

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1.  $f(x) = 2x^4 - 5x^3$

(4 marks)

- a) Find the intervals where  $f$  is increasing and where  $f$  is decreasing.  
 b) Find the intervals where  $f$  is concave up and where  $f$  is concave down.  
 c) List any inflection points.

$$f'(x) = 8x^3 - 15x^2$$

$$= x^2(8x - 15) = 0$$

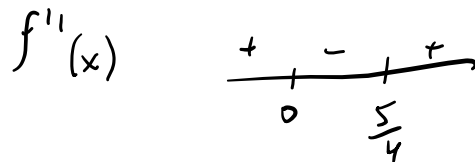
$$x = 0, \frac{15}{8}$$

INCREASING ON  $[\frac{15}{8}, \infty)$ DECREASING ON  $(-\infty, \frac{15}{8}]$ 

$$f''(x) = 24x^2 - 30x$$

$$= 6x(4x - 5) = 0$$

$$x = 0, \frac{5}{4}$$



CONCAVE UP

 $(-\infty, 0) \cup (\frac{5}{4}, \infty)$ CONCAVE DOWN ON  $(0, \frac{5}{4})$ INFLECTION POINTS  $(0, 0)$  &  $(\frac{5}{4}, -4.88)$

2.  $f(x) = xe^{x^2}$

(4 marks)

- a) Find the intervals where  $f$  is increasing and where  $f$  is decreasing.
- b) Find the intervals where  $f$  is concave up and where  $f$  is concave down.
- c) List any inflection points.

$$f'(x) = x e^{x^2} (2x) + e^{x^2} = 0$$

$$e^{x^2} (2x^2 + 1) = 0$$

NO X VALUES

$f'(x)$  +

$f(x)$  IS INCREASING ON  $(-\infty, \infty)$

NEVER DECREASING

$$f''(x) = e^{x^2} (4x) + (2x^2 + 1) e^{x^2} (2x)$$

$$= e^{x^2} 2x (2 + (2x^2 + 1)) = 0$$

$$2x e^{x^2} (2x^2 + 3) = 0$$

$$x = 0$$

$f''(x)$  - | +  
0

CONCAVE UP ON  $(0, \infty)$

CONCAVE DOWN ON  $(-\infty, 0)$

INFLECTION POINT  $(0, 0)$

3. Are the following true or false?

(1/2 mark each)

a) If  $f'(x) > 0$  on the open interval  $(a, b)$  then  $f(x)$  is increasing on  $(a, b)$ .

TRUE



POSITIVE SLOPE MEANS  $f(x)$  IS INCREASING

b) If  $f''(x) < 0$ , then  $f(x) < 0$ .

FALSE



THIS IS CONCAVE DOWN BUT SOME  $f(x)$  VALUES ARE POSITIVE.

c) If  $f''(x) = 0$ , then  $x$  is a point of inflection.

FALSE

A POINT OF INFLECTION MUST ALSO CHANGE CONCAVITY.

d) If  $f''(x) > 0$ , then  $f'(x)$  is increasing.

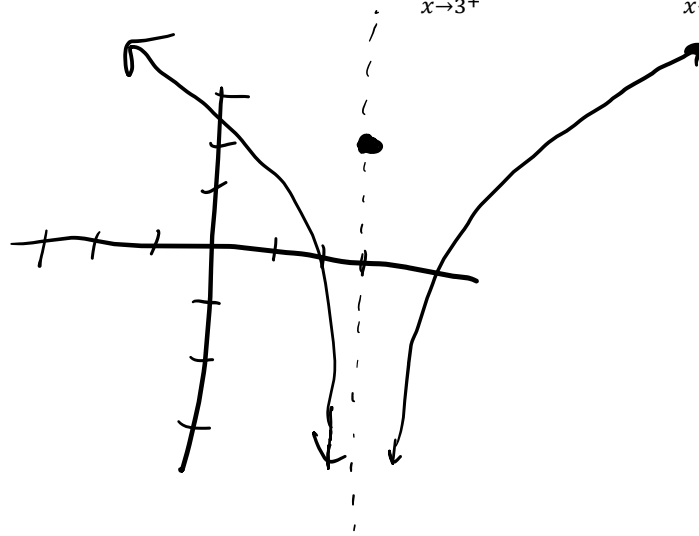
TRUE



SLOPE IS INCREASING

4. Sketch a curve having the following properties: (3 marks)

$$f(3) = 2, \quad f''(x) < 0 \text{ for } x \neq 3 \text{ and } \lim_{x \rightarrow 3^+} f(x) = -\infty, \quad \lim_{x \rightarrow 3^-} f(x) = -\infty$$



5. Find all relative extrema for  $f(x) = 2x + x^{\frac{2}{3}}$  (4 marks)

$$f'(x) = 2 + \frac{2}{3}x^{-\frac{1}{3}} = 0$$

$$2 + \frac{2}{3\sqrt[3]{x}} = 0$$

$$x \neq 0$$

$$\frac{2}{3\sqrt[3]{x}} = -2$$

$$\frac{1}{\sqrt[3]{x}} = -3$$

$$\sqrt[3]{x} = -\frac{1}{3}$$

$$x = \left(-\frac{1}{3}\right)^3 = -\frac{1}{27}$$

$$f'(x) \quad \begin{array}{c} + \quad - \quad + \\ | \quad | \quad | \\ -\frac{1}{27} \quad 0 \end{array}$$

RELATIVE MAX OF 0.037 AT  $x = -\frac{1}{27}$

RELATIVE MIN OF 0 AT  $x = 0$