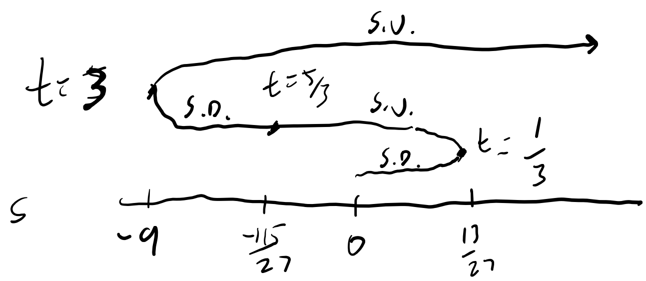
$6t = 10$

$t = \frac{5}{3}$

STOPPED AT $x = \frac{1}{3}$ AND 3

SLOWING DOWN ON $[0, \frac{1}{3})$ & $(\frac{5}{3}, 3)$

SPEEDING UP ON $(\frac{1}{3}, \frac{5}{3})$ & $(3, \infty)$



8. Approximate $-\sqrt[3]{72}$ to 4 decimal places by applying Newton's Method to the equation $x^3 + 72 = 0$.

(3 marks)

LET $f(x) = x^3 + 72 \quad f'(x) = 3x^2$

$x_1 = -4$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= -4 - \dfrac{(-4)^3 + 72}{3(x_1)^2}$

$= -4.17$

$x_3 = -4.1602$

$x_4 = -4.1602$

THE APPROXIMATE SOLUTION IS -4.1602

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9. The equation $x^3 + x^2 - 3x - 3 = 0$ has one real solution for $-2 < x < -1$. Approximate it to 4 decimal places using Newton's Method. (3 marks)

$f(x) = x^3 + x^2 - 3x - 3$ $\quad f'(x) = 6x^2 + 2x - 3$

$x_1 = -1.5$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= -1.5 - \frac{(-1.5)^3 + (-1.5)^2 - 3(-1.5) - 3}{6(-1.5)^2 + 2(-1.5) - 3}$

$= -2$

$x_3 = -1.8$

$x_4 = -1.7385$

$x_5 = -1.7321$

$x_6 = -1.7321$

THE APPROXIMATE SOLUTION IS -1.7321

10. The velocity of a particle is given by the differentiable function $v(t)$ such that $v(0) = 2$ and $v(4) = 22$. Is there a time $t, 0 \le t \le 4$, such that $v'(t) = 5$? Justify your answer. (3 marks)

$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{22 - 2}{4 - 0} = \frac{20}{4} = 5$

SINCE THE FUNCTION $v(t)$ IS DIFFERENTIABLE AND CONTINUOUS, ACCORDING TO THE MEAN VALUE THEOREM, THERE MUST EXIST AT LEAST ONE VALUE ON $[0,4]$ SUCH THAT $v'(t) = 5$.

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