

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1.  $f(x) = 5x^4 - x^5$

(4 marks)

- a) Find the intervals where  $f$  is increasing and where  $f$  is decreasing.  
 b) Find the intervals where  $f$  is concave up and where  $f$  is concave down.  
 c) List any inflection points.

$$f'(x) = 20x^3 - 5x^4$$

$$= 5x^3(4 - x) = 0$$

$$x = 0 \quad x = 4$$

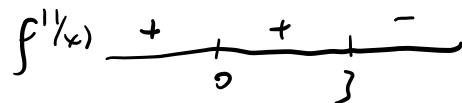


$f(x)$  is INCREASING ON  $[0, 4]$   
 AND DECREASING ON  $(-\infty, 0]$   
 AND  $[4, \infty)$

$$f''(x) = 60x^2 - 20x^3$$

$$= 20x^2(3 - x)$$

$$x = 0, 3$$



CONCAVE UP ON  $(-\infty, 3)$   
 CONCAVE DOWN ON  $(3, \infty)$   
 INFLECTION POINT  $(3, 162)$

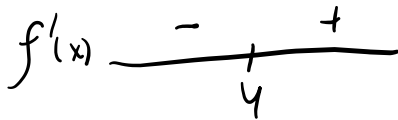
2.  $f(x) = (x - 4)^4 + 4$

(4 marks)

- a) Find the intervals where  $f$  is increasing and where  $f$  is decreasing.
- b) Find the intervals where  $f$  is concave up and where  $f$  is concave down.
- c) List any inflection points.

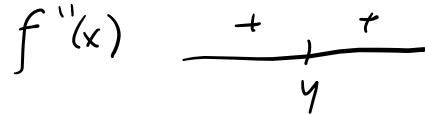
$$f'(x) = 4(x - 4)^3$$

$$x = 4$$



$f(x)$  IS INCREASING ON  $[4, \infty)$   
AND DECREASING ON  $(-\infty, 4]$

$$f''(x) = 12(x - 4)^2$$



$f(x)$  IS CONCAVE UP ON  $(-\infty, \infty)$   
AND NEVER CONCAVE DOWN.  
NO INFLECTION POINTS.

3. Are the following true or false? (1/2 mark each)

a) If  $f''(x) > 0$  on the open interval  $(a, b)$  then  $f'(x)$  is increasing on  $(a, b)$ .

TRUE



b) If  $f(x) > 0$ , then  $f'(x) > 0$ .

FALSE



c) If  $f''(x) = 0$ , then  $x$  is a point of inflection.

FALSE. QUESTION # 2 IS AN EXAMPLE

d) If  $f'(x) < 0$ , then  $f(x)$  is decreasing.

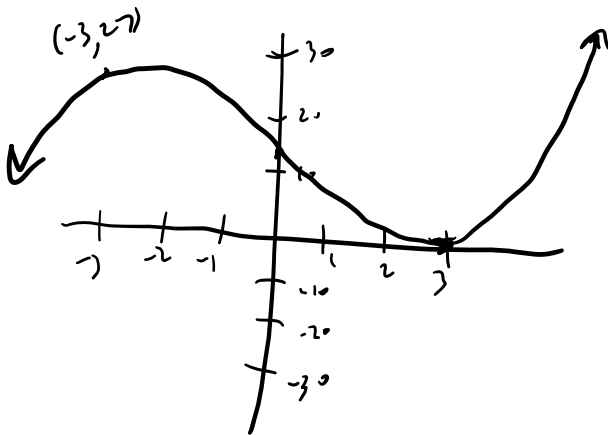
TRUE



4. Sketch a continuous curve having the following properties: (3 marks)

$$f(-3) = 27, f(0) = \frac{27}{2}, f(3) = 0, f'(x) > 0 \text{ for } |x| > 3$$

$$f'(-3) = f'(3) = 0, f''(x) < 0 \text{ for } x < 0, f''(x) > 0 \text{ for } x > 0$$



5. Find all relative extrema for  $f(x) = x + \cos(2x)$ ,  $0 < x < \pi$  (4 marks)

$$f'(x) = 1 - \sin(2x) \cdot 2 = 0$$

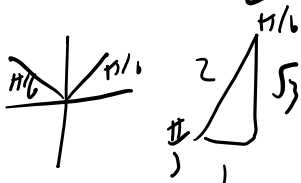
$$1 - 2\sin 2x = 0$$

$$2\sin 2x = 1$$

$$\sin 2x = \frac{1}{2}$$

$$\text{Let } \theta = 2x$$

$$\sin \theta = \frac{1}{2}$$



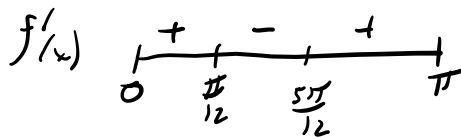
$$\theta = \frac{\pi}{6} + n2\pi$$

$$2x = \frac{\pi}{6} + n2\pi$$

$$\text{or } \theta = \frac{5\pi}{6} + n2\pi$$

$$\text{or } 2x = \frac{5\pi}{6} + n2\pi$$

$$\text{So } x = \frac{\pi}{12} + n\pi \text{ or } x = \frac{5\pi}{12} + n\pi$$



RELATIVE MAX OF 1.13  
AT  $x = \frac{\pi}{12}$

RELATIVE MIN OF 0.44  
AT  $x = \frac{5\pi}{12}$